Modeling and Computation of Security-constrained Economic Dispatch with Multi-stage Rescheduling

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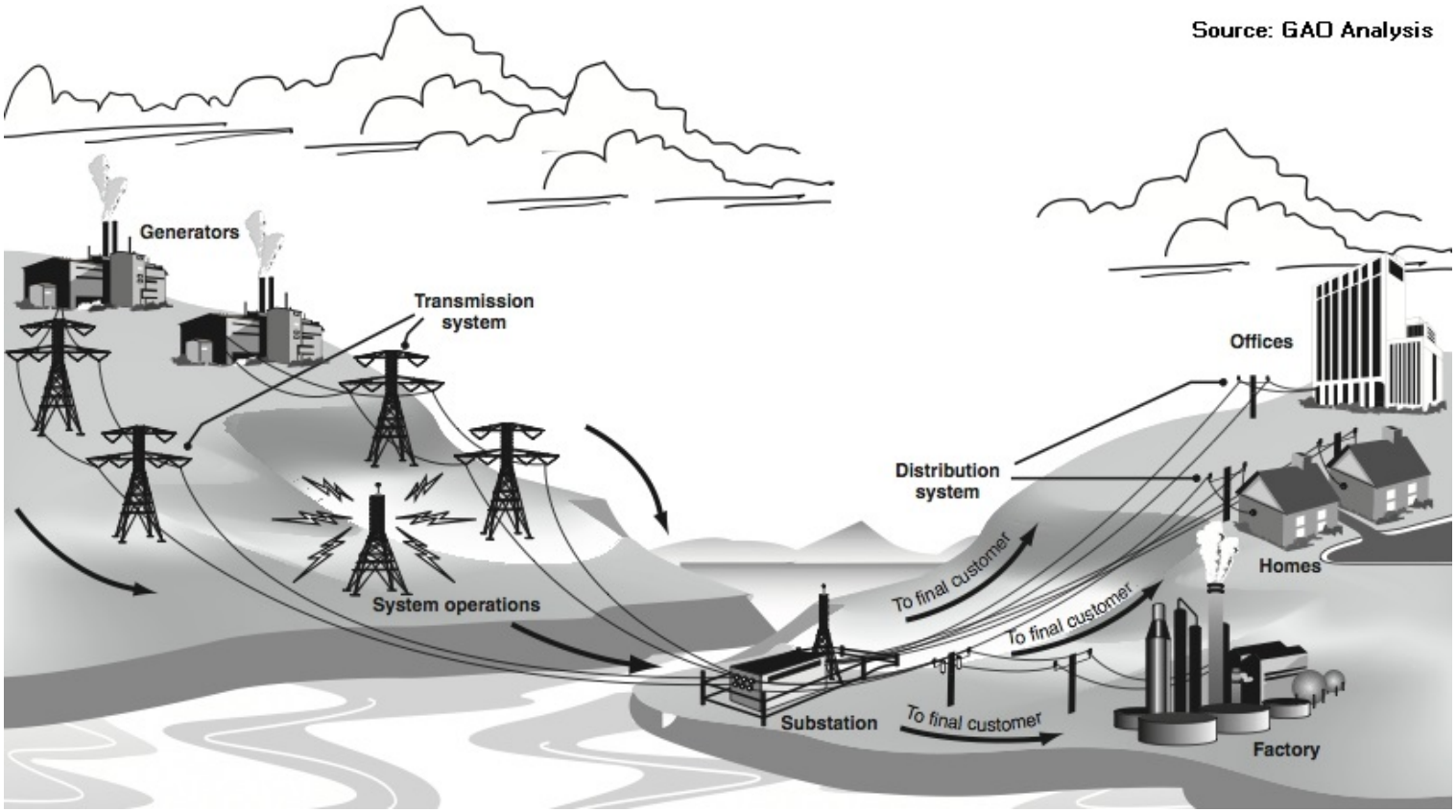
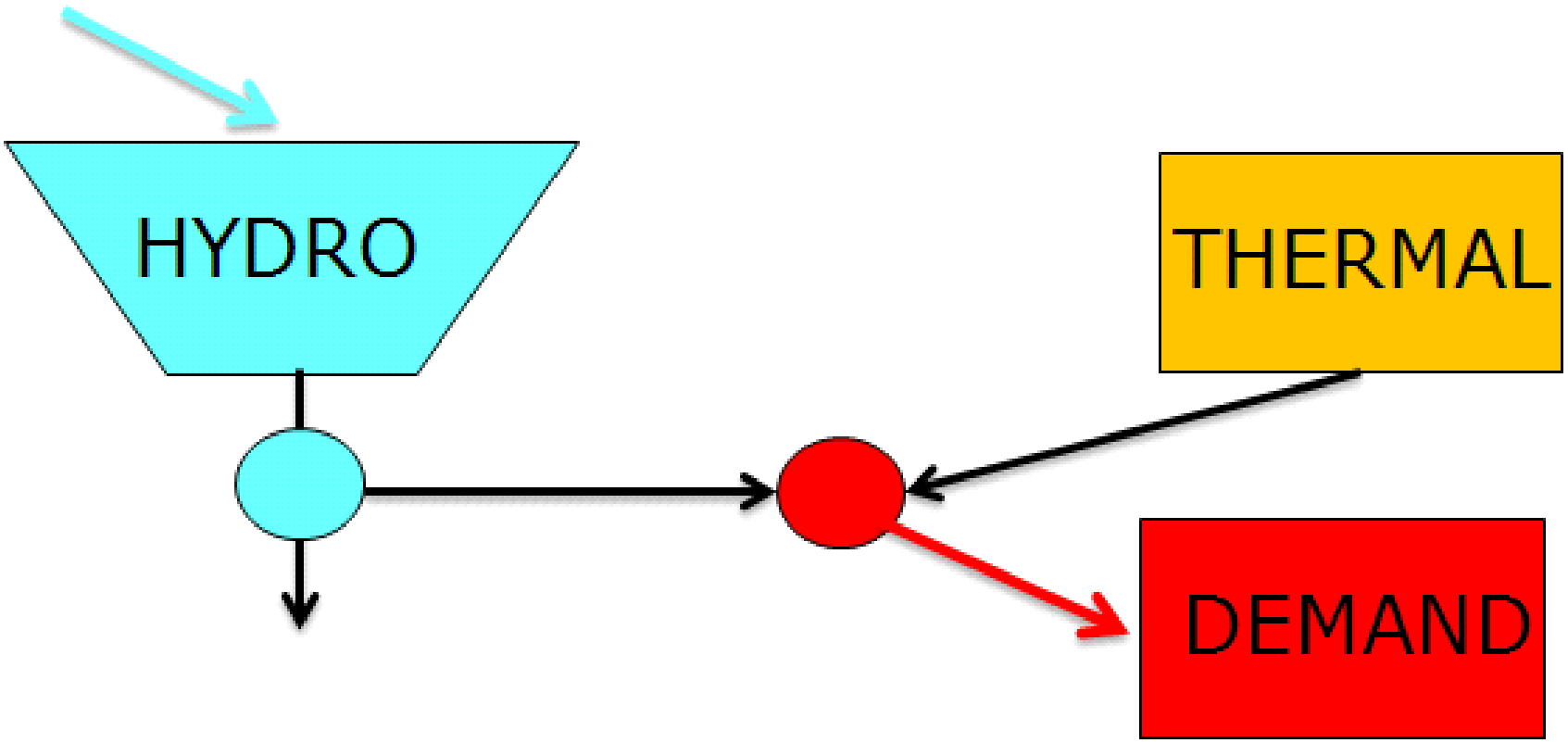
Power generation, transmission and distribution



- Determine generators' output to reliably meet the load
  - $\sum$ Gen MW $=$ $\sum$ Load MW, at all times.
  - Power flows cannot exceed lines' transfer capacity.

Hydro-Thermal System (Philpott/F./Wets)



Simple electricity "system optimization" problem

$$\text{SO:} \quad \max_{d_k, u_i, v_j, x_i \geq 0} \quad \sum_{k \in \mathcal{K}} W_k(d_k) - \sum_{j \in \mathcal{T}} C_j(v_j) + \sum_{i \in \mathcal{H}} V_i(x_i)$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{H}} U_i(u_i) + \sum_{j \in \mathcal{T}} v_j \geq \sum_{k \in \mathcal{K}} d_k,$$

$$x_i = x_i^0 - u_i + h_i^1, \quad i \in \mathcal{H}$$

- u_i water release of hydro reservoir $i \in \mathcal{H}$
- v_j thermal generation of plant $j \in \mathcal{T}$
- x_i water level in reservoir $i \in \mathcal{H}$
- prod fn U_i (strictly concave) converts water release to energy
- $C_j(v_j)$ denote the cost of generation by thermal plant
- $V_i(x_i)$ future value of terminating with storage x (assumed separable)
- $W_k(d_k)$ utility of consumption d_k

SO equivalent to CE

$$\text{Consumers } k \in \mathcal{K} \text{ solve CP}(k): \quad \max_{d_k \geq 0} \quad W_k(d_k) - p^T d_k$$

$$\text{Thermal plants } j \in \mathcal{T} \text{ solve TP}(j): \quad \max_{v_j \geq 0} \quad p^T v_j - C_j(v_j)$$

$$\text{Hydro plants } i \in \mathcal{H} \text{ solve HP}(i): \quad \max_{u_i, x_i \geq 0} \quad p^T U_i(u_i) + V_i(x_i)$$

$$\text{s.t.} \quad x_i = x_i^0 - u_i + h_i^1$$

Perfectly competitive (Walrasian) equilibrium is a MOPEC

$$\begin{aligned}
\text{CE:} \qquad d_k &\in \arg\max \text{CP}(k), && k \in \mathcal{K}, \\
v_j &\in \arg\max \text{TP}(j), && j \in \mathcal{T}, \\
u_i, x_i &\in \arg\max \text{HP}(i), && i \in \mathcal{H}, \\
0 \leq p \perp \sum_{i \in \mathcal{H}} U_i(u_i) + \sum_{j \in \mathcal{T}} v_j &\geq \sum_{k \in \mathcal{K}} d_k.
\end{aligned}$$

Nash Equilibria (as a MOPEC)

- Nash Games: x^* is a Nash Equilibrium if

$$x_i^* \in \arg\min_{x_i \in X_i} \ell_i(x_i, x_{-i}^*, p), \forall i \in \mathcal{I}$$

  x_{-i} are the decisions of other players.
- Prices p given exogenously, or via complementarity:

$$0 \leq H(x, p) \ \perp \ p \geq 0$$

- empinfo: equilibrium
  min loss(i) x(i) cons(i)
  vi H p
- Applications: Discrete-Time Finite-State Stochastic Games. Specifically, the Ericson & Pakes (1995) model of dynamic competition in an oligopolistic industry.

Key point: models generated correctly solve quickly

Here S is mesh spacing parameter

S	Var	rows	non-zero	dense(%)	Steps	RT (m:s)
20	2400	2568	31536	0.48	5	0 : 03
50	15000	15408	195816	0.08	5	0 : 19
100	60000	60808	781616	0.02	5	1 : 16
200	240000	241608	3123216	0.01	5	5 : 12

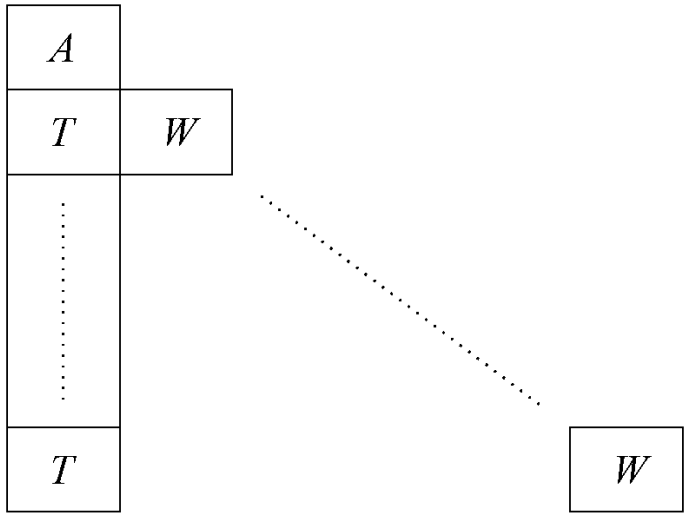
Convergence for $S = 200$ (with new basis extensions in PATH)

Iteration	Residual
0	1.56(+4)
1	1.06(+1)
2	1.34
3	2.04(−2)
4	1.74(−5)
5	2.97(−11)

Agents have stochastic recourse?

- Two stage stochastic programming, x^1 is here-and-now decision, recourse decisions x^2 depend on realization of a random variable
- ρ is a risk measure (e.g. expectation, CVaR)

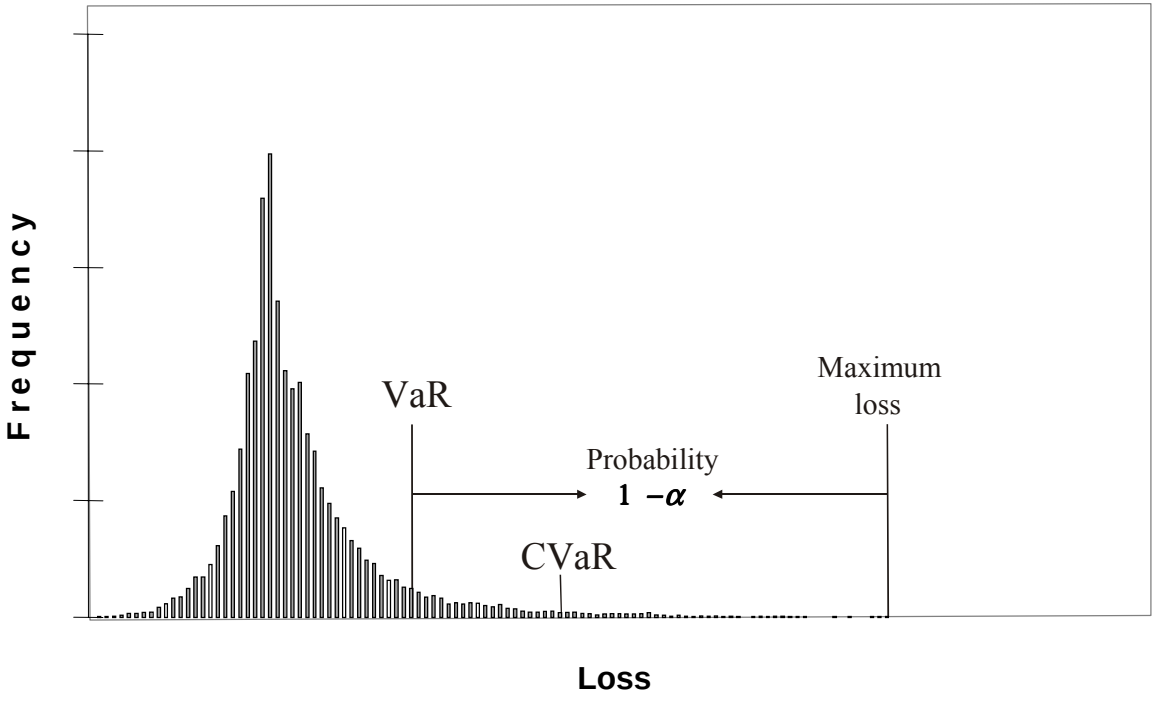
$$
\begin{aligned}
\text{SP: } \max \quad & c^T x^1 + \rho[q^T x^2] \\
\text{s.t.} \quad & Ax^1 = b, \quad x^1 \geq 0, \\
& T(\omega)x^1 + W(\omega)x^2(\omega) \geq d(\omega), \\
& x^2(\omega) \geq 0, \forall \omega \in \Omega.
\end{aligned}
$$



EMP/SP extensions to facilitate these models

Risk Measures

- Modern approach to modeling risk aversion uses concept of risk measures
- $\overline{CVaR}_\alpha$: mean of upper tail beyond α-quantile (e.g. $\alpha = 0.95$)



- mean-risk, mean deviations from quantiles, VaR, CVaR
- Much more in mathematical economics and finance literature
- Optimization approaches still valid, different objectives, varying convex/non-convex difficulty

Stochastic unit commitment: different risk measures

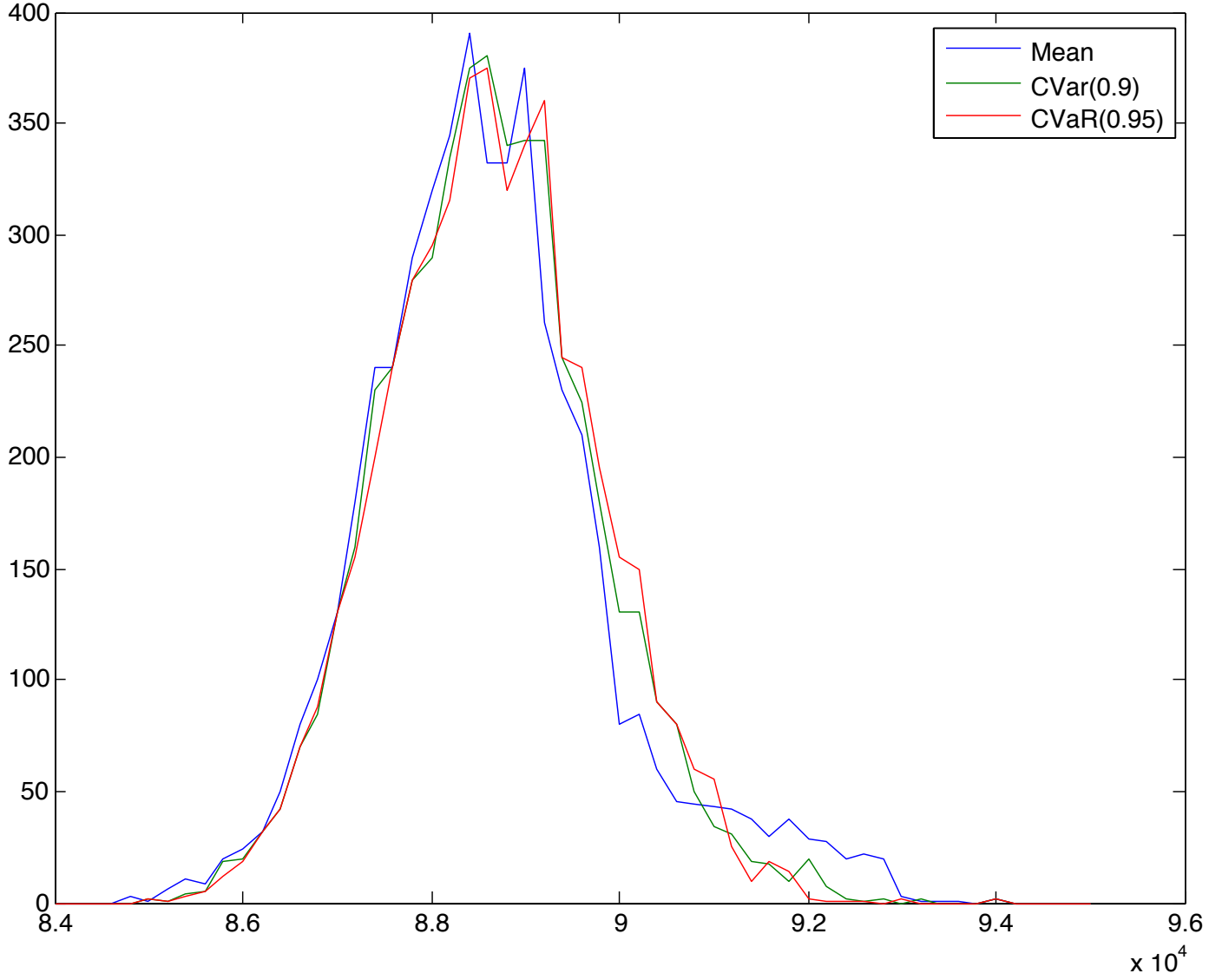


Figure : Frequency plot for cost for 5000 (out-of-sample) simulations

Equilibrium or optimization?

- Each agent has its own risk measure
- Is there a system risk measure?
- Is there a system optimization problem?

$$\min \sum_i C(x_i^1) + \rho_i \left(C(x_i^2(\omega)) \right) ????$$

- Can we modify (complete) system to have a social optimum by trading risk?
- How do we design these instruments? How many are needed? What is cost of deficiency?
- Can we solve efficiently / distributively?

Contracts in MOPEC (F./Wets)

- Competing agents (consumers, or generators in energy market)
- Each agent minimizes objective independently (cost)
- Market prices are function of all agents activities
- Additional twist: model must "hedge" against uncertainty
- Facilitated by allowing contracts bought now, for goods delivered later (e.g. Arrow-Debreu Securities)
- Conceptually allows to transfer goods from one period to another (provides wealth retention or pricing of ancilliary services in energy market)
- Can investigate new instruments to mitigate risk, or move to system optimal solutions from equilibrium (or market) solutions

Example as MOPEC: agents solve a Stochastic Program

Buy y_i contracts in period 1, to deliver $D(\omega)y_i$ in period 2, scenario ω
Each agent i:

$$\begin{aligned} \min \quad & C(x_i^1) + \rho_i \left(C(x_i^2(\omega)) \right) \\ \text{s.t.} \quad & p^1 x_i^1 + v y_i \leq p^1 e_i^1 && \text{(budget time 1)} \\ & p^2(\omega) x_i^2(\omega) \leq p^2(\omega)(D(\omega) y_i + e_i^2(\omega)) && \text{(budget time 2)} \end{aligned}$$

$$0 \leq v \perp -\sum_i y_i \geq 0 \qquad \text{(contract)}$$

$$0 \leq p^1 \perp \sum_i \left(e_i^1 - x_i^1 \right) \geq 0 \qquad \text{(walras 1)}$$

$$0 \leq p^2(\omega) \perp \sum_i \left(D(\omega) y_i + e_i^2(\omega) - x_i^2(\omega) \right) \geq 0 \qquad \text{(walras 2)}$$

Theory and Observations

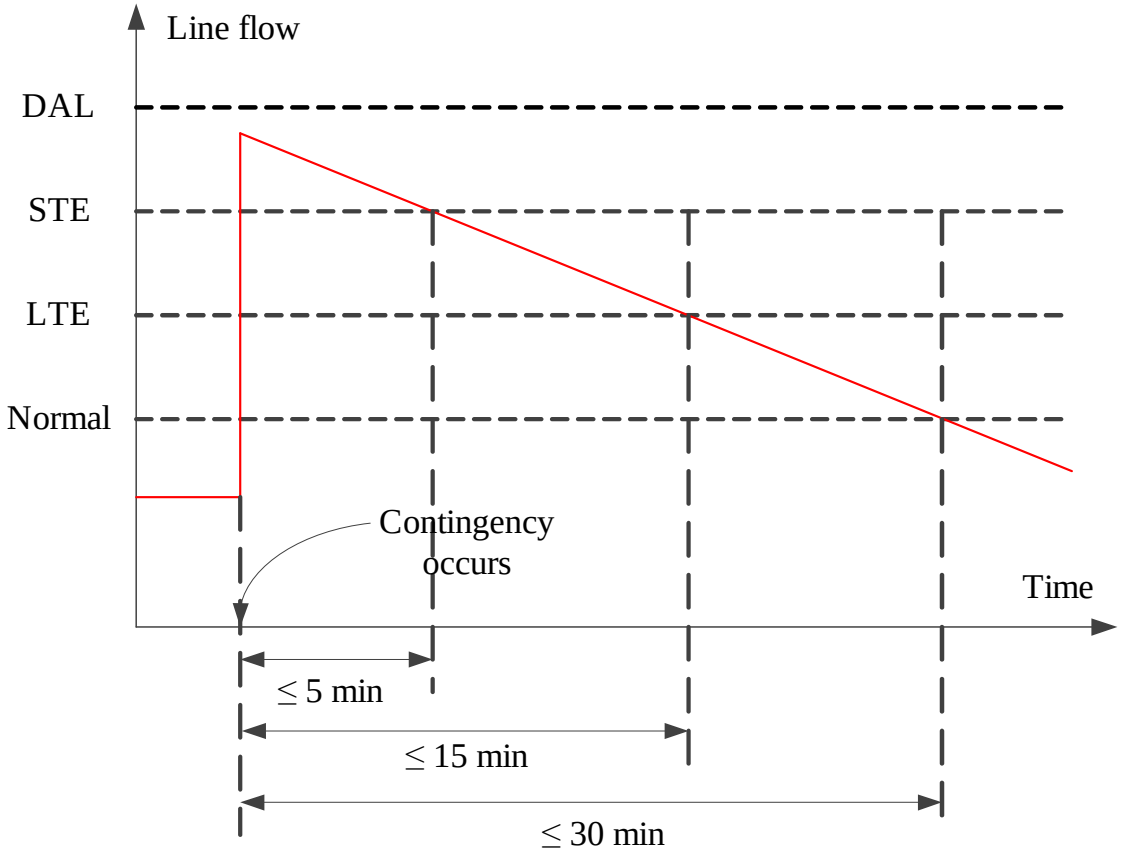
- agent problems are multistage stochastic optimization models
- perfectly competitive partial equilibrium still corresponds to a social optimum when all agents are risk neutral and share common knowledge of the probability distribution governing future inflows
- situation complicated when agents are risk averse
  - utilize stochastic process over scenario tree
  - under mild conditions a social optimum corresponds to a competitive market equilibrium if agents have time-consistent dynamic coherent risk measures and there are enough traded market instruments (over tree) to hedge inflow uncertainty
- Otherwise, must solve the stochastic equilibrium problem
- Solution possible via disaggregation only seems possible in special cases
  - When problem is block diagonally dominant (Wathen/F./Rutherford)
  - When overall (complementarity) problem is monotone
  - (Pang): when problem is a potential game
- Research challenge: develop reliable algorithms for large scale decomposition approaches to MOPEC

Security-constrained Economic Dispatch

- Base-case network topology g_0 and line flow x_0.
- If the k-th line fails, line flow jumps to x_k in new topology g_k.
- Ensure that x_k is within limit, for all k.
- SCED model:

$$
\begin{array}{lll}
\min\limits_{u,x_0,\ldots,x_k} & c^T u + \rho(u) & \triangleright \text{ Total cost} \\
\text{s.t.} & 0 \le u \le \bar{u} & \triangleright \text{ GEN capacity const.} \\
 & g_0(x_0, u) = 0 & \triangleright \text{Base-case network eqn.} \\
 & -\bar{x} \le x_0 \le \bar{x} & \triangleright \text{Base-case flow limit} \\
 & g_k(x_k, u) = 0, \quad k = 1, \ldots, K & \triangleright \text{Ctgcy network eqn.} \\
 & -\bar{x} \le x_k \le \bar{x}, \quad k = 1, \ldots, K & \triangleright \text{Ctgcy flow limit}
\end{array}
$$

Reality offers a sweeter deal...



Operating procedure (ISO-NE) requires post-contingency line loadings be:

- $\leq$ STE (short time emergency) rating in 5 minutes;
- $\leq$ LTE (long time emergency) rating in 15 minutes;
- $\leq$ Normal rating in 30 minutes.

What we will contribute

Research issues:

- Corrective actions are not modeled in ISO's dispatch software.
- Because it was "insolvable" due to its large size ($\geq$ 10GB LP).
  - "We looked into SCED with corrective actions before, and were hindered by the computational challenge." – Feng Zhao, senior analyst at ISO-NE, via private correspondence.

Our contributions:

- We **model** the *multi-period* corrective rescheduling in SCED; solutions much better quality
- **Enhance** the Benders' **algorithm** to solve the problem faster
- **Achieve** about 50$\times$ **speedup** compared to traditional approaches

Our model (K contingencies, T periods)

$$
\begin{array}{lll}
\min\limits_{x_0,\ldots,x_k,u_0,\ldots,u_k} & c^T u_0 & \\
\text{s.t.} & g_0(x_0, u_0) = 0 & \\
 & h_0(x_0, u_0) \leq 0 & \\
 & g_k(x_k^t, u_k^t) = 0 & k = 1, \ldots, K,\ t = 0, \ldots, T \\
 & h_k(x_k^t, u_k^t) \leq 0 & k = 1, \ldots, K,\ t = 0, \ldots, T \\
 & |u_k^t - u_k^{t-1}| \leq \Delta_t & k = 1, \ldots, K,\ t = 1, \ldots, T \\
 & u_k^0 - u_0 = 0 & k = 1, \ldots, K
\end{array}
$$

- Subscript 0 indicates a quantity in the base-case network topology.
- This is a large-scale linear program.
- What special structure does it have?

Model structure

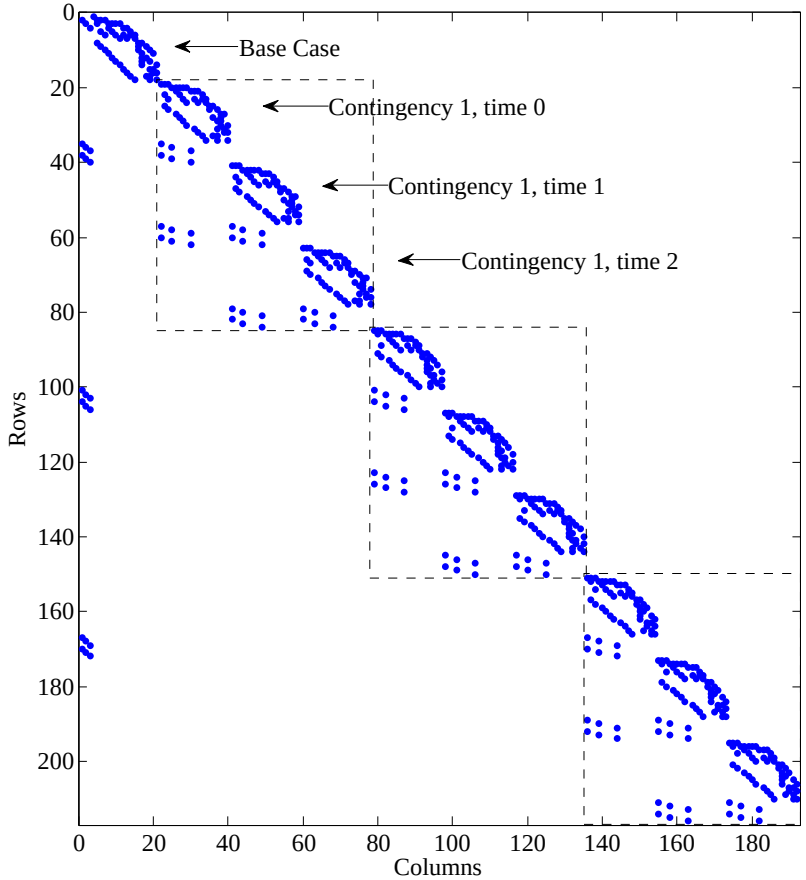


Figure : Sparsity structure of the Jacobian matrix of a 6-bus case, considering 3 contingencies and 3 post-contingency checkpoints.

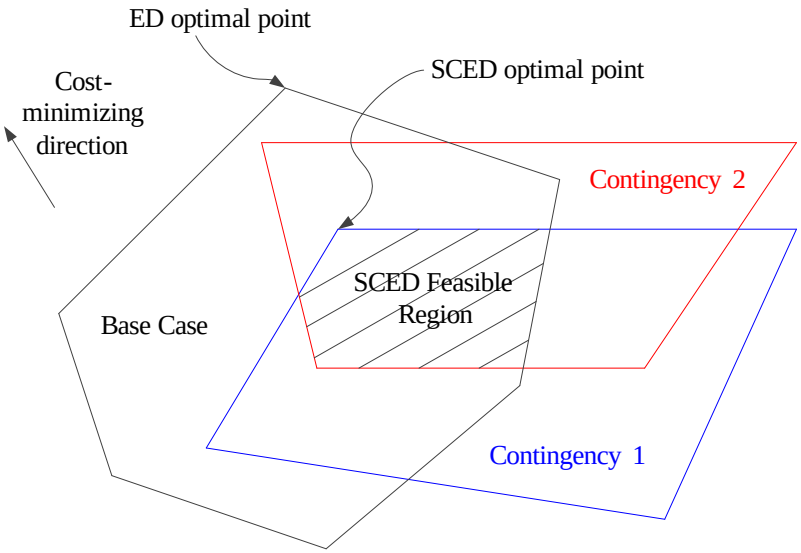


Figure : On the u_0 plane, the feasible region of a SCED is the intersection of K+1 polyhedra.

Current state of the art (unsatisfactory)

Table : CPLEX v.s. Vanilla Benders Algorithm

Case	Ctgcy	Big LP (time)		Vanilla Benders		
		Simplex	Barrier[1]	Iter	LPs	Time
118-bus	183	207.8	13.8	8	1464	123.5
2383-bus	20	175.0	205.5	52	1040	1281.2
2383-bus	50	1403.2	123.1	49	2450	2799.3
2383-bus	100	3621.8	240.6	32	3200	3688.6
2383-bus	400	-	2354.5	-	-	-

- Three time-periods: 5-min STE, 15-min LTE and 30-min Normal.
- Vanilla Benders' algorithm is inferior to the big LP formulation.
- Big LP cannot handle large instances.

[1] Barrier method without crossover. Crossover may take even more time.

How we enhanced the Benders' algorithm ...

1. Reduce the number of LPs
2. Solve subproblem LPs faster
3. Parallel computing
4. Add difficult contingencies to master model

Case	Ctgcy	Big LP (time)		**Enhanced Benders**		
		Simplex	Barrier	Iter	LPs	Time
118-bus	183	207.8	13.8	12	755	13.5
2383-bus	20	175.0	205.5	11	60	41.5
2383-bus	50	1403	123.1	11	135	46.5
2383-bus	100	3621	240.6	12	245	79.4
2383-bus	400	-	2354.5	13	879	197.8
2383 wp	2349			21	9529	515.7
2736 sp	2749			4	5500	220.9
2737 sop	2753			1	2753	100.5
2746 wop	2794			1	2794	118.5
2746 wp	2719			14	5558	333.5

Illustration

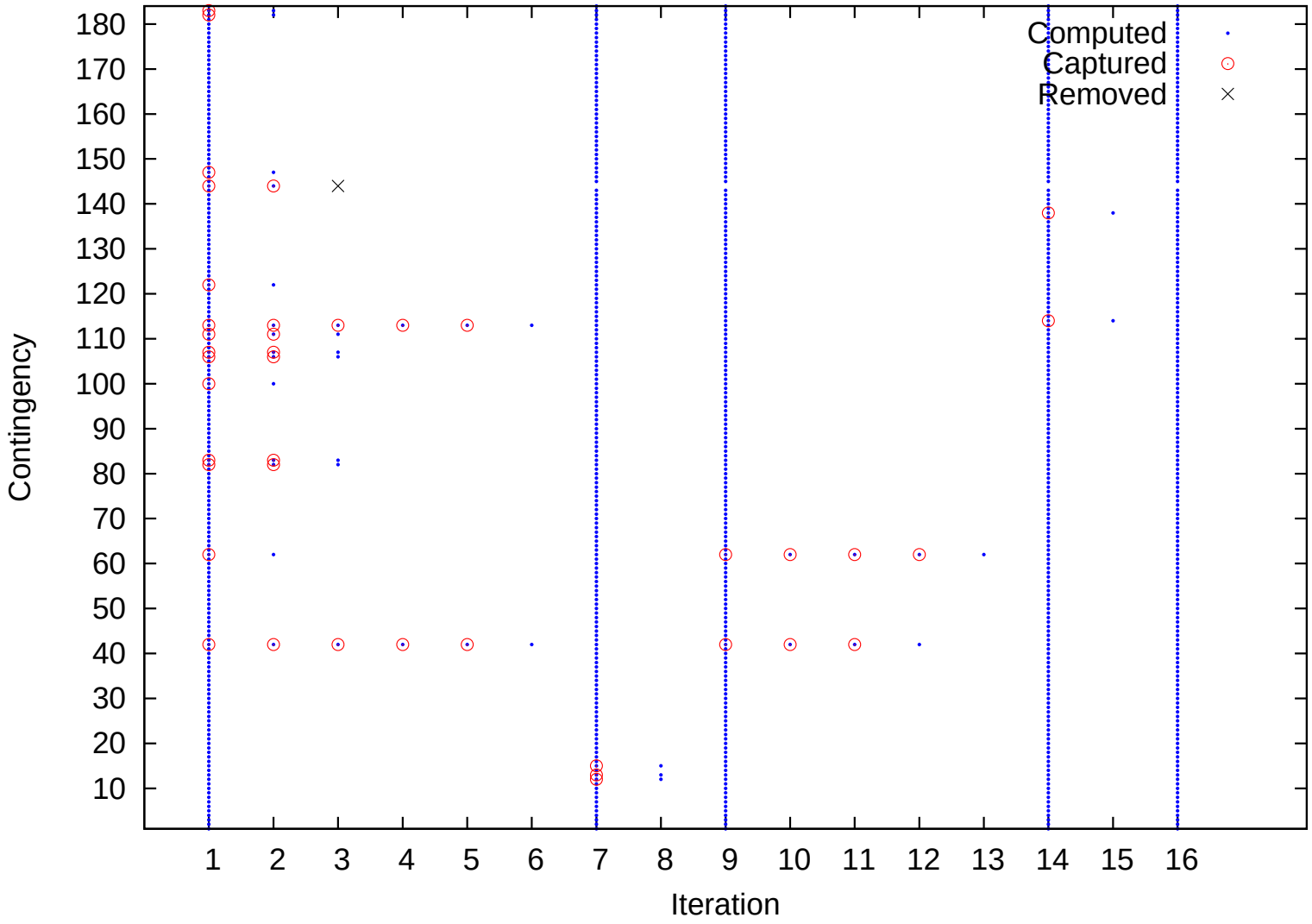


Figure : Benders' algorithm with reduced number of subproblem LPs, 118-bus case

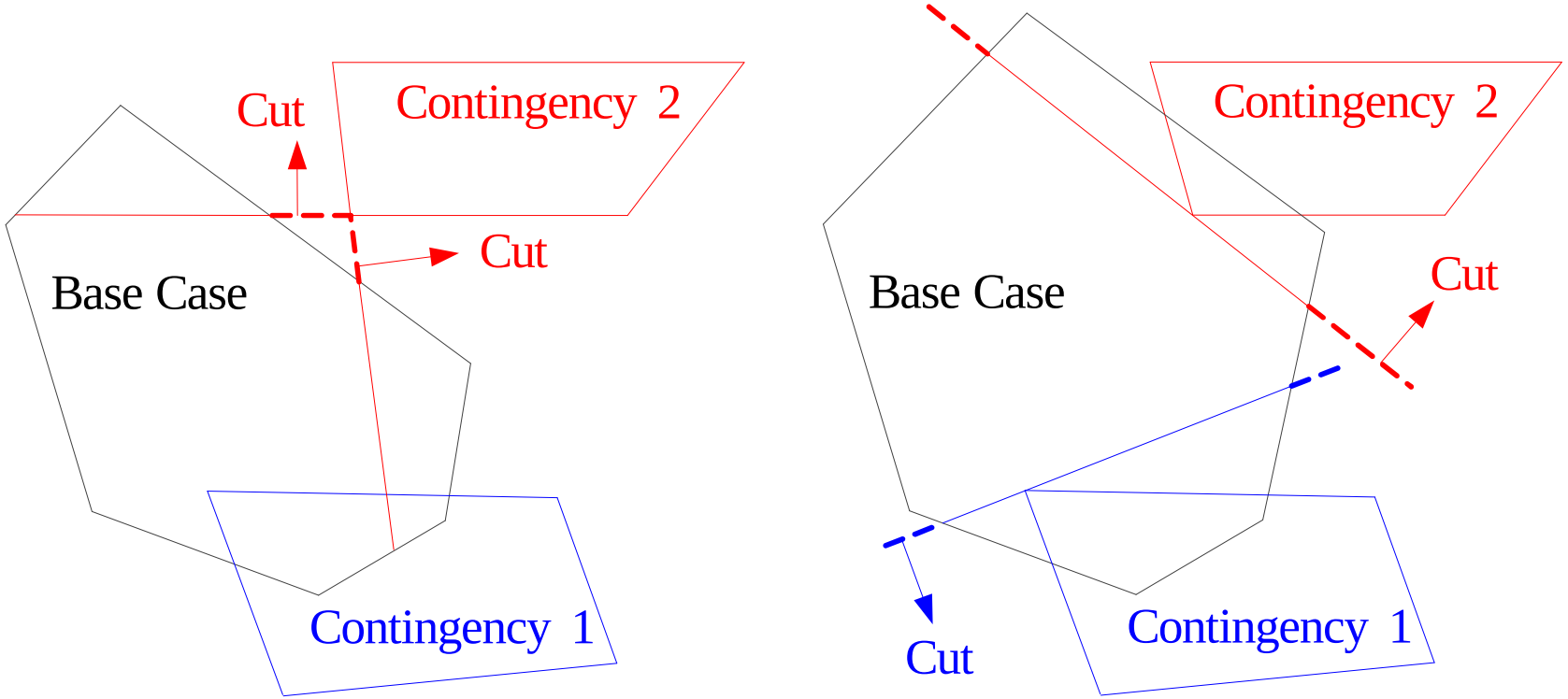
Computational Results

Case	Ctgcy	RedLP+Opt			Paraguss (8)			Fatmaster (5)		
		Iter	LPs	Time	Iter	LPs	Time	Iter	LPs	Time
118-bus	183	10	764	72.6	14	776	15.1	12	755	13.5
2383 wp	20	46	115	99.8	48	117	95.4	11	60	41.5
2383 wp	50	48	193	160.3	48	193	101.7	11	135	46.5
2383 wp	100	33	289	226.0	32	288	96.3	12	245	79.4
2383 wp	400	35	953	913.3	38	956	218.0	13	879	197.8

Case	Ctgcy	RedLP+Opt			Paraguss (40)			Fatmaster (5)		
		Iter	LPs	Time	Iter	LPs	Time	Iter	LPs	Time
2383wp	2349	106	12123	12165	104	9788	770	21	9529	516
2736sp	2749	45	5543	5836	44	5542	366	4	5500	221
2737sop	2753	1	2753	2801	1	2753	100	1	2753	101
2746wop	2794	1	2794	3046	1	2794	118	1	2794	119
2746wp	2719	262	8646	9738	278	8622	1428	14	5558	334

- Big LP for 2383-bus 2349-contingency case generates a 18GB LP. CPLEX could not solve it in 3 hours.
- Computer used for the lower table: Dell R710 (opt-a006) 2 3.46G Chips 12 Cores, 288G Memory.

Dealing with Infeasibility



(a) Contingency 2 is intrinsically infeasible. Either the corresponding subproblem is infeasible or its Benders' cuts will render the master problem infeasible.

(b) Each individual contingency is feasible, but they are not simultaneously feasible. Their Benders' cuts will render the master problem infeasible.

Figure : Two cases of infeasibility.

Identifying infeasible contingencies in Benders' algorithm

- If a subproblem is infeasible (in the first iteration), the corresponding contingency is intrinsically infeasible. Remove (tabu) it.
  - Typically line failure results in an islanded load node or sub-network.
- Master problem infeasible: solve a modified master model to find the "minimal" set of problematic contingencies using sparse optimization.

$$
\begin{aligned}
\min_{x_0, u_0} \quad & f_0(x_0, u_0) + \sum_{k \in K} M v_k \\
\text{s.t.} \quad & g_0(x_0, u_0) = 0, h_0(x_0, u_0) \leq 0 \\
& \bar{w}_k^i + \bar{\lambda}_k^i (u_0 - \bar{u}_0^i) - v_k \leq 0, v_k \geq 0 \qquad \forall (k, i) \in \text{CUT}
\end{aligned}
$$

  - Solution of this model indicates the violated cuts.
  - Tabu the contingency that has contributed one or more violated cuts.
- Start a pre-screening daemon in parallel when the Active List size is smaller than L^{fc}.
  - Tabu infeasible ones, and add feasible ones to the master problem.

Computational Results

Table : Solution for big cases on opt-a006, 80 threads, $L^{\text{fc}} = 5$

Case	Ctgcy	Iter	LPs	Time	To Master	Tabu
2383 wp	2896	15	7694	522.1	6	547
2736 sp	3269	4	6020	252.9	1	520
2737 sop	3269	4	6023	242.2	0	516
2746 wop	3307	4	6102	280.2	0	513
2746 wp	3279	8	6053	334.3	4	560
2383 wp	2353	16	7156	460.6	6	4
2736 sp	2749	4	5498	245.9	1	0
2737 sop	2753	1	2753	110.8	0	0
2746 wop	2794	1	2794	131.7	0	0
2746 wp	2719	14	5558	354.4	4	0

- Upper: all lines are in the Contingency List (N-1 security).
- Lower: all pre-screened lines are in the Contingency List.

SCED with SDP subproblems

- Economic dispatch is a short-term planning problem, so a "DC" model is OK.
- Contingency response is an operational problem, and should be studied on full AC network representation.
- But AC power flow gives a nonconvex problem, which cannot generate valid cuts from a Benders' subproblem.

Idea

Relaxing the AC feasibility problem using semi-definite programming (SDP) to obtain a convex subproblem.

Goal

Producing a base-case dispatch solution such that contingencies are "really" controllable in the AC context.

SDP relaxation of AC feasibility problem

Model ACF-SDP:

$$
\begin{array}{lll}
\min\limits_{W \succeq 0} & A_0 \bullet W & \\
\text{s.t.} & \sum\limits_{g \in \mathcal{G}_i} \underline{G}_g^{\text{real}} - D_i^{\text{real}} \leq A_{1i} \bullet W \leq \sum\limits_{g \in \mathcal{G}_i} \bar{G}_g^{\text{real}} - D_i^{\text{real}} & \forall i \in BUS \\
& \sum\limits_{g \in \mathcal{G}_i} \underline{G}_g^{\text{imag}} - D_i^{\text{imag}} \leq A_{2i} \bullet W \leq \sum\limits_{g \in \mathcal{G}_i} \bar{G}_g^{\text{imag}} - D_i^{\text{imag}} & \forall i \in BUS \\
& -\bar{F}_{i,j} \leq A_{3ij} \bullet W \leq \bar{F}_{i,j} & \forall (i,j) \in LINE \\
& (\underline{V}_i)^2 \leq A_{4i} \bullet W \leq (\bar{V}_i)^2 & \forall i \in BUS \\
& \sum\limits_{g \in \mathcal{G}_i} (G_g^0 - \Delta_g) \leq A_{5i} \bullet W \leq \sum\limits_{g \in \mathcal{G}_i} (G_g^0 + \Delta_g) & \forall i \in BUS
\end{array}
$$

- It is a convex optimization problem and weak (strong) duality holds.
- It is a relaxation because the requirement that W has rank 1 is dropped.

Experiments

Case	Cont	Solution				Performance		
		Model	Tabu	Cost	Time	IF	FS	FT
14	20	LP	0	13253.3	4.2	12	12	0
		SDP	6	16065.8	18.4	6	0	0
		SDP0	6	16003.4	11.9	6	0	0
30	40	LP	0	582.0	4.0	1	1	0
		SDP	1	585.0	20.1	1	0	0
		SDP0	1	600.5	22.1	1	0	0
57	20	LP	0	12508.0	1.9	1	1	0
		SDP	1	12508.0	13.2	1	0	0
		SDP0	1	12560.0	50.9	1	0	0
118	15	LP	0	139716.8	54.0	16	16	0
		SDP	0	141372.2	2414.3	1	1	0
		SDP0	0	144220.1	11951.1	0	0	0

- SDP subproblem is "exact" in contingency response, no False Secure, no False Tabu.
- It takes longer time to solve (with room for improvement).

Summary

1. SCED is a million-dollar problem for system operators.
2. SCED with corrective actions can save money, but is hard to solve.
   - Too big for CPLEX
   - Original Benders' decomposition algorithm is slow.
3. Our algorithmic enhancements yield significant speedup.
4. Potential for practical deployment.
5. SDP extension allows for more accurate operational modeling.

Extension

1. Decomposition approach is useful in many applications.
2. Currently in collaboration with ISO-NE to deploy our algorithm.

Conclusions

- Optimization critical for understanding of power system markets
- Different behaviors are present in practice and modeled here
- Modern optimization within applications requires multiple model formats, computational tools and sophisticated solvers
- Policy implications addressable using MOPEC
- Stochastic MOPEC models capture behavioral effects (as an EMP)
- Extended Mathematical Programming available within the GAMS modeling system
- Modeling, optimization, statistics and computation embedded within the application domain is critical

What is EMP?

Annotates existing equations/variables/models for modeler to provide/define additional structure

- equilibrium
- vi (agents can solve min/max/vi)
- bilevel (reformulate as MPEC, or as SOCP)
- disjunction (or other constraint logic primitives)
- randvar
- dualvar (use multipliers from one agent as variables for another)
- extended nonlinear programs (library of plq functions)

Currently available within GAMS