Load-side
Frequency Control

Changhong Zhao*
Enrique Mallada*
Steven Low
EE, CMS, Caltech

Ufuk Topcu
Elec & Sys Engr
U Penn

Lina Li
LIDS/MIT
Harvard

Jan 2015

Outline

Motivation

Network model

Load-side frequency control

Simulations

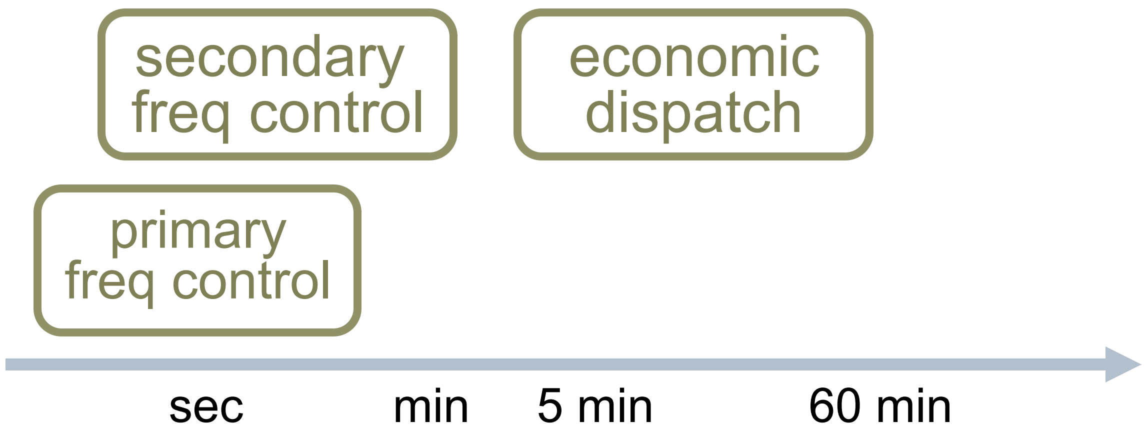
Main references:

Zhao, Topcu, Li, Low, TAC 2014
Mallada, Zhao, Low, Allerton 2014
Zhao, Low, CDC 2014

Why frequency regulation

Control signal to balance supply & demand

Andersson's talk in am



Why frequency regulation

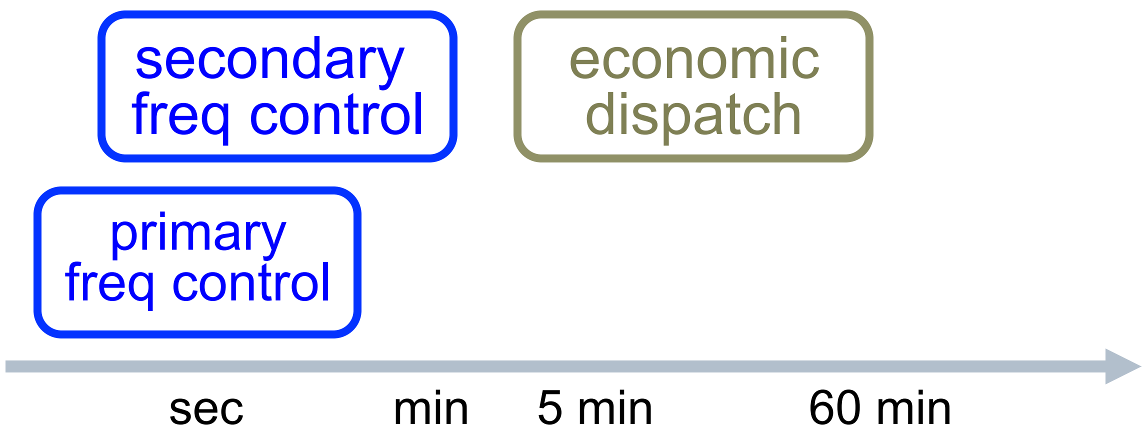
Traditionally done on generator-side

- Frequency control: Lu and Sun (1989), Qu et al (1992), Jiang et al (1997), Wang et al (1998), Guo et al (2000), Siljak et al (2002)
- Stability analysis: Bergen and Hill (1981), Hill and Bergen (1982), Arapostathis et al (1982), Tsolas et al (1985), Tan et al (1995), …
- Recent analysis: Andreasson et al (2013), Zhang and Papachristodoulou (2013), Li et al (2014), Burger et al (2014), You and Chen (2014), Simpson-Porco et al (2013), Dorfler et al (2014), Zhao et al (2014)

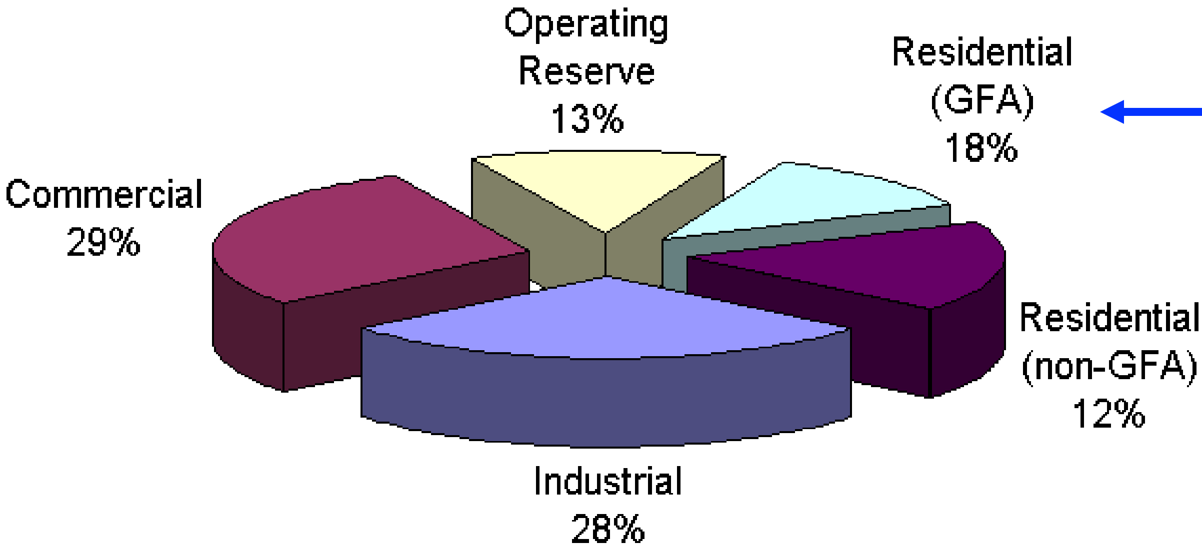
Why load-side participation

Ubiquitous continuous load-side control can supplement generator-side control

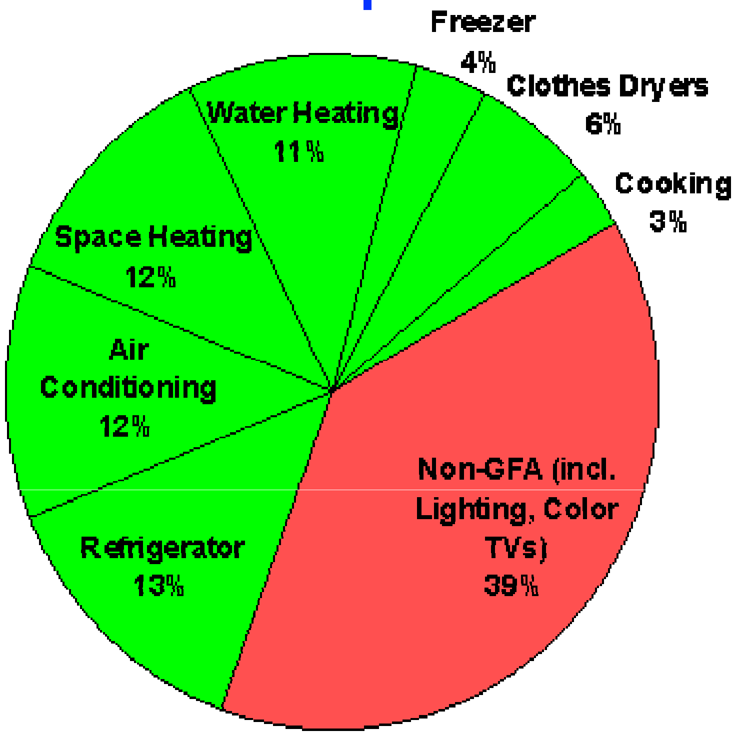
- faster (no/low inertia)
- no extra waste or emission
- more reliable (large #)
- better localize disturbances
- reducing generator-side control capacity



What is the potential

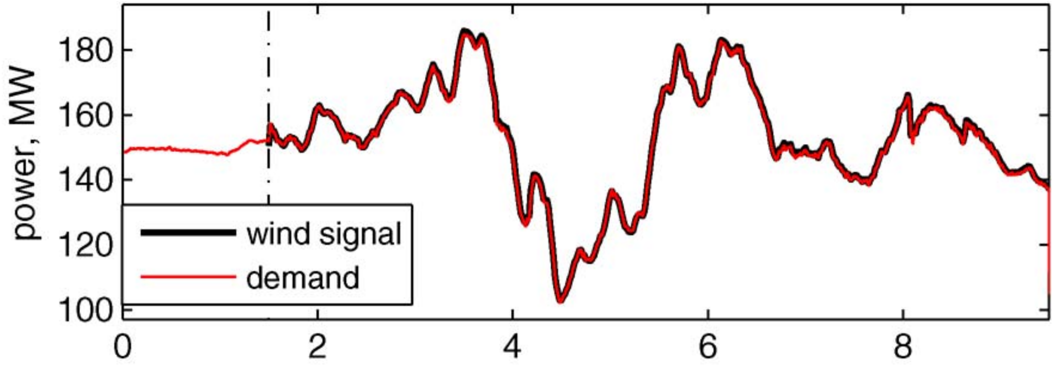


- Residential load accounts for ~1/3 of peak demand
- 61% residential appliances are Grid Friendly



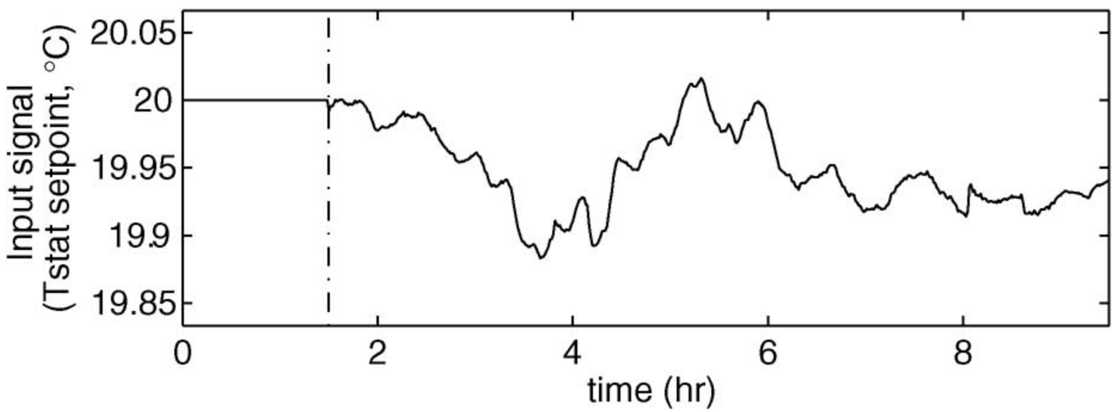
US:
operating reserve: 13% of peak
total GFA capacity: 18%

Lu & Hammerstrom (2006), PNNL



Can household Grid Friendly appliances follow its own PV production?

- 60,000 AC
- avg demand ~ 140 MW
- wind var: +- 40MW
- temp var: 0.15 degC



Dynamically adjust thermostat setpoint

Fig. 7. ***Load control example for balancing variability from intermittent renewable generators, where the end-use function—in this case, thermostat setpoint—is used as the input signal.***

Callaway, Hiskens (2011)
Callaway (2009)

How

How to design load-side frequency control ?

How does it interact with generator-side control ?

Literature: load-side control

Original idea

- Schweppe et al 1979, 1980

Small scale trials around the world

- D.Hammerstrom et al 2007, UK Market Transform Programme 2008

Numerical studies

- Trudnowski et al 2006, Lu and Hammerstrom 2006, Short et al 2007, Donnelly et al 2010, Brooks et al 2010, Callaway and I. A. Hiskens, 2011, Molina-Garcia et al 2011

Analytical work

- Zhao et al (2012/2014), Mallada and Low (2014), Mallada et al (2014)
- Simpson-Porco et al 2013, You and Chen 2014, Zhang and Papachristodoulou (2014), Zhao, et al (2014)

Outline

Motivation

Network model

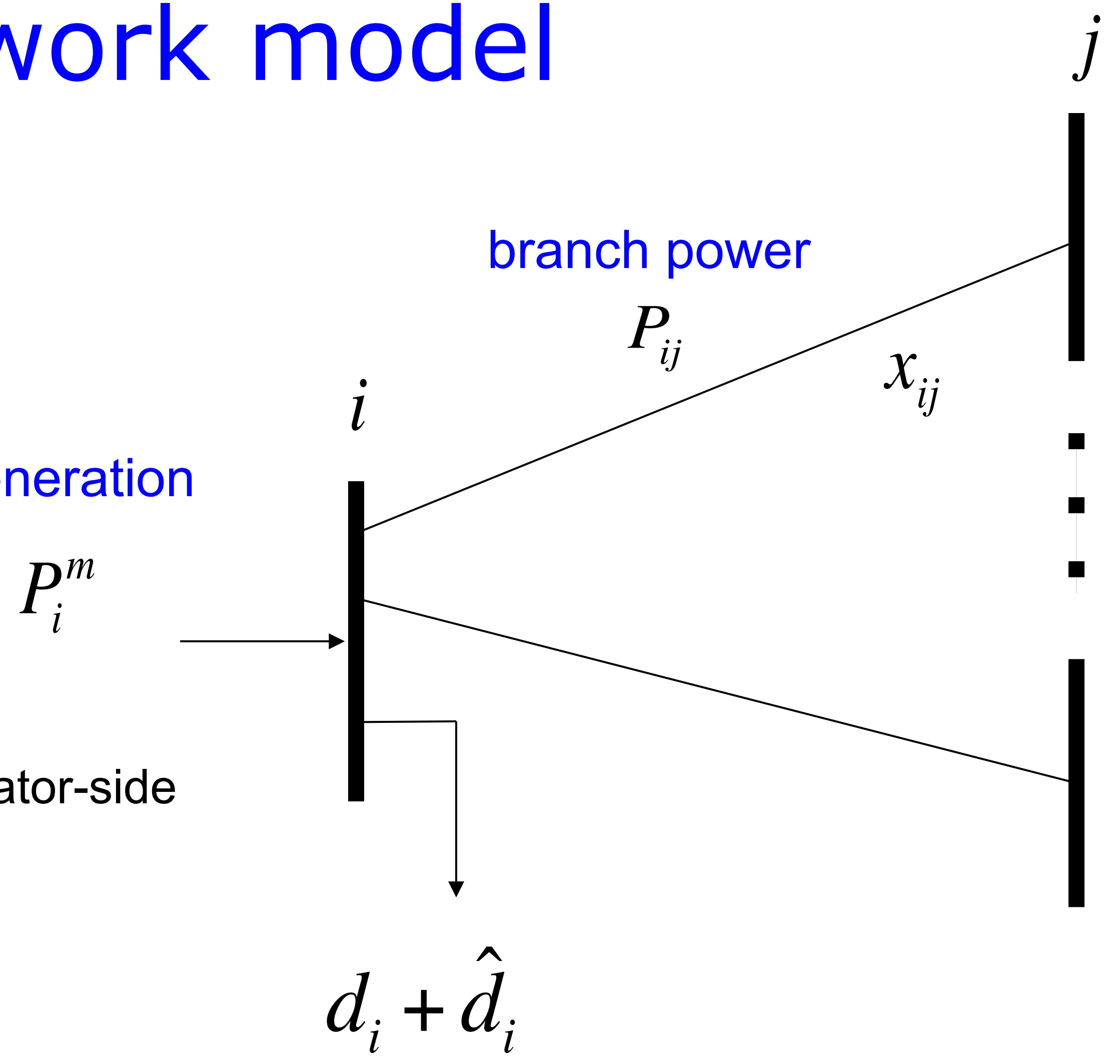
Load-side frequency control

Simulations

Main references:

Zhao, Topcu, Li, Low, TAC 2014
Mallada, Zhao, Low, Allerton 2014
Zhao, Low, CDC 2014

Network model



Will include generator-side control later

loads:
controllable + freq-sensitive

i : region/control area/balancing authority

Network model

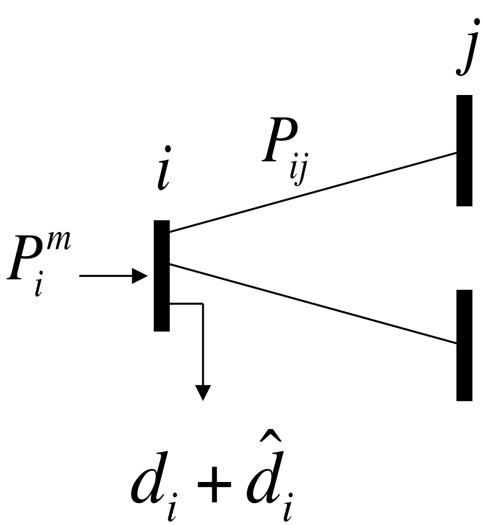
$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

Generator bus: $M_i > 0$

Load bus: $M_i = 0$

Damping/uncontr loads: $\hat{d}_i = D_i \omega_i$

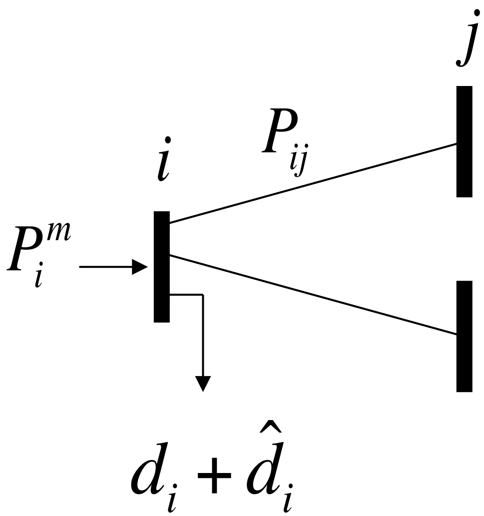
Controllable loads: d_i



Network model

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right) \qquad \forall\ i \rightarrow j$$



- swing dynamics
- all variables are deviations from nominal
- nonlinear : Mallada, Zhao, Dorfler

Frequency control

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right) \qquad \forall\, i \to j$$

Suppose the system is in steady state

$$\dot{\omega}_i = 0 \qquad \dot{P}_{ij} = 0 \qquad \omega_i = 0$$

and suddenly …

Frequency control

Given: disturbance in gens/loads

Current: adapt remaining generators P_i^m

- re-balance power
- restore nominal freq and inter-area flows (zero ACE)

Our goal: adapt controllable loads d_i

- re-balance power
- restore nominal freq and inter-area flows
- … while minimizing disutility of load control

Questions

How to design load-side frequency control ?

How does it interact with generator-side control ?

Limitations

- Modeling assumptions
- Preliminary design and analysis

Outline

Motivation

Network model

Load-side frequency control

Simulations

Main references:

Zhao, Topcu, Li, Low, TAC 2014
Mallada, Zhao, Low, Allerton 2014
Zhao, Low, CDC 2014

Frequency control

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right) \qquad \forall\ i \rightarrow j$$

current approach

new approach

Load-side controller design

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij} \left(\omega_i - \omega_j \right) \qquad \forall \; i \rightarrow j$$

How to design feedback control law

$$d_i = F_i \left(\omega(t), P(t) \right)$$

Load-side controller design

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij} \left(\omega_i - \omega_j \right) \qquad \forall \ i \rightarrow j$$

Control goals

Zhao, Topcu, Li, Low
TAC 2014

Mallada, Zhao, Low
Allerton, 2014

- Rebalance power
- Stabilize frequency
- Restore nominal frequency
- Restore scheduled inter-area flows

Load-side controller design

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij} \left(\omega_i - \omega_j \right) \qquad \forall \, i \rightarrow j$$

Desirable properties of $d_i = F_i \left(\omega(t), P(t) \right)$

- simple, scalable
- decentralized/distributed

Motivation: reverse engineering

Dj interpreted power flows as solution of an optimization problem

- PF equations = stationarity condition

We interpret swing dynamics as algorithm for an optimization problem

- eq pt of swing equations = optimal sol
- dynamics = primal-dual algorithm

Other examples: Internet congestion control (2000s), …
What are the advantages of this design approach?

Motivation: reverse engineering

$$M_i \dot{\omega}_i = P_i^m \qquad - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right) \qquad \forall\ i \rightarrow j$$

primal-dual algorithm

Equilibrium point is unique optimal of:

$$\min_{\hat{d},P} \quad \sum_i \frac{\hat{d}_i^2}{2D_i}$$

$$\text{s. t.} \quad P_i^m - \hat{d}_i - \sum_j C_{ij} P_{ij} = 0 \quad \forall i$$

demand = supply

Load-side controller design

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij} \left(\omega_i - \omega_j \right) \qquad \forall \; i \rightarrow j$$

Proposed approach: forward engineering

- formalize control goals into OLC objective
- derive local control as distributed solution

Outline

Motivation

Network model

Load-side frequency control

- Primary control   Zhao et al SGC2012, Zhao et al TAC2014
- Secondary control
- Interaction with generator-side control

Simulations

Optimal load control (OLC)

$$\min_{d,\hat{d},P} \quad \sum_i \left(c_i(d_i) + \frac{\hat{d}_i^2}{2D_i} \right)$$

$$\text{s. t.} \quad P_i^m - \left(d_i + \hat{d}_i \right) - \sum_e C_{ie} P_{ie} = 0 \quad \forall i$$

demand = supply

disturbances

controllable loads

Decoupled dual (DOLC)

$$\max_{\nu} \quad \sum_i \Phi_i(\nu_i)$$

$$\text{s. t.} \quad \nu_i = \nu_j \quad \forall\, i \sim j$$

decouples areas/buses i

$$\Phi_i(\nu_i) \quad := \quad \min_{d_i,\, \hat{d}_i} \; \text{Lagrangian}\,(d_i, \hat{d}_i, \nu_i)$$

$$c_i(d_i) + \frac{1}{2D_i}\hat{d}_i^2 - \nu_i\left(d_i + \hat{d}_i - P_i^m\right)$$

primal objective

constraint penalty

Decoupled dual (DOLC)

$$\max_{v} \quad \sum_i \Phi_i(v_i)$$
$$\text{s. t.} \quad v_i = v_j \quad \forall\, i \sim j$$

Lemma

A unique optimal $v^* := (v^*, \ldots, v^*)$ is attained

There is no duality gap (assuming Slater's condition)

- ➔ solve DOLC and recover optimal solution to primal (OLC)

system dynamics + load control = primal dual alg

swing dynamics

$$\dot{\omega}_i = -\frac{1}{M_i}\left(d_i(t) + D_i\omega_i(t) - P_i^m + \sum_{i\to j} P_{ij}(t) - \sum_{j\to i} P_{ji}(t) \right)$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i(t) - \omega_j(t)\right)$$

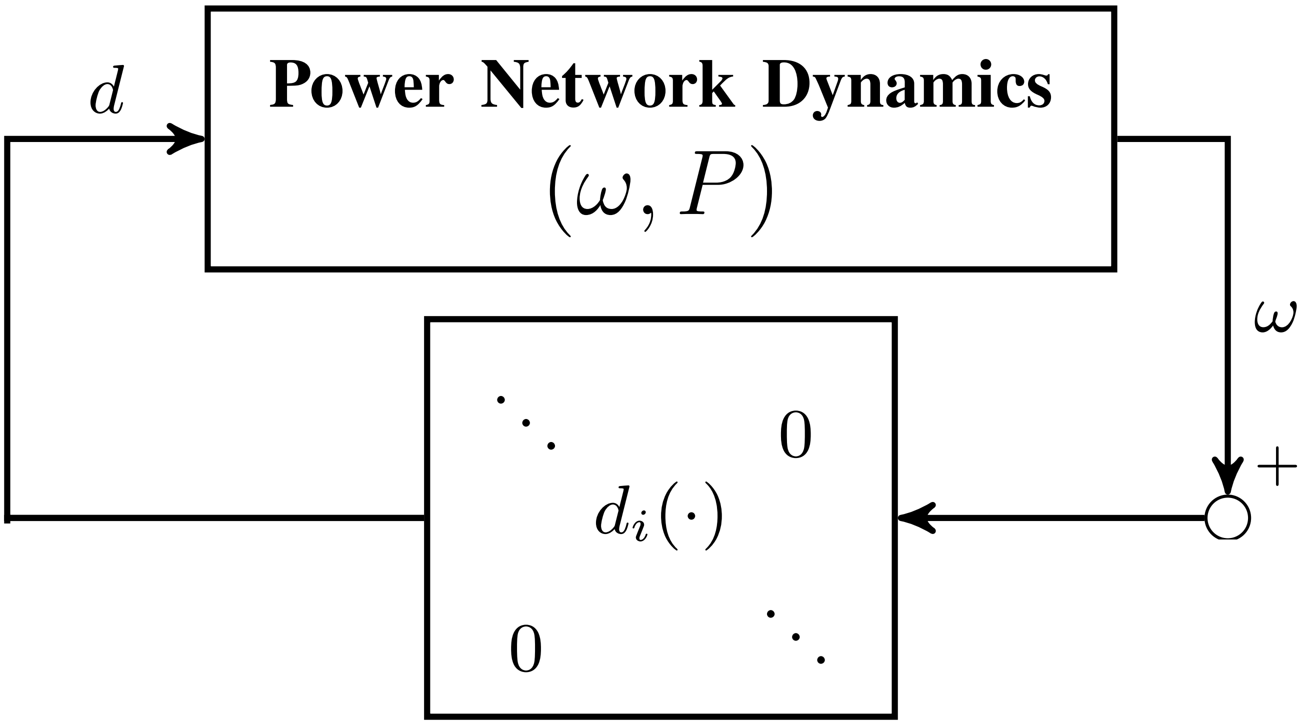
implicit

load control

$$d_i(t) := \left[c_i^{'-1}\left(\omega_i(t)\right) \right]_{\underline{d}_i}^{\overline{d}_i}$$

active control

Control architecture



Load-side primary control works

Theorem

Starting from any $\left(d(0),\ \hat{d}(0),\ \omega(0),\ P(0)\right)$

system trajectory $\left(d(t),\ \hat{d}(t),\ \omega(t),\ P(t)\right)$

converges to $\left(d^*,\ \hat{d}^*,\ \omega^*,\ P^*\right)$ as $t \to \infty$

- $\left(d^*,\ \hat{d}^*\right)$ is unique optimal of OCL
- ω^* is unique optimal for dual

- completely decentralized
- frequency deviations contain right info for local decisions that are globally optimal

Implications

- Freq deviations contains right info on global power imbalance for local decision

Implications

- Freq deviations contains right info on global power imbalance for local decision

- Decentralized load participation in primary freq control is stable

Implications

- Freq deviations contains right info on global power imbalance for local decision

- Decentralized load participation in primary freq control is stable

- ω^*: Lagrange multiplier of OLC

  info on power imbalance

Implications

- Freq deviations contains right info on global power imbalance for local decision

- Decentralized load participation in primary freq control is stable

- ω^*: Lagrange multiplier of OLC

  info on power imbalance

- P^*: Lagrange multiplier of DOLC

  info on freq asynchronism

Recap: control goals

Yes ■ Rebalance power

Yes ■ Stabilize frequencies

No ■ Restore nominal frequency $\left(\omega^* \neq 0\right)$

No ■ Restore scheduled inter-area flows

Proposed approach: forward engineering

- formalize control goals into OLC objective
- derive local control as distributed solution

Outline

Motivation

Network model

Load-side frequency control

- Primary control
- Secondary control
- Interaction with generator-side control

Mallada, Low, IFAC 2014
Mallada et al, Allerton 2014

Simulations

Recall: OLC for primary control

$$\min_{d,\hat{d},P} \quad \sum_i \left(c_i(d_i) + \frac{1}{2D_i}\hat{d}_i^2 \right)$$

$$\text{s. t.} \quad P^m - (d + \hat{d}) = CP$$

demand = supply

OLC for secondary control

$$\min_{d,\hat{d},P,v} \quad \sum_i \left(c_i(d_i) + \frac{1}{2D_i}\hat{d}_i^2 \right)$$

$$\text{s.t.} \quad P^m - (d + \hat{d}) = CP$$

demand = supply

key idea: "virtual flows"

$$BC^T v$$

in steady state: virtual = real flows

$$BC^T v = P$$

OLC for secondary control

$$\min_{d,\hat{d},P,v} \quad \sum_i \left(c_i(d_i) + \frac{1}{2D_i}\hat{d}_i^2 \right)$$

$$\text{s. t.} \quad P^m - (d + \hat{d}) = CP$$

demand = supply

$$P^m - d = CBC^T v$$

restore nominal freq

in steady state: virtual = real flows

$$BC^T v = P$$

OLC for secondary control

$$\min_{d,\hat{d},P,v} \quad \sum_i \left(c_i(d_i) + \frac{1}{2D_i}\hat{d}_i^2 \right)$$

$$\text{s.t.} \quad P^m - (d + \hat{d}) = CP$$

demand = supply

$$P^m - d = CBC^T v$$

restore nominal freq

$$\hat{C}BC^T v = \hat{P}$$

restore inter-area flow

$$\underline{P} \le BC^T v \le \overline{P}$$

respect line limit

in steady state: virtual = real flows

$$BC^T v = P$$

Recall: primary control

swing dynamics:

$$\dot{\omega}_i = -\frac{1}{M_i}\left(d_i(t) + D_i\omega_i(t) - P_i^m + \sum_{e\in E} C_{ie}P_e(t)\right)$$

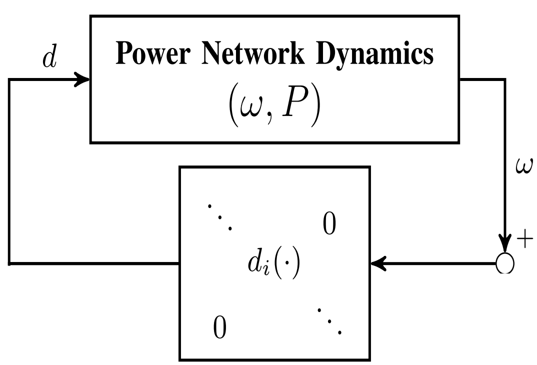
$$\dot{P}_{ij} = b_{ij}\left(\omega_i(t) - \omega_j(t)\right)$$

implicit

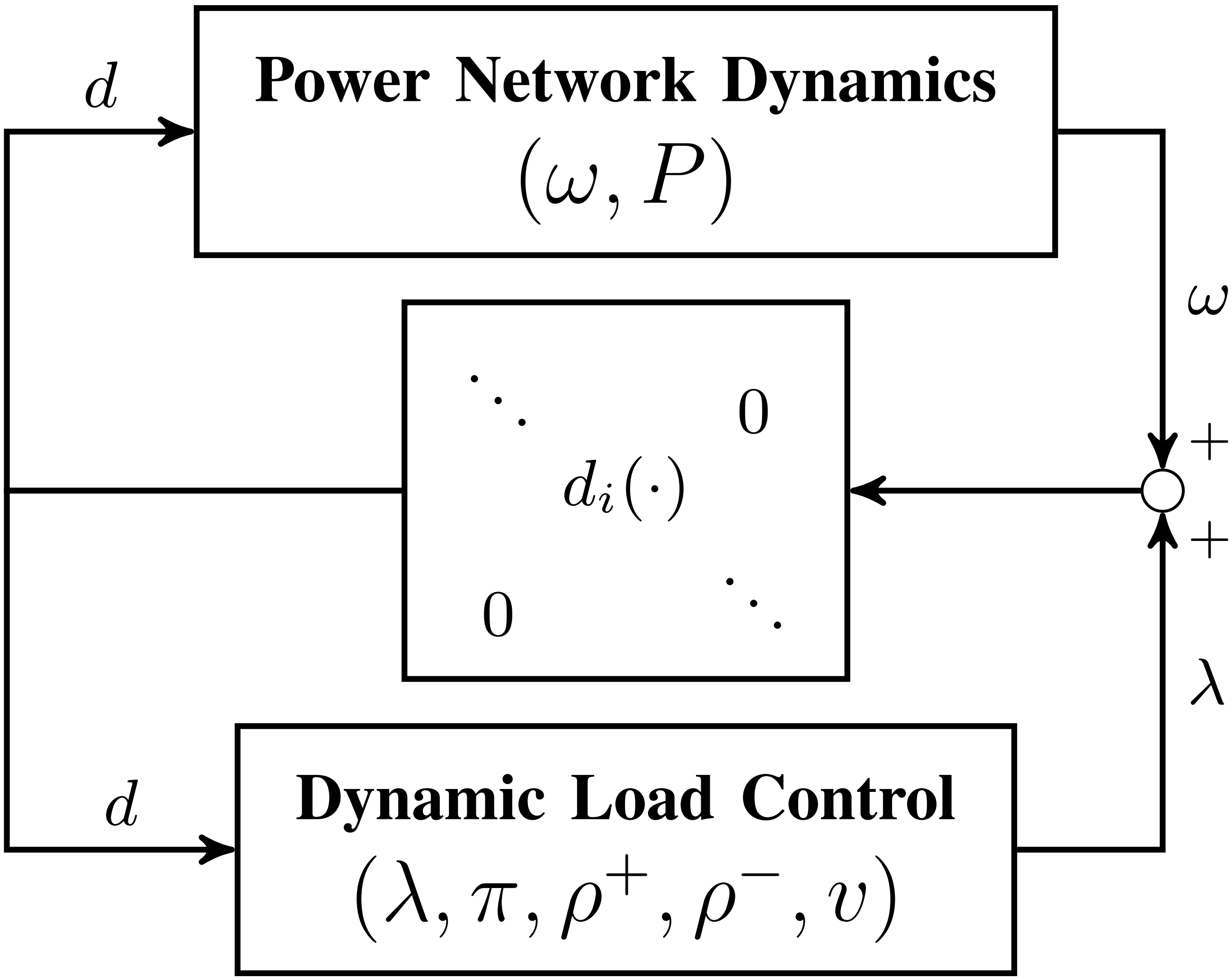
load control:

$$d_i(t) := \left[c_i'^{-1}\left(\omega_i(t)\right)\right]_{\underline{d}_i}^{\bar{d}_i}$$

active control



Control architecture



Secondary frequency control

load control: $$d_i(t) := \left[c_i'^{-1} \left(\omega_i(t) + \lambda_i(t) \right) \right]_{\underline{d}_i}^{\overline{d}_i}$$

computation & communication:

primal var: $$\dot{v} = \chi^v \left(L_B \lambda - C D_B \hat{C}^T \pi - C D_B (\rho^+ - \rho^-) \right)$$

dual vars: $$\dot{\lambda} = \zeta^\lambda \left(P^m - d - L_B v \right)$$

$$\dot{\pi} = \zeta^\pi \left(\hat{C} D_B C^T v - \hat{P} \right)$$

$$\dot{\rho}^+ = \zeta^{\rho^+} \left[D_B C^T v - \bar{P} \right]_{\rho^+}^+$$

$$\dot{\rho}^- = \zeta^{\rho^-} \left[\underline{P} - D_B C^T v \right]_{\rho^-}^+$$

Secondary control works

Theorem

starting from any initial point, system trajectory converges s. t.

- $\left(d^*,\, \hat{d}^*, P^*, v^*\right)$ is unique optimal of OLC
- nominal frequency is restored $\omega^* = 0$
- inter-area flows are restored $\hat{C}P^* = \hat{P}$
- line limits are respected $\underline{P} \le P^* \le \overline{P}$

Recap: control goals

- Yes ■ Rebalance power
- Yes ■ Resynchronize/stabilize frequency

Zhao, et al TAC2014

- Yes ■ Restore nominal frequency $\left(\omega^* \neq 0\right)$
- Yes ■ Restore scheduled inter-area flows

Mallada, et al Allerton2014

Secondary control restores nominal frequency but requires local communication

Outline

Motivation

Network model

Load-side frequency control

- Primary control
- Secondary control
- Interaction with generator-side control

Zhao and Low, CDC2014

Simulations

Generator-side control

New model: nonlinear PF, with generator control

$$\dot{\theta}_i = \omega_i$$

$$M_i\dot{\omega}_i = -D_i\omega_i + \boxed{p_i} - \sum_e C_{ie}P_e$$

$$P_{ij} = b_{ij}\sin\left(\theta_i - \theta_j\right) \qquad \forall\, i \to j$$

Recall model: linearized PF, no generator control

$$M_i\dot{\omega}_i = -D_i\omega_i + \boxed{P_i^m - d_i} - \sum_e C_{ie}P_e$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right) \qquad \forall\, i \to j$$

Generator-side control

New model: nonlinear PF, with generator control

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + p_i - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \rightarrow j$$

generator bus: real power injection
load bus: controllable load

Generator-side control

New model: nonlinear PF, with generator control

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + p_i - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \to j$$

generator buses:

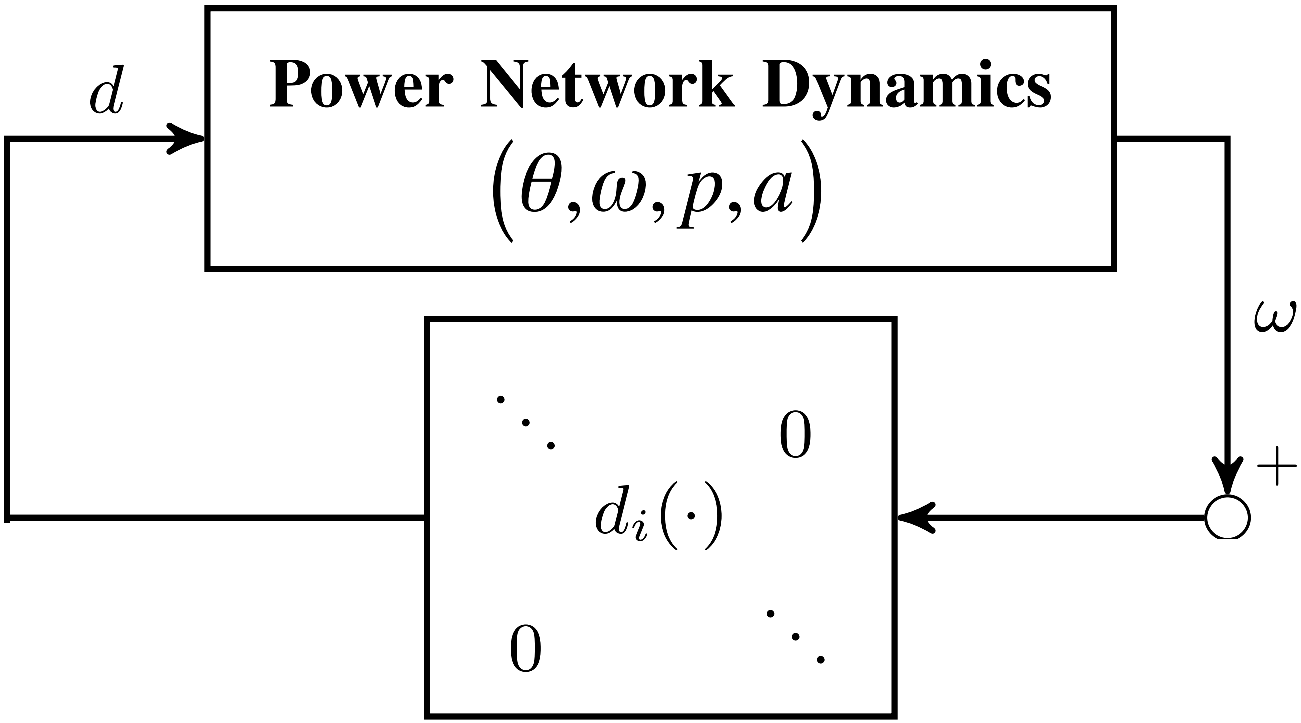
$$\dot{p}_i = -\frac{1}{\tau_{bi}}\left(p_i + a_i\right)$$

primary control $p_i^c(t) = p_i^c\left(\omega_i(t)\right)$

e.g. freq droop $p_i^c\left(\omega_i\right) = -\beta_i \omega_i$

$$\dot{a}_i = -\frac{1}{\tau_{gi}}\left(a_i + p_i^c\right)$$

Load-side (primary) control



load-side control

$$d_i(t) := \left[c_i'^{-1}\left(\omega_i(t)\right) \right]_{\underline{d}_i}^{\overline{d}_i}$$

Load-side primary control works

Theorem

- Every closed-loop equilibrium solves OLC and its dual

Suppose $\left| p_i^c(\omega) - p_i^c\left(\omega^*\right) \right| \le L_i \left| \omega - \omega^* \right|$

$$\text{near } \omega^* \text{ for some } L_i < D_i$$

- Any closed-loop equilibrium is (locally) asymptotically stable provided

$$\left| \theta_i^* - \theta_j^* \right| < \frac{\pi}{2}$$

Outline

Motivation

Network model

Load-side frequency control

Simulations

Main references:

Zhao, Topcu, Li, Low, TAC 2014
Mallada, Zhao, Low, Allerton 2014
Zhao, Low, CDC 2014

Simulations

Dynamic simulation of IEEE 39-bus system

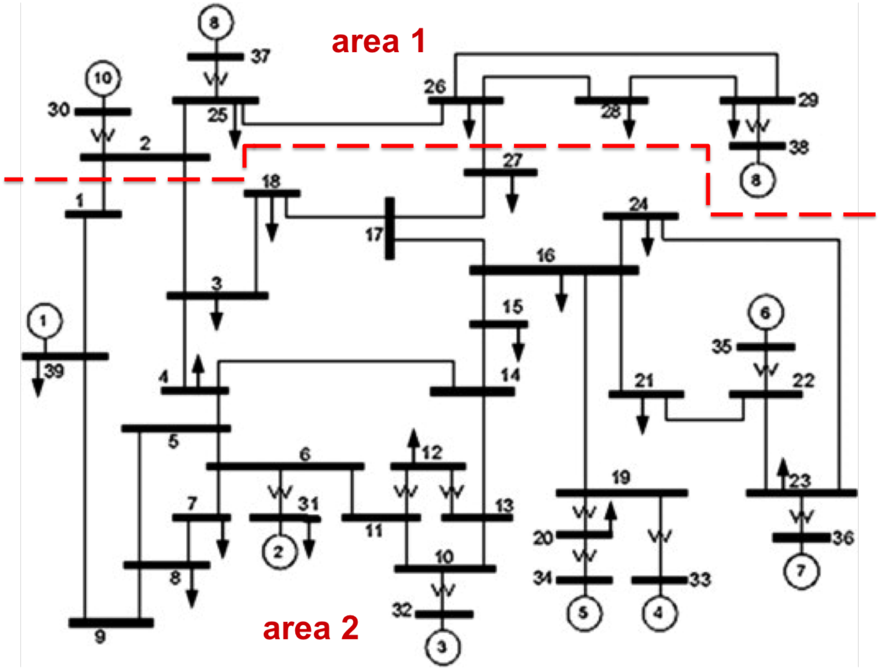
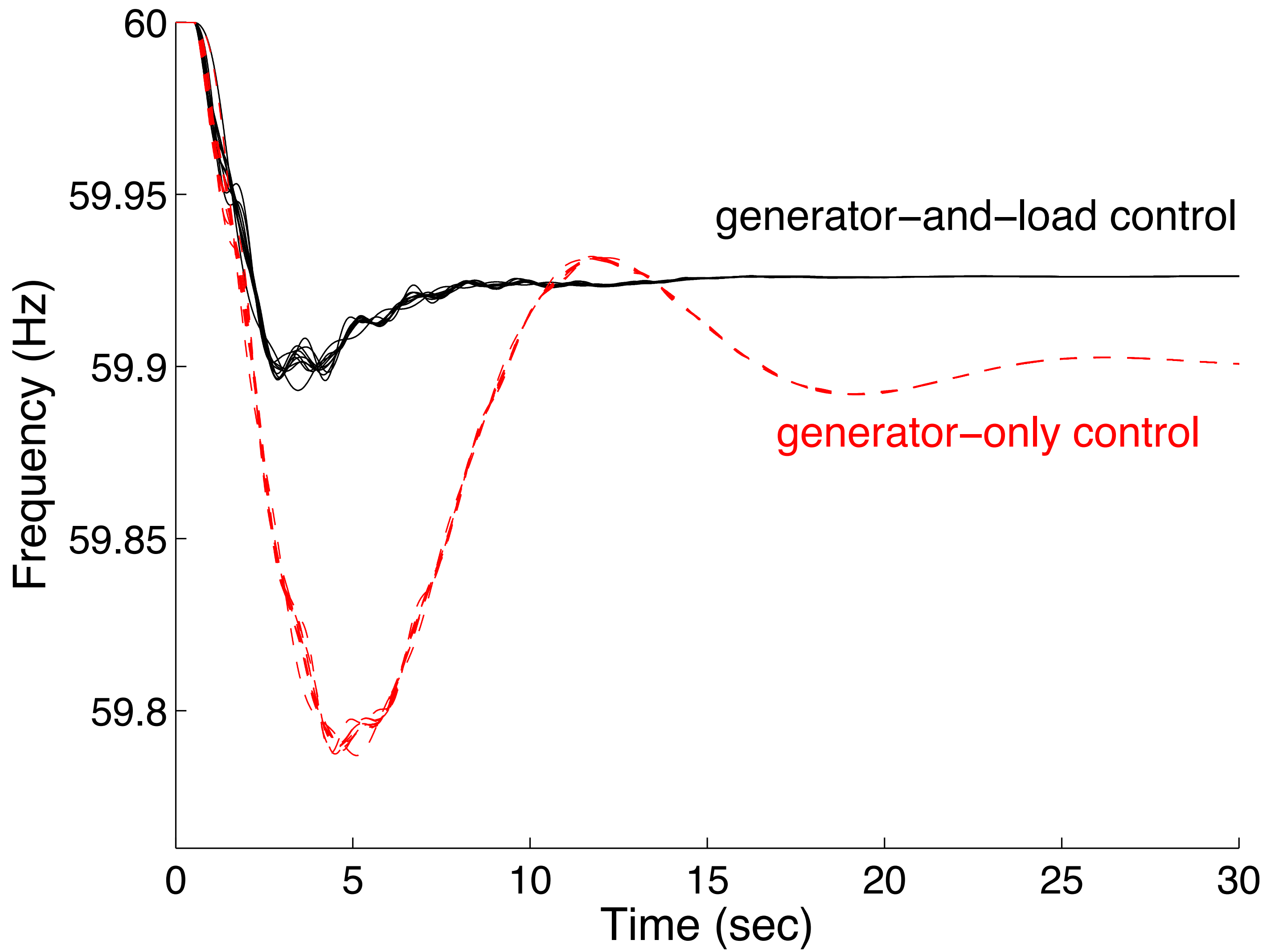


Fig. 2: IEEE 39 bus system: New England

- Power System Toolbox (RPI)
- Detailed generation model
- Exciter model, power system stabilizer model
- Nonzero resistance lines

Primary control



Secondary control

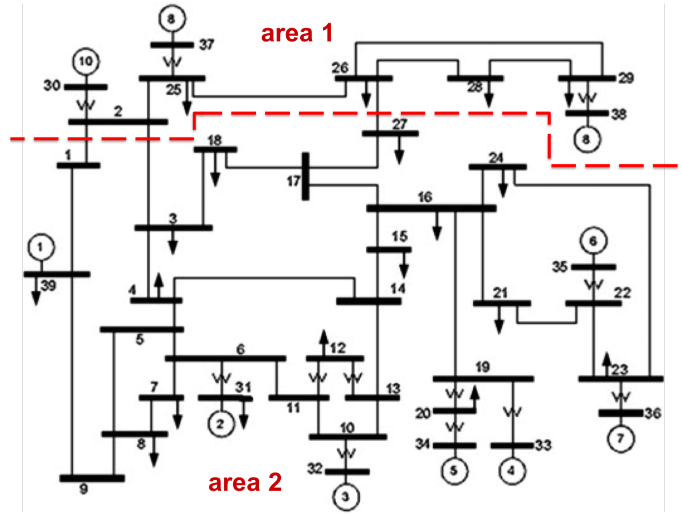
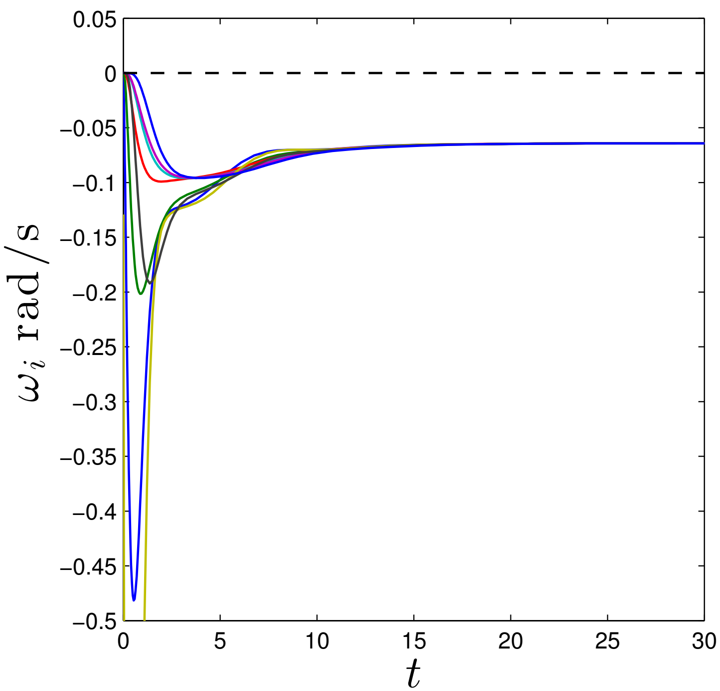
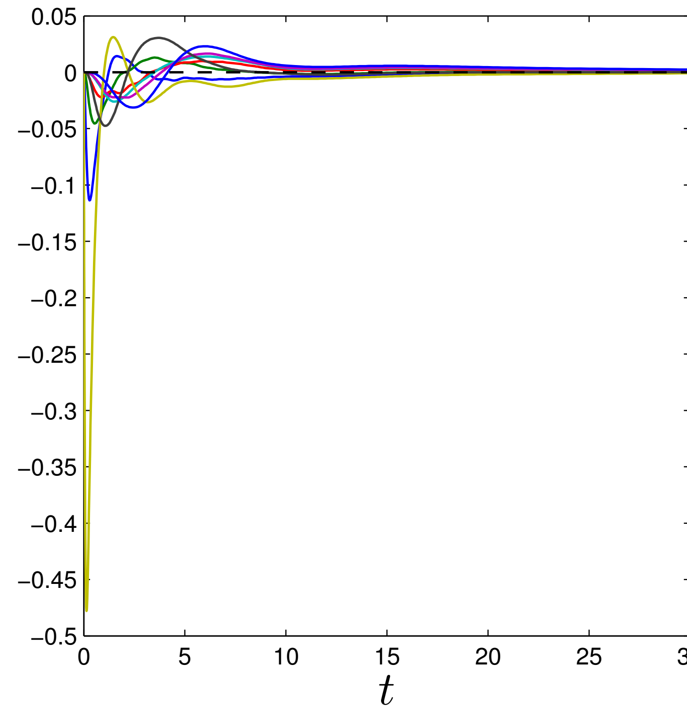


Fig. 2: IEEE 39 bus system: New England

swing dynamics



with OLC



area 1

Conclusion

Forward-engineering design facilitates

- explicit control goals
- distributed algorihtms
- stability analysis

Load-side frequency regulation

- primary & secondary control works
- helps generator-side control