EPR-Bohr and Quantum Trajectories

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Presented at

CNLS/LANL Workshop on Quantum Trajectories

28–30 July 2008

Abstract

Quantum trajectories are used to investigate the EPR-Bohr paradox in a modern sense by examining entanglement and nonlocality. We synthesize a single "EPR-molecule" from the two scattered particles of the EPR experiment. Herein, we explicitly investigate the behavior of the EPR-molecule rather than the behaviors of the two scattered particles to gain insight into the EPR-Bohr paradox. We develop the EPR-molecule's wave function in polar form and its reduced action, both of which manifest entanglement. We next apply Jacobi's theorem to the reduced action to generate the equation of motion for the EPR-molecule to produce its quantum trajectory. The resultant quantum trajectory manifests entanglement and has retrograde segments interspersed between segments of forward motion in the quantum trajectory. This alternating of forward and retrograde segments generates nonlocality. Analysis of the equation of motion renders an additional component manifesting entanglement while also rendering a classical behavior of the two scattered particles. The behavior of the entanglement component and its relationship to nonlocality are discussed.

ROAD MAP

1\. BACKGROUND

1\. RECITAL — Limited mostly to quantum trajectory methodology needed herein.

2\. ENTANGLEMENT — In one-dimension, epr-molecule is formed by two particles interacting to become entangled and subsequently recoiling away from each other in opposite directions.

- Synthesize epr-molecule's wave function & reduced action.
- Generate trajectory for the epr-molecule.

3\. APPLICATION — Consider a specific example. Analyze trajectory into its components.

4\. FINDINGS & DISCUSSION — Insight into nonlocality. Impact upon foundations of QM.

BACKGROUND

In 1935 both EPR & Bohr entitled their positions, "Can Quantum Mechanical Description of Physical Reality Be Considered Complete?" [PR **47**, 777 (1935); **48**, 696 (1935)]

- Circa 1935, issues were **reality** and **completeness of** $\boldsymbol{\psi}$.
- Modern issues are **entanglement** and **nonlocality**.
  - Entanglement: EPR explicitly accepted separability of the particles while Bohr straddled accepting separability in the debate
  - Nonlocality: Einstein abhorred; Bohr advanced "ψ-collapse".

MOTIVATION — Resolve the EPR-Bohr debate with deterministic, quantum trajectories.

- What is the impact of entanglement and nonlocality on foundations of QM?

ALERT — Herein, we capitalize on nonlocality.

- Investigate motion of the entangled, synthesized "epr-molecule"
  - Entanglement maintains epr-molecule until some subsequent interaction.
- This investigation is extra to EPR & Bohr.

RECITAL

Underlying the trajectory theory in one dimension is the Quantum Stationary Hamilton-Jacobi Equation (QSHJE)

$$\underbrace{\frac{1}{2m}\left(\frac{\partial W}{\partial x}\right)^2 + V - E}_{\text{classical HJE}} = \underbrace{-\frac{\hbar^2}{4m}\overbrace{\langle W;x\rangle}^{\text{Schwarzian derivative}}}_{\text{quantum effects}} = \underbrace{-\frac{\hbar^2}{4m}\left[\frac{\partial^3 W/\partial x^3}{\partial W/\partial x} - \frac{3}{2}\left(\frac{\partial^2 W/\partial x^2}{\partial W/\partial x}\right)^2\right]}_{\text{negative of Bohm's quantum potential, Q}}$$

Historically derived from Schrödinger equation, but Faraggi & Matone derived QSHJE by quantum equivalence principle for greater generality.

M&F's QEP without any use of axiomatic interpretations of the wave function makes quantum trajectories the more fundamental representation of QM.

Conjugate momentum, $\partial W/\partial x$, is generally not mechanical momentum, $\partial W/\partial x \neq m\dot{x}$ in contrast to Bohmian mechanics.

The equation of motion (Jacobi's theorem): $t - \tau = \partial W/\partial E$ in contrast to Bohmian mechanics.

QSHJE is third-order differential equation $\Rightarrow$ Heisenberg uncertainty principle implies Copenhagen assumes an insufficient subset of initial conditions for QM [IJMPA **15**, 1363 (2000), quant-ph/9907092; Faraggi & Matone, IJMPA **15**, 1869 (2000), hep-th/9809127].

RECITAL (continued)

Algorithm for Entangled Ensembles of complex wave functions

Entangled Wave Function, $\psi_{\mathcal{E}}$, synthesized from the ensemble of wave functions, $\psi_j(x_j)$, $j = 1, 2, \cdots, N$, by {inspired by Bohm [PR **85**, 166 (1953)]}

$$\psi_{\mathcal{E}} = (\mathcal{X}^2 + \mathcal{Y}^2)^{1/2} \exp[i \arctan(\mathcal{Y}/\mathcal{X})]$$

with

$$\mathcal{X} = \Re\left[\sum_{j=1}^{N} \psi_j\right] = \sum_{j=1}^{N} \Re[\psi_j] \text{ and } \mathcal{Y} = \Im\left[\sum_{j=1}^{N} \psi_j\right] = \sum_{j=1}^{N} \Im[\psi_j].$$

F&M's QEP makes $\psi_{\mathcal{E}}$ superfluous herein. $\psi_{\mathcal{E}}$ still presented (1) as a recipe for reversible mapping a multi-polar ansatz into a polar asnsatz, (2) for completeness, and (3) for Copenhagen insight.

Reduced Action, $W_{\mathcal{E}}$, for ensemble

$$W_{\mathcal{E}} = \hbar \arctan(\mathcal{Y}/\mathcal{X}) \underbrace{\neq \sum_{j=1}^{N} W_j(x_j)}_{\text{iff } =, \text{ then unentangled}}$$

ENTANGLEMENT

WAVE FUNCTION

INITIAL CONDITIONS IN LABORATORY SYSTEM: For EPR, two particles, $\psi_1(x_1)$ & $\psi_2(x_2)$ interact instantaneously at $x_1, x_2 = 0;\ t = 0$ and then recoil from each other such that

$$\psi_1(x_1) = \exp(ikx_1), \qquad \psi_2(x_2) = \alpha \exp(-ikx_2 + i\beta); \qquad t > 0$$

where $1 \geq \alpha > 0$, $-\pi < \beta < \pi$.

The factor α in ψ_2 is inserted arbitrarily as a convenient tool by which we approach EPR in the limit $\alpha \to 1$.

- ψ_1 & ψ_2 are not identical particles consistent with EPR unless $\alpha = 1$.

EPR & Bohr assumed that $x_1 + x_2 \gg 1$ sufficiently so to ensure **separability**.

But we herein assume that the two particles remain **entangled no matter how far apart** as confirmed by Aspect experiments.

Conservation of relative position: $x_1 = -x_2$. [A. Fine, <http://plato.stanford.edu/archives/sum2004/entries/qt-epr/>.]

- May drop subscript i in x_i.

WAVE FUNCTION (continued)

Under entanglement we may synthesize an epr-molecule as a simple polar wave function, ψ_{epr}, from the entangled pair (bipolar wave function) , ψ_1 & ψ_2, by [Found. Phys. **37**, 1386 (2007), quant-ph/0605120]

$$\begin{aligned}
\psi_{epr}(x) &= \overbrace{\exp(ikx) + \alpha\exp(-ikx - i\beta)}^{\text{bipolar wave function}} \\
&= \underbrace{[1 + \alpha^2 + 2\alpha\cos(2kx + \beta)]^{1/2}\exp\left[i\arctan\left(\frac{\sin(kx) - \alpha\sin(kx + \beta)}{\cos(kx) + \alpha\cos(kx + \beta)}\right)\right]}_{\text{polar wave function is still an eigenfuncion for } E = \hbar^2k^2/(2m).}
\end{aligned}$$

Just superpositional principle at work.

From the above equation, ψ_{epr} has the same form dichromatic wave function $\psi_{dichromatic}$ [Found. Phys. **37**, 1386 (2007), quant-ph/0605120].

- But ψ_{epr} & $\psi_{dichromatic}$ represent different physics.

WAVE FUNCTION (continued)

ψ_{epr} is inherently **NONLOCAL** as

$$\psi_{epr} \neq K \underbrace{\exp(ikx)}_{\text{particle 1}} \underbrace{\alpha \exp(-ikx - i\beta)}_{\text{particle 2}}, \quad K \text{ is a constant}$$

The ψ_{epr} is not the wave function representing EPR landscape.

The actual wave function for the EPR-molecule, ψ_{EPR}, for identical particles is given by $\psi_{EPR} = \lim_{\alpha \to 1}(\psi_{epr})$.

DIGRESSION: In general, we shall investigate EPR phenomena, where $\alpha = 1$, by

$$\lim_{\alpha \to 1} \Big(\text{epr-phenomenon}\Big) \to \text{EPR-phenomenon.}$$

- This avoids directly working with standing waves.

REDUCED ACTION FOR epr-MOLECULE

Reduced action (Hamilton's characteristic function)[Found. Phys. **37**, 1386 (2007), quant-ph/0605120]

$$W_{epr} = \hbar \arctan\left(\frac{\sin(kx) - \alpha \sin(kx+\beta)}{\cos(kx) + \alpha \cos(kx+\beta)}\right)$$

- Absolute value of W_{epr} increases monotonically with x.

Conjugate momentum for epr-molecule:

$$\partial W_{epr}/\partial x = \frac{\hbar k}{[1+\alpha^2+2\alpha\cos(2kx+\beta)]}.$$

- $\partial W_{epr}/\partial x \neq m\dot{x}$ in contrast to Bohmian mechanics

EQUATION OF MOTION FOR epr-MOLECULE.

Equation of motion (Jacobi's Theorem): $t_{epr} - \tau = \dfrac{\partial W_{epr}}{\partial E} = \dfrac{mx(1-\alpha^2)}{\hbar k[1+\alpha^2+2\alpha\cos(2kx+\beta)]}$

MOTION, $x(t)$, FOR THE epr-MOLECULE

PARTICULAR CASE:

$\hbar = 1,\ m = 1,\ k = \pi/2,$

$\alpha = 0.5$, & $\tau = 0$.

$\beta = 0$, solid line.

$\beta = \pi$, dashed line

Retrograde motion manifests nonlocality

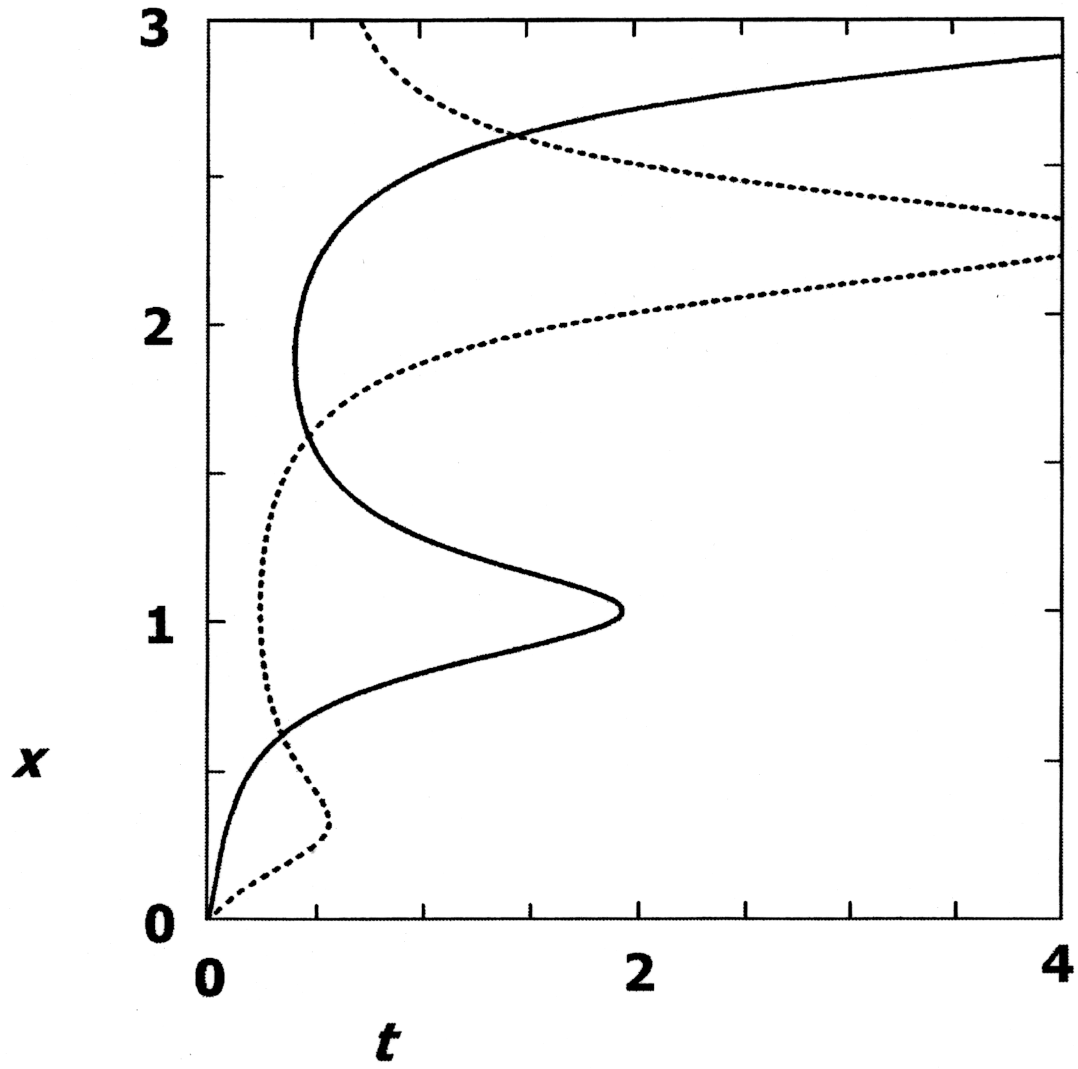
$\dot{x} \to \pm\infty$ at extrema in t.

The quantum trajectory is restricted approximately to a wedge given by

$$\frac{mx}{3\hbar k} \le t \le \frac{3mx}{\hbar k}.$$

Generalized to

$$\frac{(1-\alpha)mx}{(1+\alpha)\hbar k} \le t \le \frac{(1+\alpha)mx}{(1-\alpha)\hbar k}.$$



QUANTUM TRAJECTORIES FOR EPR-MOLECULE

ESTABLISH THE TRAJECTORY FOR A STANDING WAVE BY A LIMITING PROCESS.

The 21st Century equivalent of squaring the circle?

Still assuming that $\hbar = 1,\ m = 1,\ k = \pi/2,\ \beta = 0$

For $\beta = 0$, W_{epr} simplifies to $W_{epr} = \hbar\left[\arctan\left(\frac{1-\alpha}{1+\alpha}\tan(kx)\right)\right]$

Let α approaches 1 from below.

The $\lim_{\alpha\to(1-)}(\psi_{epr}) = 2\cos(kx) = \psi_{EPR}$.

- Both edges of the wedge become orthogonal
    - Wedge spans the entire quadrant $t, x \geq 0$ of t, x-plane.

The equation of motion, $t_{EPR} = \partial W_{EPR}/\partial E$ for a launch point (initial position) of $x = 0$ at $t = 0$ for a EPR-molecule in the limit $\alpha \to 1$ from below would render [Found. Phys. **37** 1386 (2007), quant-ph/0605120]

$$\lim_{\alpha\to(1-)} t_{epr} = t_{EPR} = \sum_{n=1}^{\infty}\delta[x-(2n-1)\pi/(2k)] = \sum_{n=1}^{\infty}\{\delta[x-(2n-1)],\ \ x>0,\ \tau_{EPR}=0.$$

QUANTUM TRAJECTORIES (continued)

For $x < 0$, let $\alpha > 1$ and use a limiting process $\alpha \to 1$ from above rendering [Found. Phys. **37** 1386 (2007), quant-ph/0605120]

$$\lim_{\alpha \to (1+)} t_{-epr} + \tau_{-EPR} = -\sum_{n=1}^{\infty} \delta[x - (2n-1)\pi/(2k)] = -\sum_{n=1}^{\infty} \{\delta[x - (2n-1)], \quad x < 0, \tau_{-EPR} = 0.$$

For launch point at $x = 0$, EPR-molecule has positive infinite velocity for $x > 0$ and negative infinite velocity for $x < 0$

- except at the nulls, $x = \pm 1, \pm 2, \pm 3, \cdots$, of $\psi_{EPR} = 2\cos(kx)$, where it has nil velocity.

Faraggi & Matone's effective quantum mass, $m_{Q_{EPR}} = m(1 - \partial Q_{EPR}/\partial E)$ here becomes [Found. Phys. **37**, 1386 (2007)]

$$\begin{aligned} \lim_{\alpha \to 1} m_{Q_{EPR}} &= 0, \quad x \neq \pm 1, \pm 3, \pm 5, \cdots \\ &= \infty, \quad x = \pm 1, \pm 3, \pm 5, \cdots . \end{aligned}$$

Note that $m_{Q_{EPR}}$ here becomes infinite where the velocity of the EPR-molecule is nil and becomes nil where the velocity is infinite.

- Consistent with conjugate momentum remaining finite.

ANALYZES OF EQUATION OF MOTION

Equation of motion for epr-molecule may be re-expressed as

$$\begin{aligned} t_{epr} - \tau &= \frac{mx(1-\alpha^2)}{\hbar k[1+\alpha^2+2\alpha\cos(2kx+\beta)]} \\ &= \underbrace{\frac{mx}{\hbar k}\frac{1}{1+\alpha^2}}_{\text{particle 1}} - \underbrace{\frac{mx}{\hbar k}\frac{2\alpha\frac{1-\alpha^2}{1+\alpha^2}\cos(2kx+\beta)}{1+\alpha^2+2\alpha\cos(2kx+\beta)}}_{\text{entanglement term}} - \underbrace{\frac{mx}{\hbar k}\frac{\alpha^2}{1+\alpha^2}}_{\text{particle 2}} \end{aligned}$$

Both lines above in the limit $\alpha \to 1$ exhibit δ-function behavior at $x = N\pi/(2k)$, $N = 1, 3, 5, \cdots$ for $\beta = 0$.

Entanglement term induces retrograde motion, which manifests **nonlocality**. **QED**

FURTHER DISCUSSION OF ENTANGLEMENT TERM

Entanglement term appears to have the characteristic that would put it in a class with phonons, photons, gravitons, electrons, etc.

- It provides the means to hold the epr-molecule coherent.
- Its retrograde and forward segments are reminiscent of John G. Cramer's Transactional Interpretation of QM.
  - Advanced & retarded waves of the Transactional Interpretation.

Common characteristics of entanglement term with gluons:

- Neither exists in isolation.
- As range increases, entanglement term spontaneously develops multipaths.

COPENHAGEN RESPONSE

Yes, ψ_{epr} does manifest entanglement.

Operating on or measuring of epr-molecule is concurrent on both particles.

Any single measurement of ψ_{epr} disturbs ψ_{epr}.

- And also concurrently disturbs the two particles individually.
    - Are concurrent individual particle disturbances correlated ???
        - Yes! (Extra to Copenhagen.)

Any subsequent non-commuting measurement on epr-molecule or individual particles will have uncertainty.

A hierarchy of entanglement?

- Yes. $\Longrightarrow$ David Bohm & Basil Hiley's *Wholeness and Implicate Order*. (Extra to Copenhagen.)

REBUTTAL FROM QUANTUM TRAJECTORY

NOTHING NEW HERE — JUST ANOTHER RECITAL

ψ is an incomplete description of phenomena.

- QSHJE is a third-order differential equation

Heisenberg uncertainty principle is founded on an **insufficient** subset {position,momentum} of the set of necessary and sufficient initial values {position,velocity,acceleration,jerk} to establish solution. [PR **D 29**, 1842 (1984); Faraggi & Matone, IJMPA **15** 1869 (2000), hep-th/9909127]

For Copenhagen, "quantum mechanics in Hilbert space is imprecise by construction." [Carroll, JCP **77**, 319 (1977), quant-ph/9903081].

Philosophically, Copenhagen is in a position-momentum domain while the underlying quantum Hamilton-Jacobi formulation of the quantum trajectory representation is in the configuration space-time domain.

So looking back, who was right in 1935 on the EPR paradox?

de Broglie