Detecting Topological Inefficiency

Seth Blumsack

John and Willie Leone Family Department of Energy and Mineral Engineering

The Pennsylvania State University

(Currently on leave at the Santa Fe Institute)

Credit and no blame: Luis Ayala (Penn State), Clayton Barrows (NREL), Russell Bent (LANL), Temitope Phillips (Chevron)

LANL Winter Grid Conference

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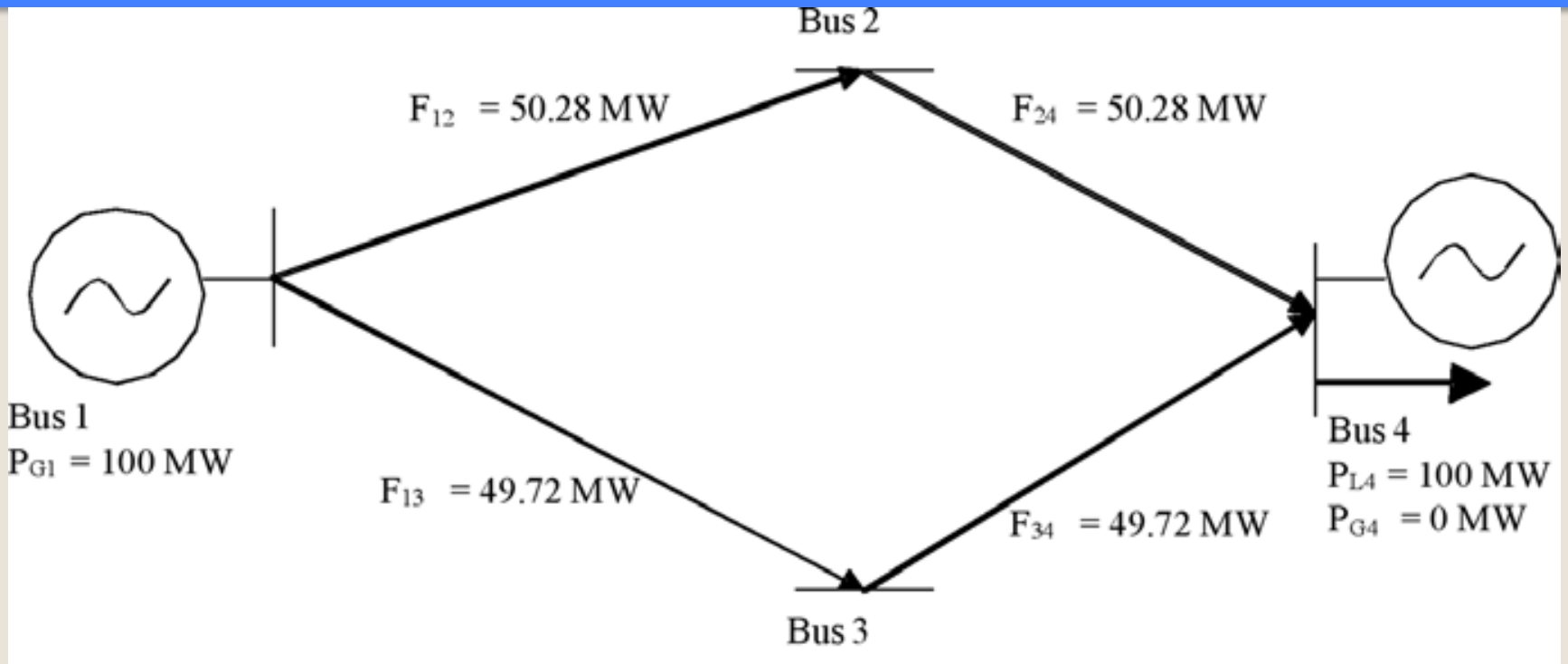
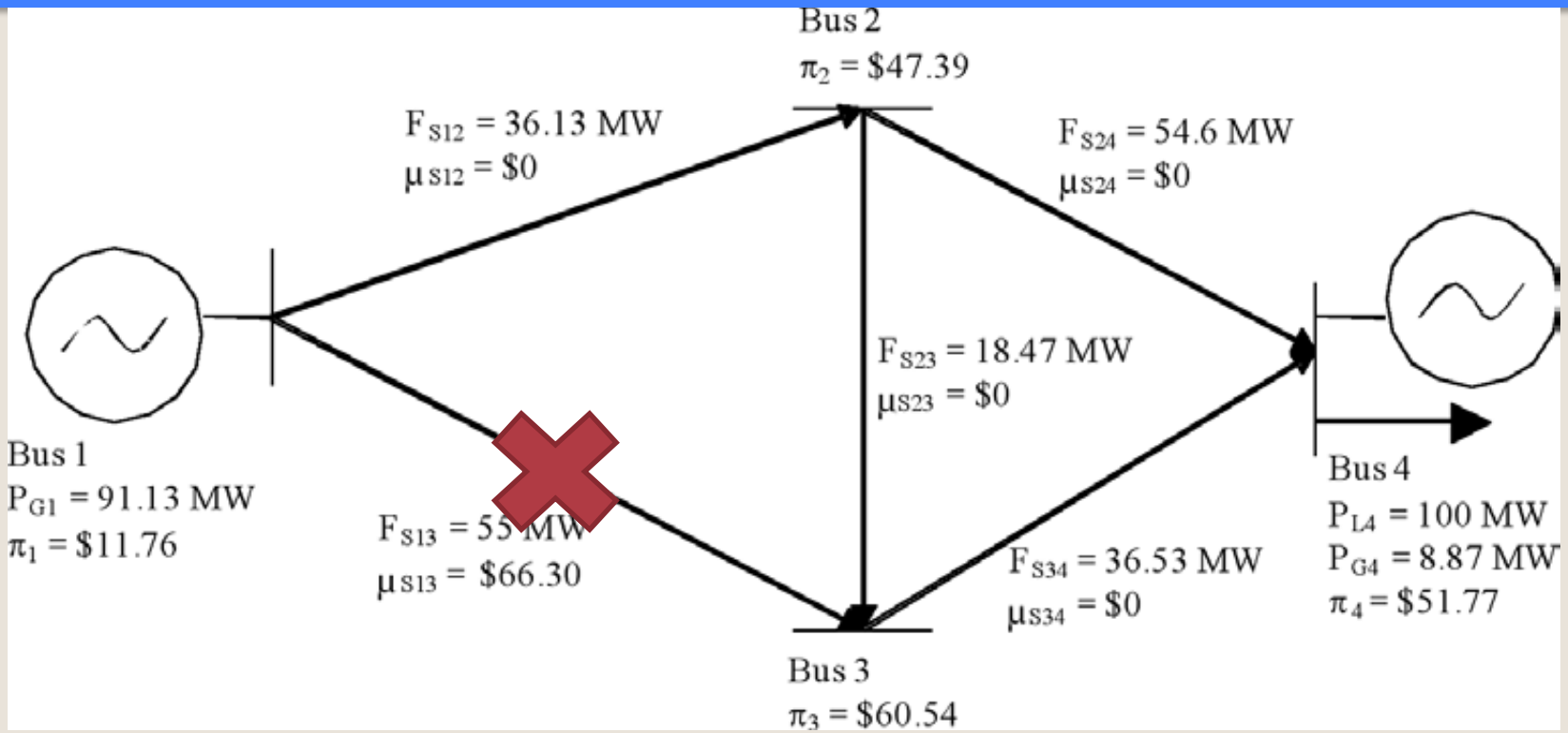
Motivating (i.e. Unrealistic) Toy Example



Rules of the toy example:

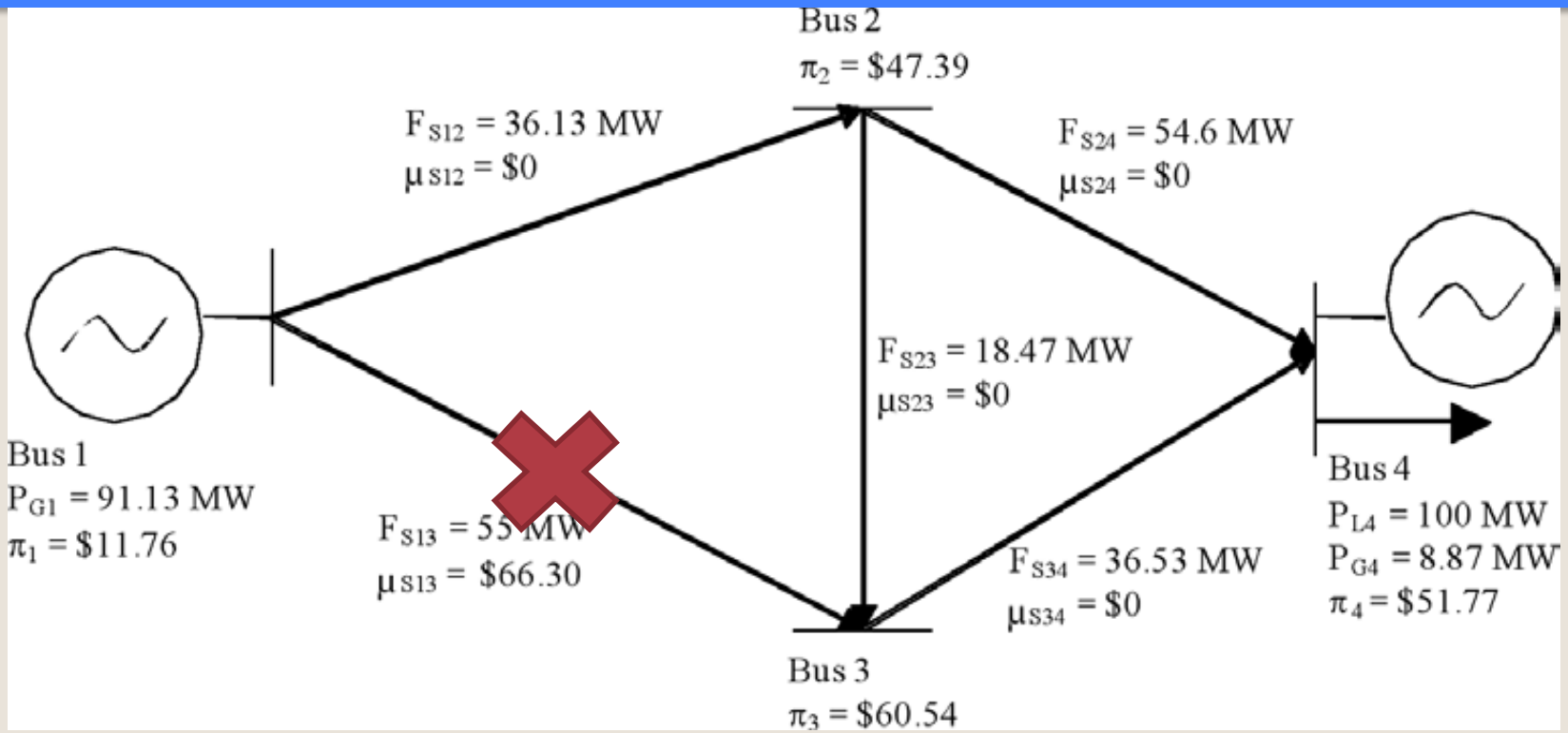
1. Cheap generation at node 1; expensive generation and customers (100 MW) at node 4;
2. All lines can carry the same fixed load (55 MW);
3. Parallel edges have the same resistance.

Building More is Not Always Better



- The link between buses 2 and 3 overloads line (1,3)
- Congestion -> Out of merit dispatch -> Higher system cost

Building More is Not Always Better



This is a cutesy example of "Braess' Paradox" in an electric transmission circuit.

(Reviewer #3: is this really a Paradox, or just Kirchhoff's Laws coming back to bite us?)

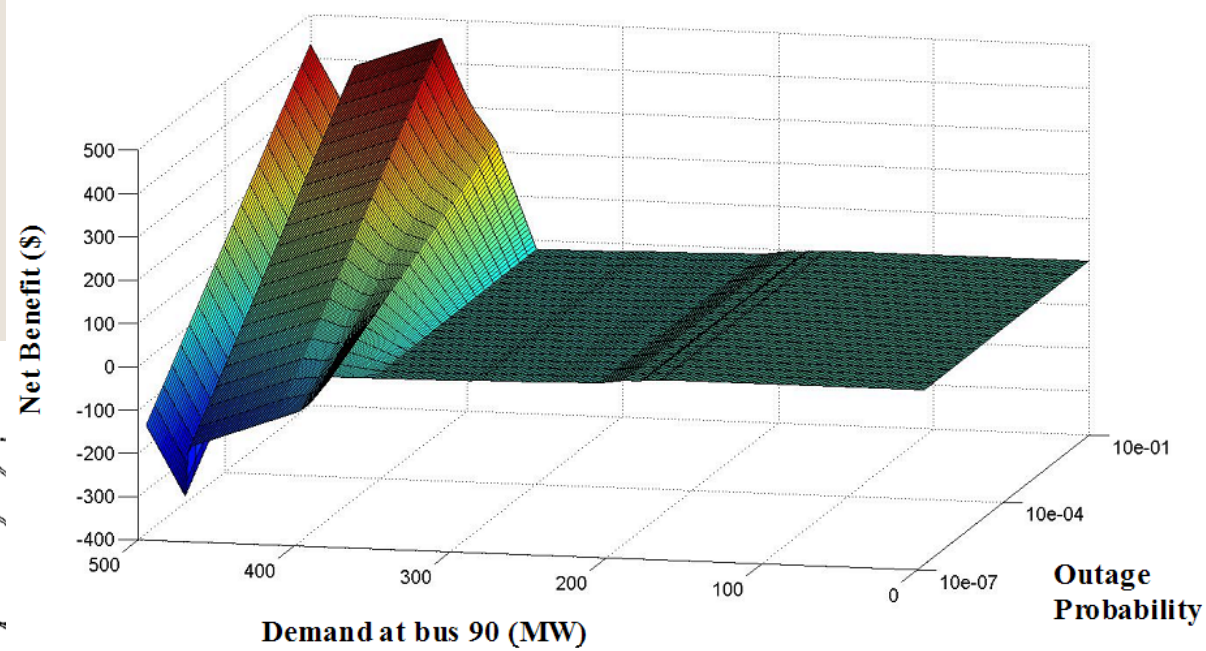
All Braess, All the Time

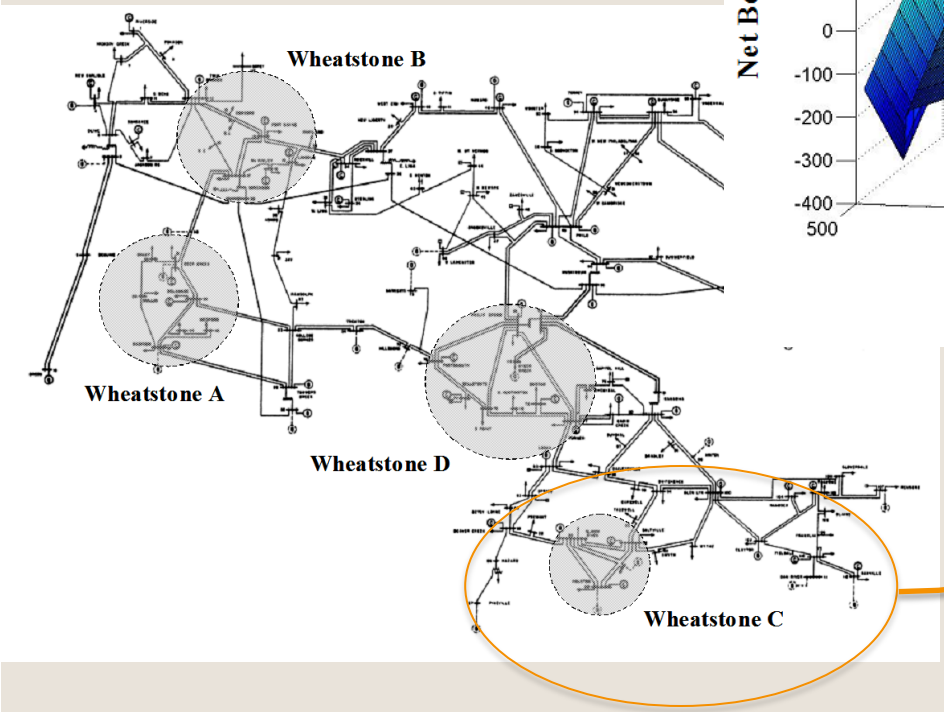
Braess (1968): Traffic paradoxes

Every system that could possibly exhibit behavior remotely Braess-like:

- Computer networks (Korilis, Lazar, Orda; 1997, 1999);
- General pipes (Calvert and Keady, 1991);
- Springs (Penchina and Penchina, 2003);
- Semiconductors (Pala, et al., 2012);
- Biological Cell Networks
- Crowd Control (Hughes, 2003);
- Basketball Teams (Simmons, 1999);
- Multi-agent Systems (Wolpert, 2002);
- Newcomb's Problem (Irvine, 1998);
- May be self-resolving (Nagurney, 2012);

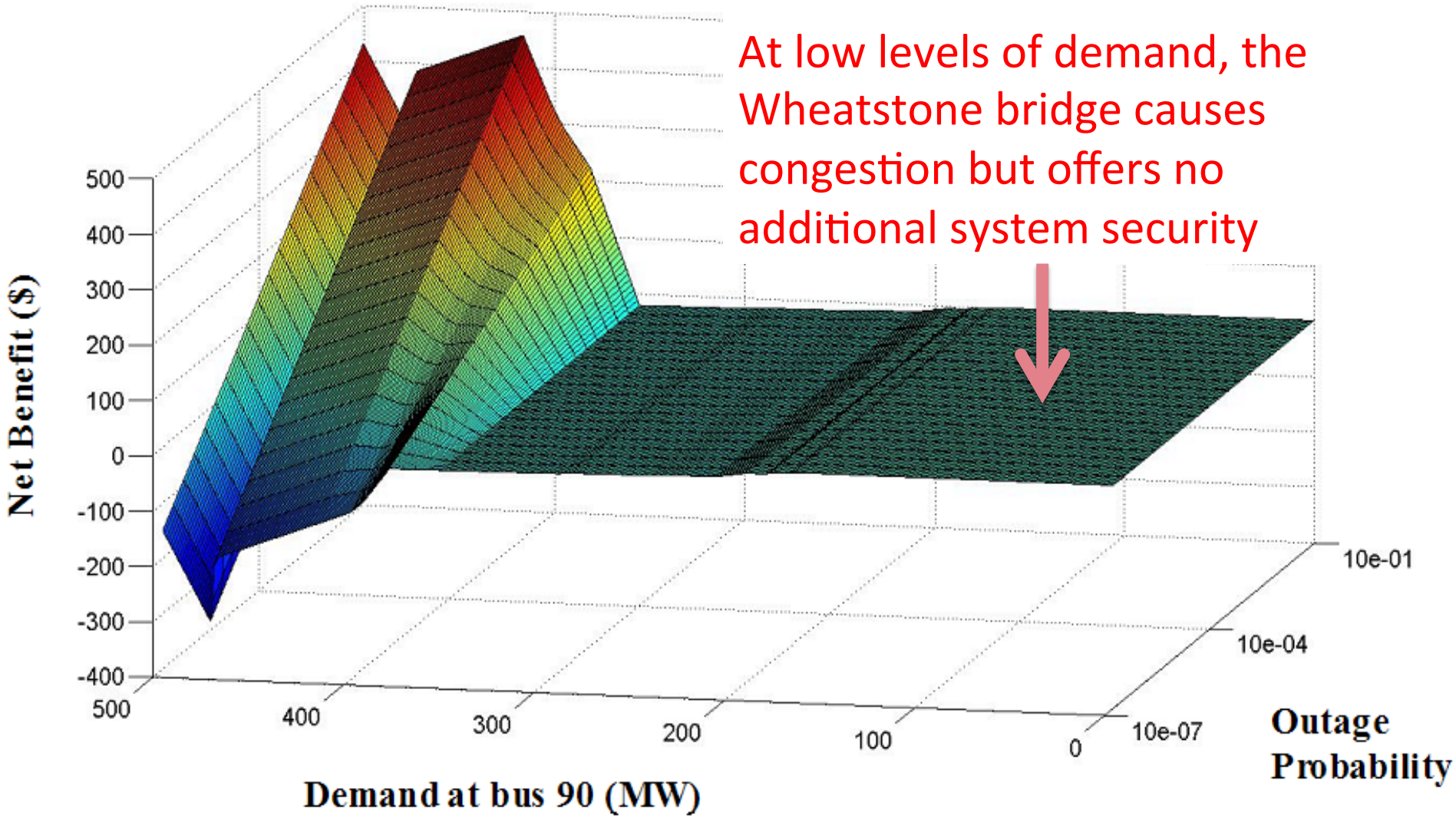
More Realistic: IEEE 118 Bus





Source: Blumsack, Lave, Ilic, *Energy Journal* 2007.

The Risk-Cost Nature of Transmission Security



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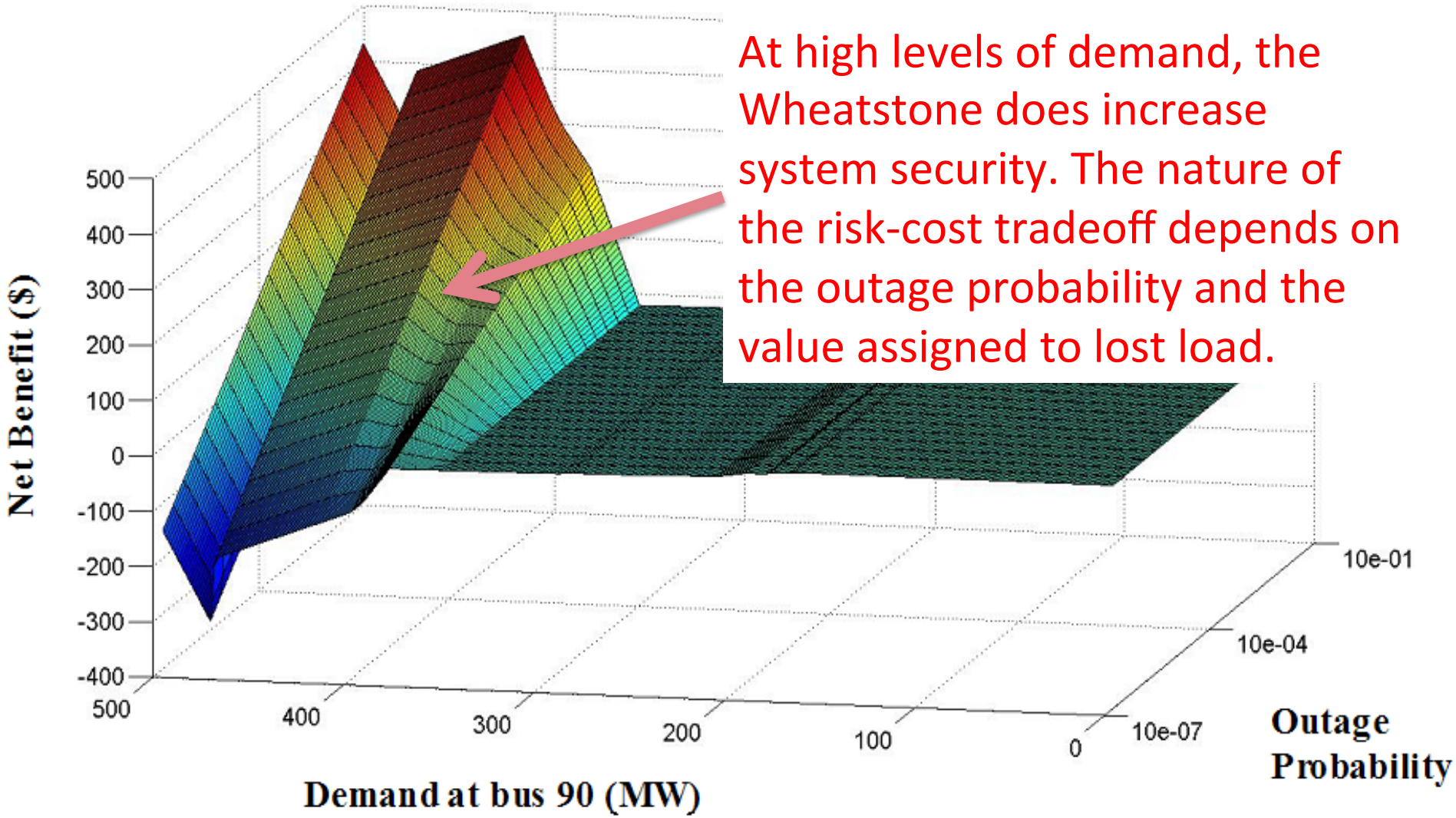
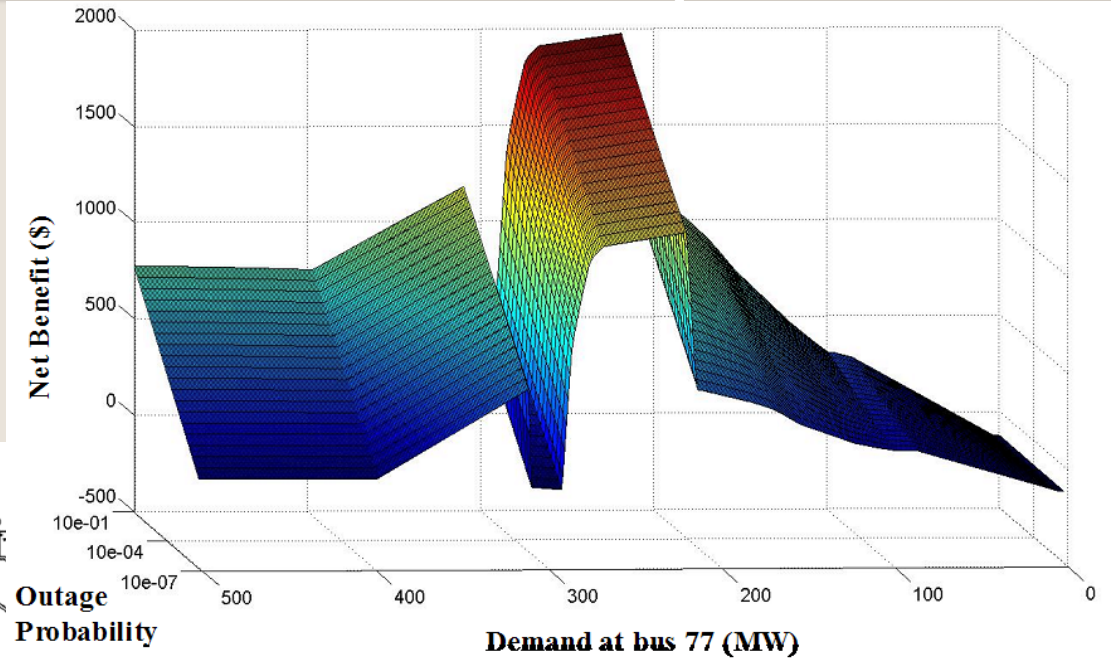
The Risk-Cost Nature of Transmission Security

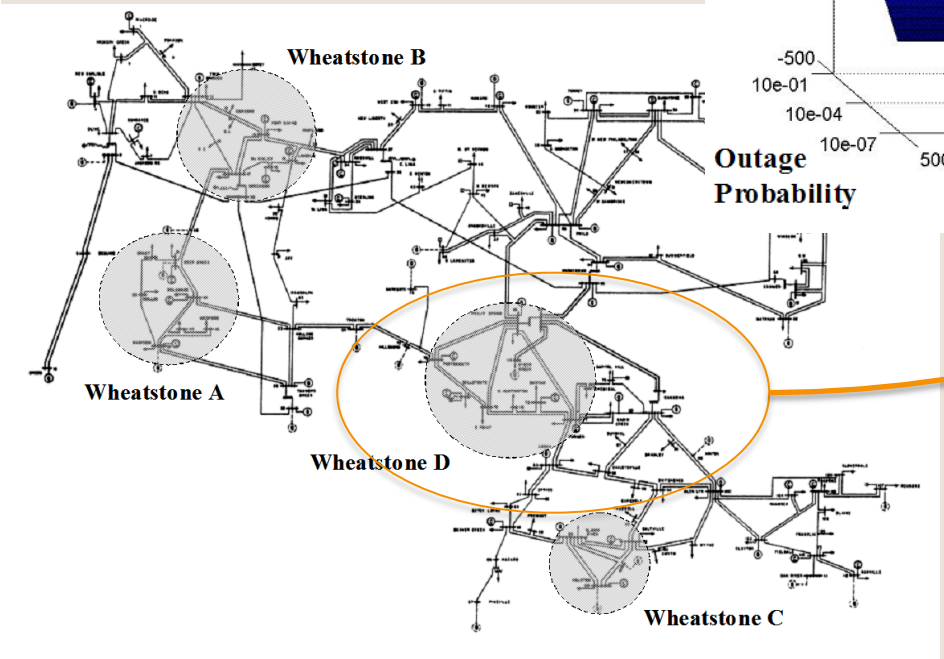


At high levels of demand, the Wheatstone does increase system security. The nature of the risk-cost tradeoff depends on the outage probability and the value assigned to lost load.

Source: Blumsack, Lave, Ilic, *Energy Journal* 2007.

The Risk-~~Cost~~ Benefit Nature of Transmission Security?





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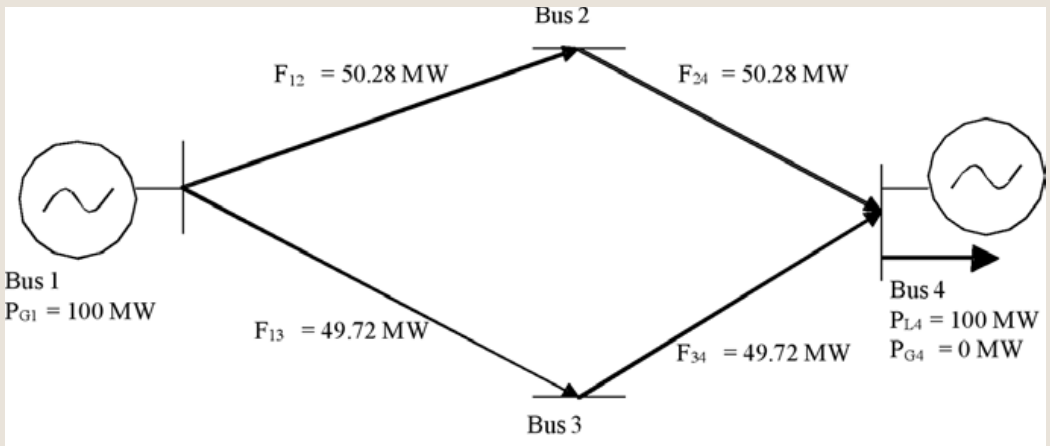
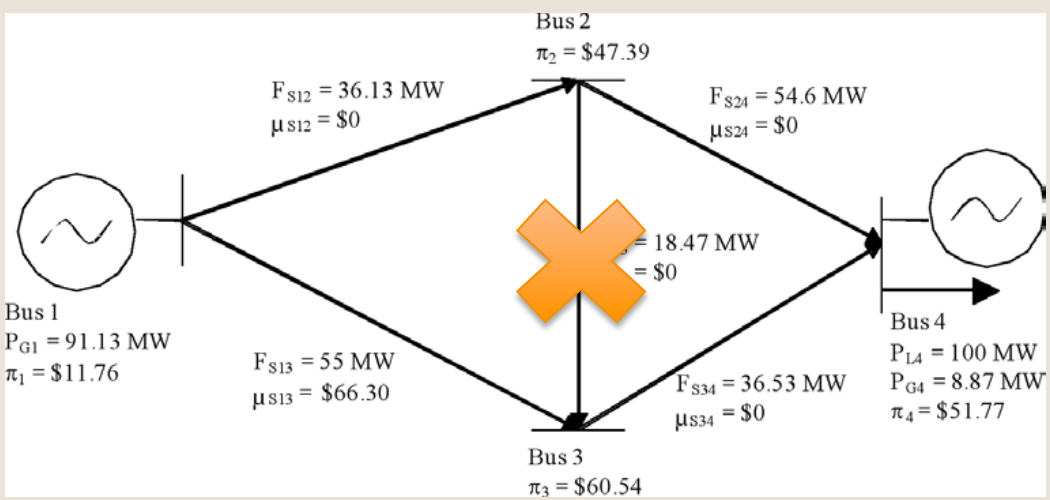
Discrete (Optimal) Topology Control





- Opening redundant circuits for economic reasons, unless a failure occurs elsewhere in the system.
- Some security cost, but hopefully not too large if done smartly.

Achieving Optimal Topology Control

- Discrete topology control is a hard optimization problem. So we could find clever new ways to solve large MILPs.

- Use off-line screening to identify areas of the network that are more likely to exhibit Braess type behavior (or to exhibit risk-cost security properties).

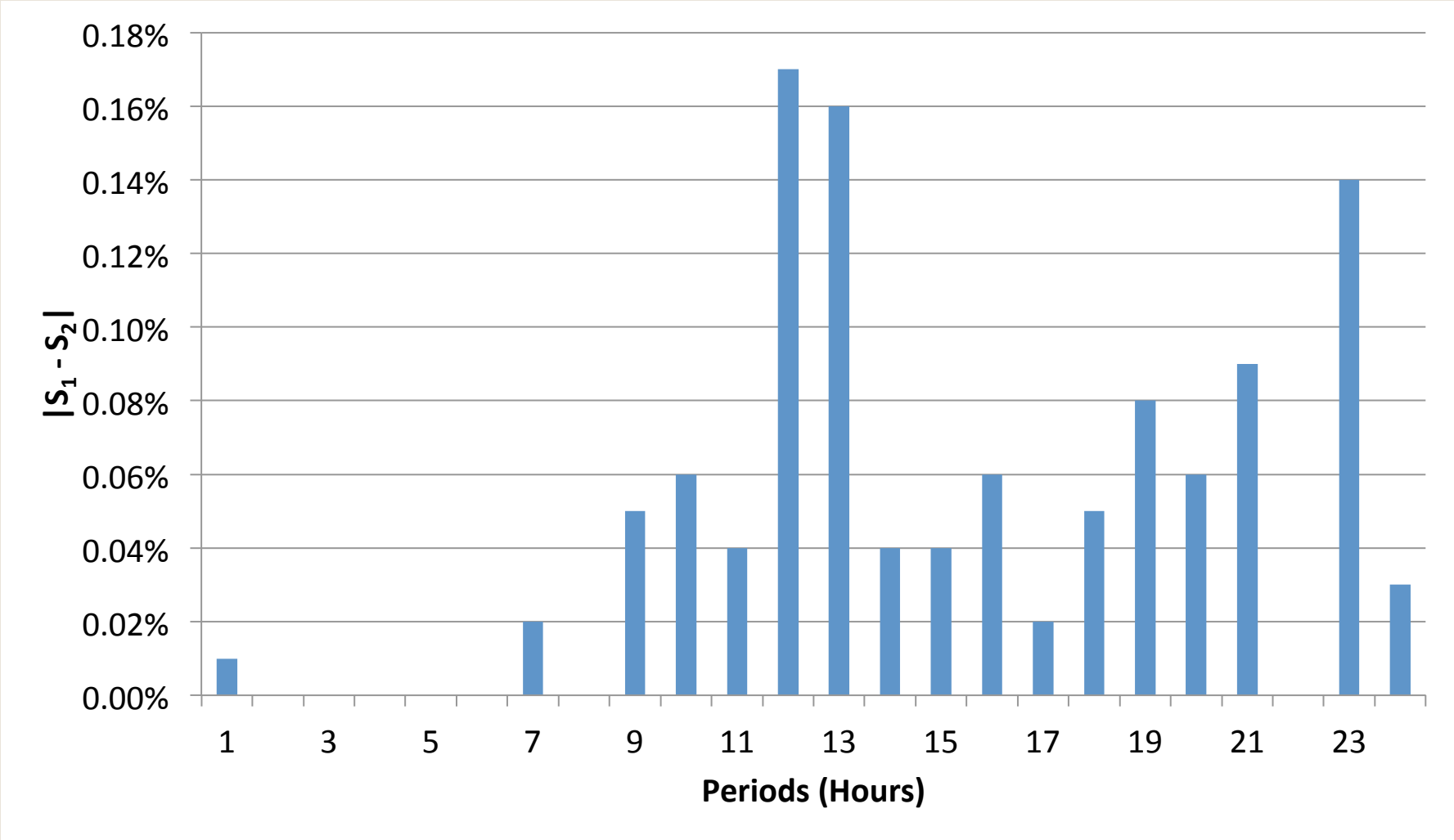
Who Needs a Big Optimization Problem?

Lines	Periods (Hours) 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
109-111	-.001	---	---	---	---	---	-.001	---	.193	.420	.463	.470	.463	.470	.470	.445	.436	.436	.387	.372	.372	---	---	---
112-113	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	.011
113-215	-.027	.000	.000	.000	.000	.000	-.027	---	---	---	-.251	---	---	---	-.252	---	---	---	---	---	---	---	-.335	---
201-202	.000	---	---	---	---	---	.000	---	---	---	---	---	---	---	-.002	---	---	---	---	---	---	---	---	-.006
209-211	---	---	---	---	---	---	.035	---	---	---	.374	---	---	---	---	---	---	---	---	---	---	---	.199	---
215-216	---	---	---	---	---	---	---	---	---	---	---	.015	.010	---	.015	-.002	---	---	---	---	---	---	---	---
215-221	---	---	---	---	---	---	---	---	---	---	---	-.024	-.024	---	---	---	---	---	---	---	---	---	---	---
217-218	---	.000	.000	.000	.000	.000	---	.017	.024	.033	.032	---	---	.032	---	---	---	---	---	.035	.035	.034	---	.016
218-221	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	.003	---	---	---	---	---	---	---	---
218-221	---	---	---	---	---	---	.000	---	---	---	---	---	---	---	.003	---	.003	---	.003	---	---	---	---	---
219-220	---	---	---	---	---	---	---	---	.025	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---
219-220	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	.050	---	---	---	---
220-223	---	---	---	---	---	---	---	---	.018	---	---	.027	.028	.027	---	.029	.030	---	---	---	---	---	---	---
220-223	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	.034	---	---	---	---
309-311	.020	---	.000	---	---	---	.020	.175	.342	.429	.450	.444	.450	.444	.444	.447	.439	.439	.412	.402	.402	---	---	.106
310-311	---	.000	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---
318-321	.000	---	---	---	---	---	---	---	-.001	---	---	-.003	-.003	-.003	-.003	-.003	---	---	---	---	---	---	---	---
318-321	---	---	---	---	---	---	.000	-.001	---	---	---	-.003	-.003	-.003	---	-.003	---	---	-.003	-.003	---	---	---	---
320-323	-.001	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	.002
320-323	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	.017	.018	---	---	---	---	---
# Sw'd[1]	6	3	3	2	2	2	7	3	6	3	5	7	7	6	7	7	4	3	5	6	3	1	2	5
Cost $k[2]	7.27	7.26	7.25	7.25	7.25	7.25	7.27	7.34	7.44	7.54	7.59	7.60	7.59	7.60	7.60	7.56	7.55	7.55	7.51	7.50	7.50	7.51	7.44	7.31
$[S_1]$[3]	-0.01	0.00	0.00	0.00	0.00	0.00	0.03	0.19	0.60	0.88	1.07	0.93	0.92	0.97	0.68	0.92	0.91	0.89	0.82	0.89	0.81	0.03	-0.14	0.13
$[S_2]$[4]	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.19	0.55	0.82	1.03	0.76	0.76	0.93	0.72	0.86	0.89	0.84	0.74	0.83	0.72	0.03	0.00	0.10

[1]#switched lines produced by Optimal Transmission Switching. [2]Total system cost of the un-switched system in thousands of dollars.
[3] S_1 represents the hourly sum of the marginal % savings of all of the switched lines. [4] S_2 is the Optimal Transmission Switching % savings.

Source: Barrows and Blumsack, *IEEE Trans. Power Sys.*, 2012

How About Little Optimization Problems?



Source: Barrows, Blumsack and Bent, *Proc. HICSS-46*, 2013

Who's on Braess?

Screening for Braess' Paradox

- Toy examples
  - Four-bus power network
  - Four-node gas pipeline network
- Larger networks
  - Electrical networks: clustering and sensitivity based screens
  - Gas networks: spanning trees

Four Interesting Observations

1. Detecting Braess' Paradox efficiently is impossible (Roughgarden, 2004);
2. For networks obeying Kirchhoff's Laws, Braess' Paradox can only be observed in Wheatstone Bridge sub-structures (Milchtaich, 2005);
3. For Hazen-Williams networks, the two-terminal Wheatstone Bridge is the simplest structure to exhibit Braess' Paradox (Calvert and Keady; 1991);
4. Every network can be decomposed into series-parallel and Wheatstone Bridge subgraphs (Duffin, 1965).

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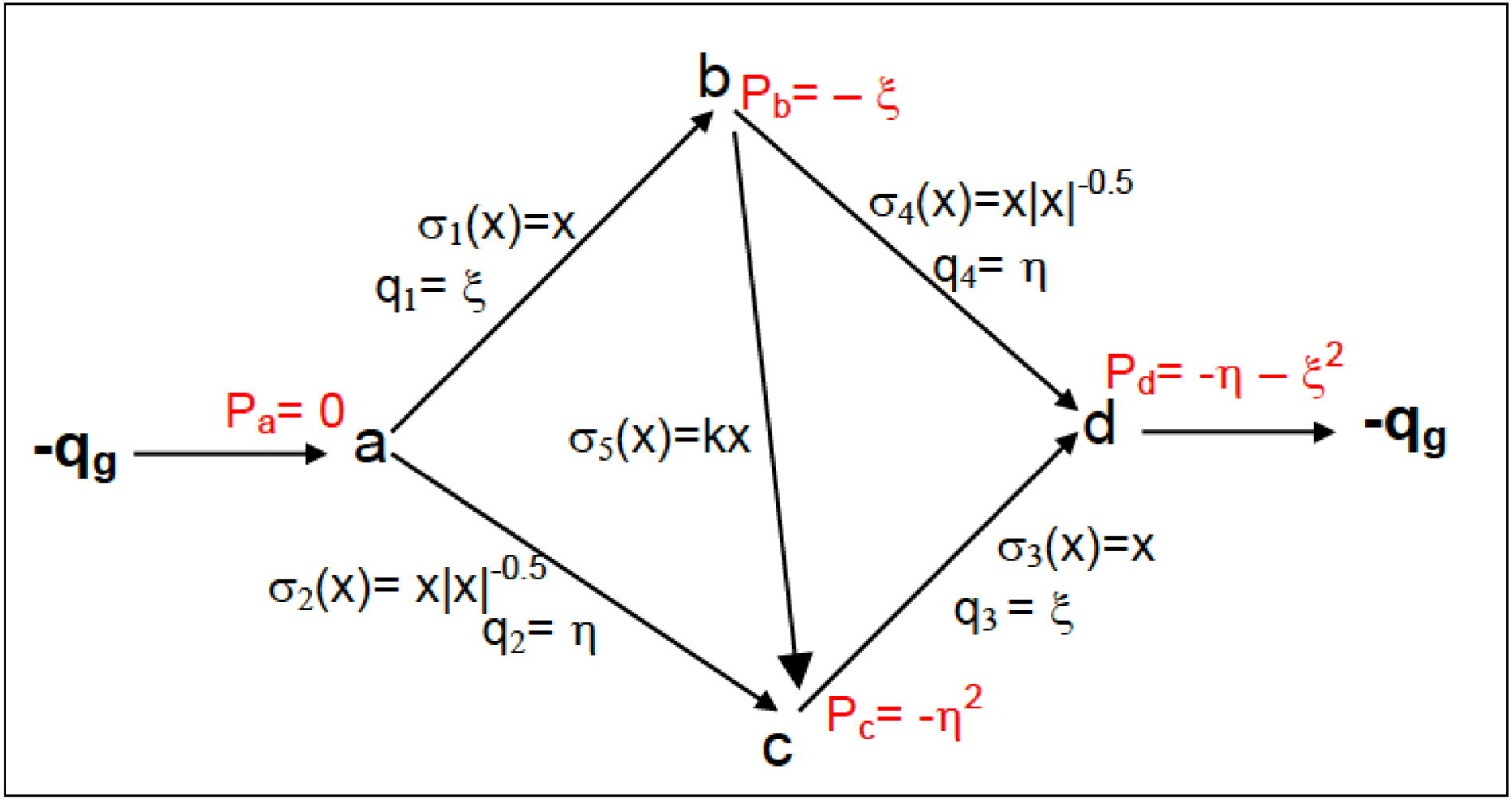
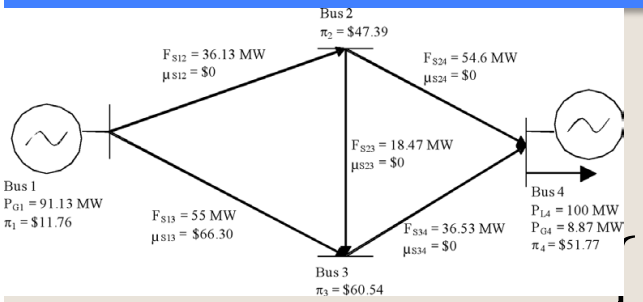
Calvert-Keady Framework



Detecting Braess: Toy Power Network



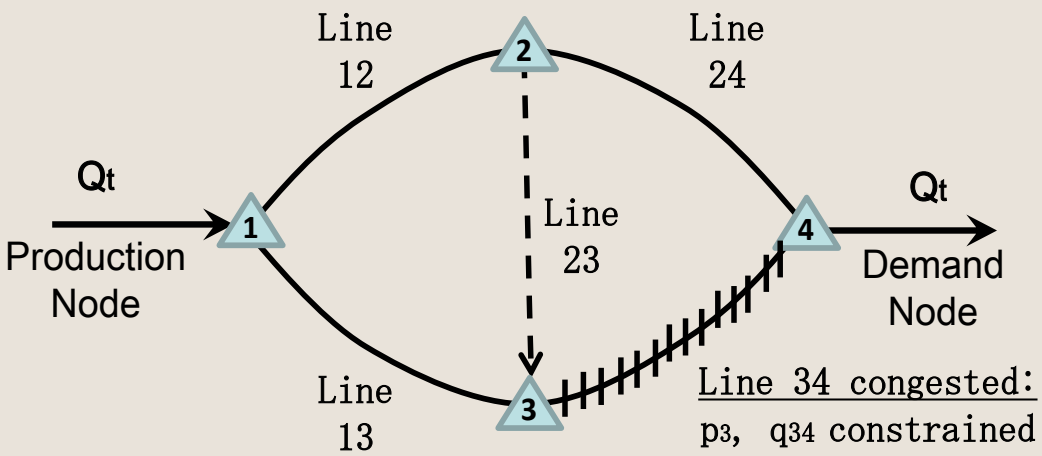
$$F_l^{new} = \begin{cases} F_l^{old} + b_k^{-1}\mathbf{A}'_l\,\mathbf{B}^{-1}\mathbf{A}_k B_l \delta_k^{old}, & l \neq k \\ \left(F_l^{old} - \Delta B_l \delta_l^{old}\right)(1 - b_l^{-1}\mathbf{A}'_l\,\mathbf{B}^{-1}\mathbf{A}_l), & l = k, \end{cases}$$

$$F_l^{old} + b_k^{-1}\mathbf{A}'_l\,\mathbf{B}^{-1}\mathbf{A}_k B_l \delta_k^{old} \geq F_l^{\max}$$

$$\Rightarrow \Delta B_k^{-1} \geq \frac{\mathbf{A}'_l\,\mathbf{B}^{-1}\mathbf{A}_k B_l \delta_k^{old}}{F_l^{\max} - F_l^{old}} - \mathbf{A}_k'\mathbf{B}^{-1}\mathbf{A}_k.$$

Source: Blumsack and Ilic. *Proc. PES*. 2008.

Detecting Braess: Toy Gas Network



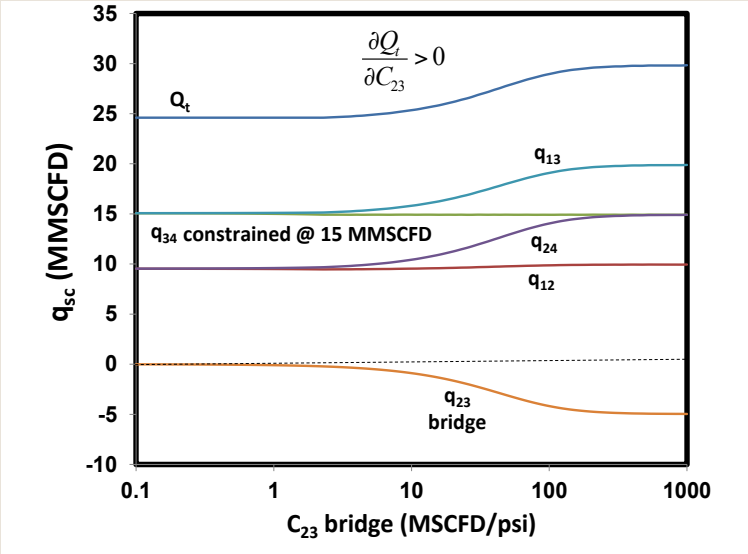
Linear conductivity analog (Ayala and Leong, 2012)

$$q_{ij} = L_{ij} \cdot (p_i - p_j)$$

$$L_{ij} = T_{ij} \cdot C_{ij}$$

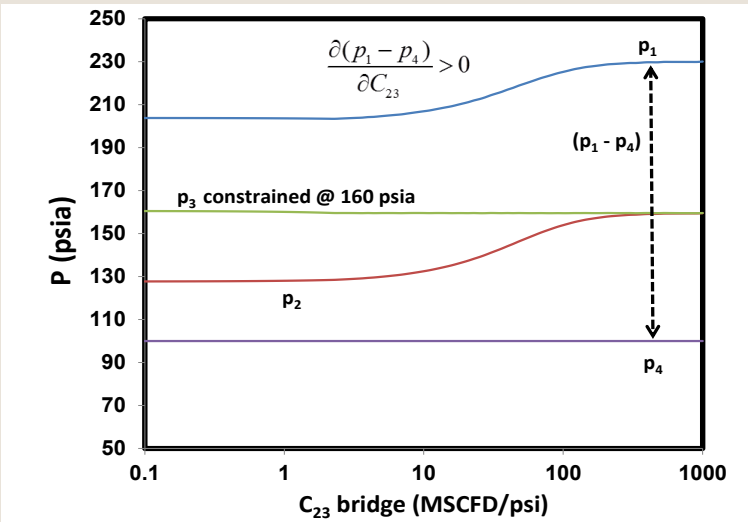
$$T_{ij} = \sqrt{1 + \frac{2}{r_{ij} - 1}}$$

Detecting Braess: Toy Gas Network



Pipe transportation capacity versus Wheatstone Bridge pipe conductivity (C_{23}):

$$\partial Q_t / \partial C_{23} > 0$$

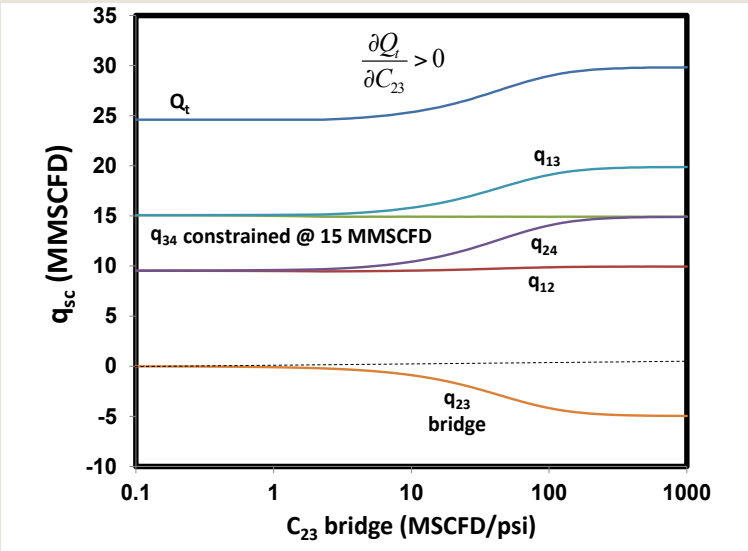


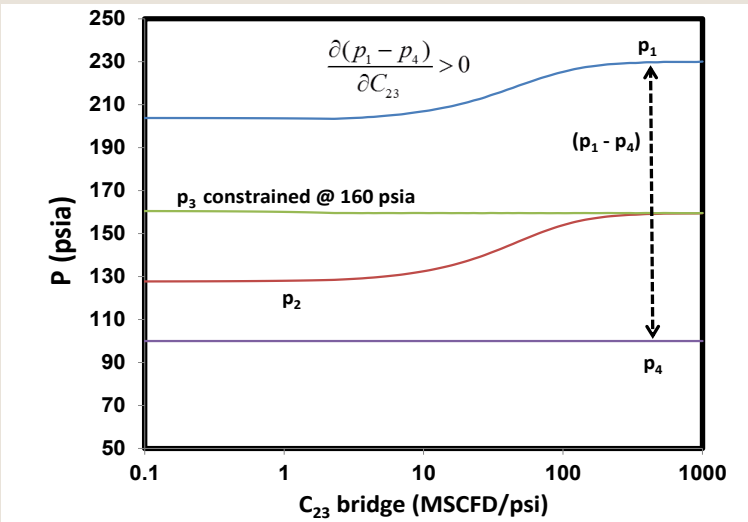
Network pressure loss versus Wheatstone Bridge pipe conductivity (C_{23}):

$$\left(\frac{\partial(p_1 - p_4)}{\partial C_{23}} \right) > 0$$

Source: Ayala and Blumsack, *Oil and Gas Facilities* 2012.

Detecting Braess: Toy Gas Network





Network condition:

$$T_{12}T_{34} \cdot C_{12}C_{34} - T_{24}T_{13} \cdot C_{24}C_{13} = 0$$

Equivalently (since parallel pipe pressure ratios are the same):

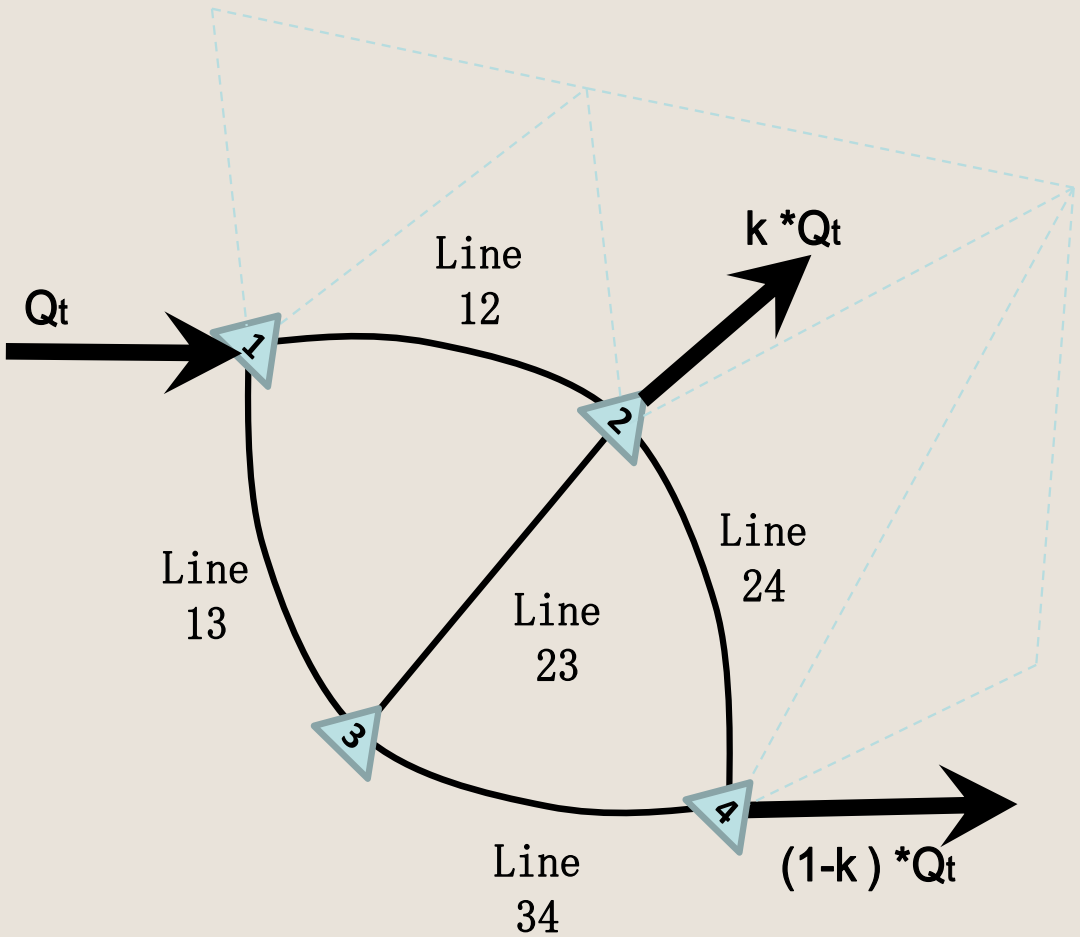
$$C_{12}C_{34} - C_{24}C_{13} = 0$$

$$C_{12}C_{34} > C_{24}C_{13} \text{ leads to } \left(\partial Q_t / \partial C_{23}\right) < 0$$

$$C_{24}C_{13} > C_{12}C_{34} \text{ leads to } \left(\partial Q_t / \partial C_{23}\right) > 0$$

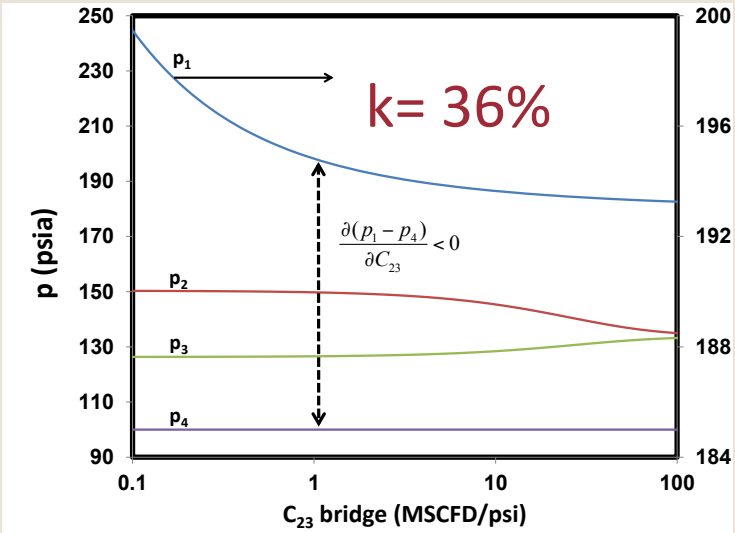
Source: Ayala and Blumsack, *Oil and Gas Facilities* 2012.

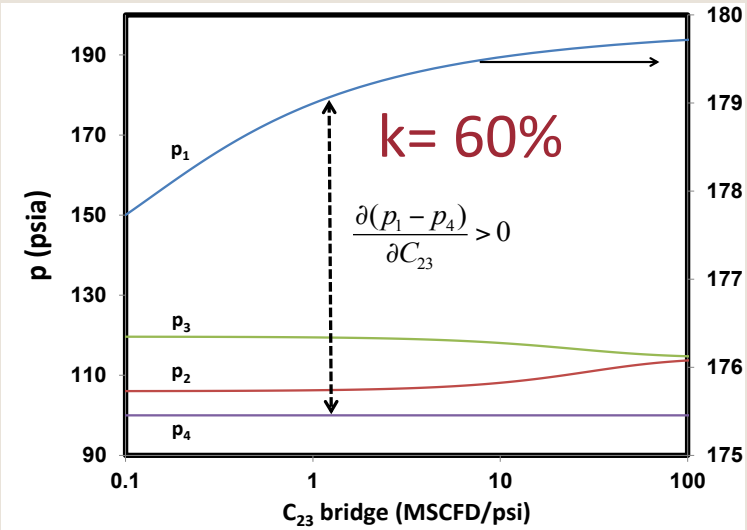
The Complexity of Braess' Paradox in Pipeline Networks



The existence of a Wheatstone Bridge topology within a larger network may induce larger pressure drops even without causing "congestion" in the pipeline system.

The Complexity of Braess' Paradox in Pipeline Networks





If the fraction of fluid take-off to one point versus another (the parameter *k*) exceeds a critical threshold:

$$k_c = \frac{L_{12}L_{34} - L_{23}L_{13}}{L_{34}(L_{12} + L_{13})}$$

$$= \frac{T_{12}T_{34} \cdot C_{12}C_{34} - T_{23}T_{13}C_{23}C_{13}}{T_{34}C_{34}(T_{12}C_{12} + T_{13}C_{13})}$$

Screening for Braess' Paradox in Large Networks via Clustering

Factoid of the day:

The clustering coefficient of the Wheatstone network is 5/6.

Second factoid of the day:

No other four-node network (with minimum geodesic path length equal to two) has the same clustering coefficient.

Clustering-Based Algorithm

Step 1: Using the node-edge adjacency matrix, reduce all simple series and parallel connections. (This step may need to be iterated.)

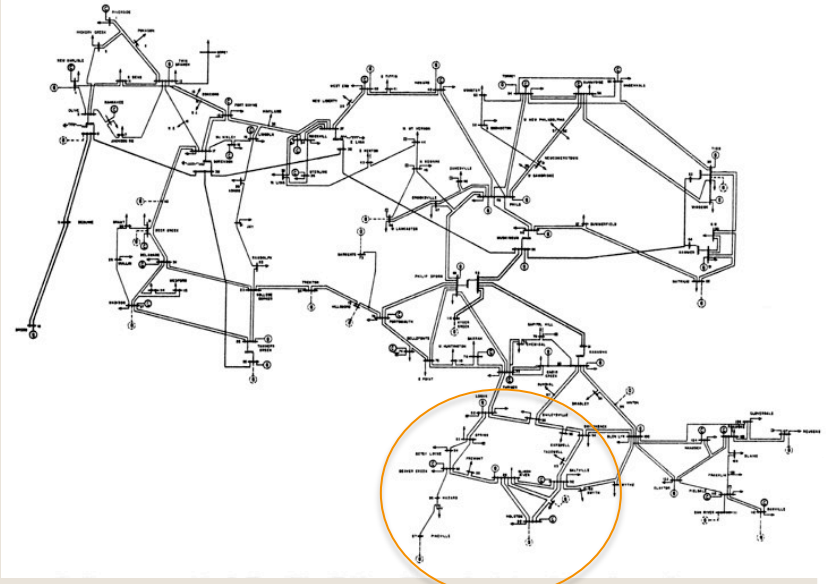
Step 2: Define R_1 as the set of all node pairs with geodesic path length two, and R_2 as a subset of R_1 such that there are two such geodesic paths.

Step 3: Calculate $WS = T \cap D \cap R_2 \cap R_3$

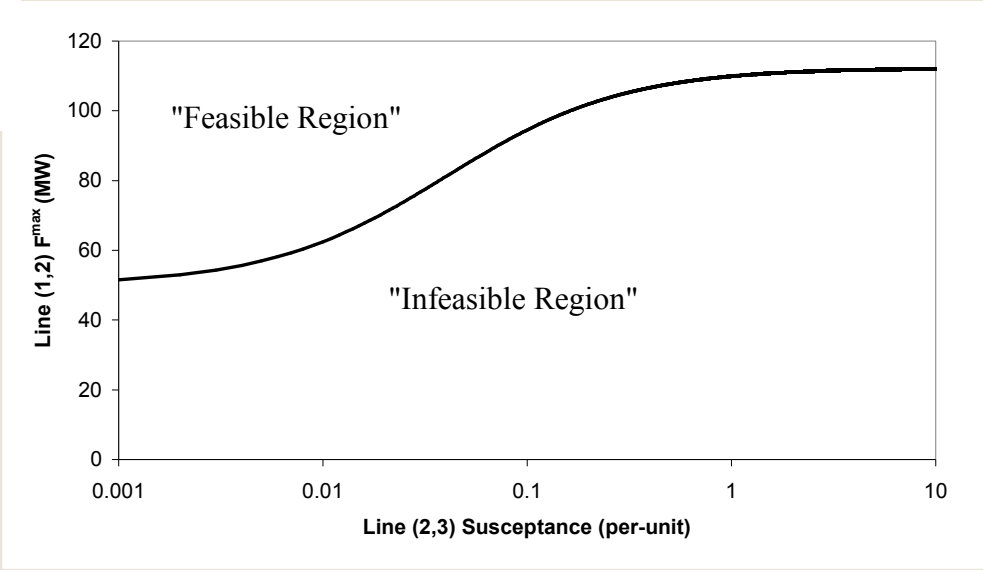
Step 4: For all node pairs in WS, construct the adjacency matrix consisting of the node pairs and all neighboring nodes.

Step 5: Calculate the clustering coefficient for each subgraph in Step 4. Those with a clustering coefficient equal to 5/6 are Wheatstone Networks

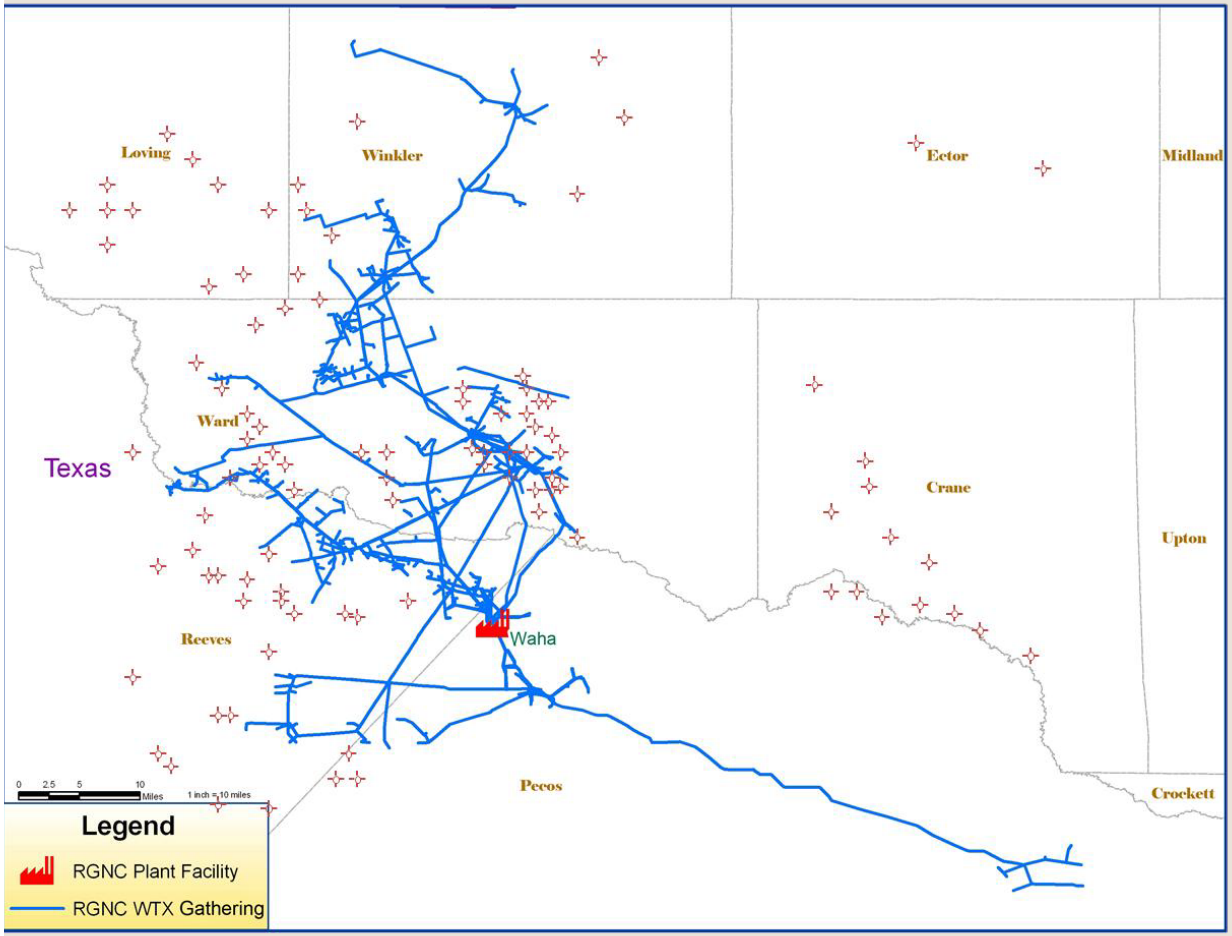
Implementation on 118 Bus Network



Clustering-based algorithm plus some network equivalencing produces screening curves like the one below.



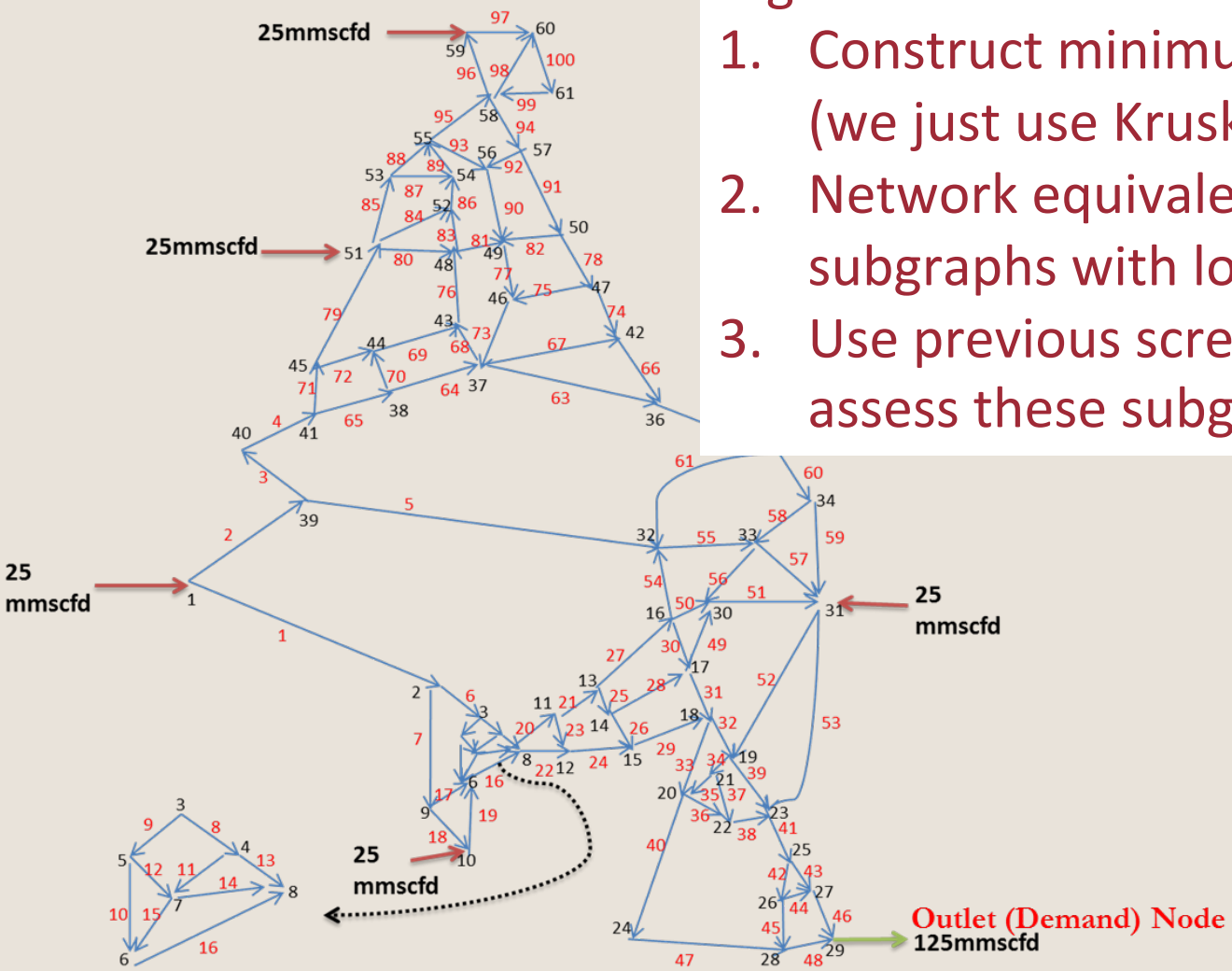
Another Approach: Spanning Trees



Waha gathering system:

- 61 nodes (some of which have compressors, others just have valves)
- 100 edges
- 5 supply nodes, one consumption node

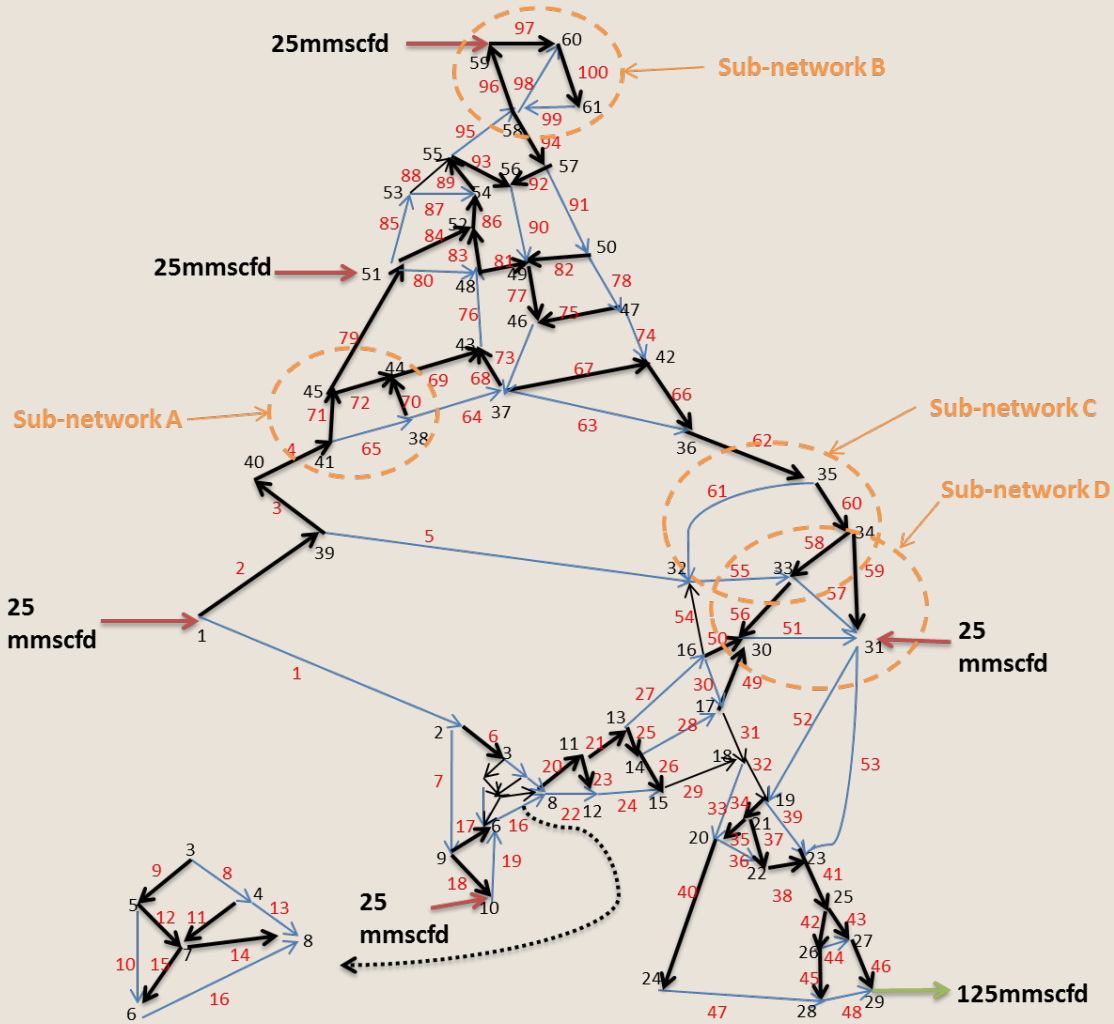
Another Approach: Spanning Trees



Algorithm:

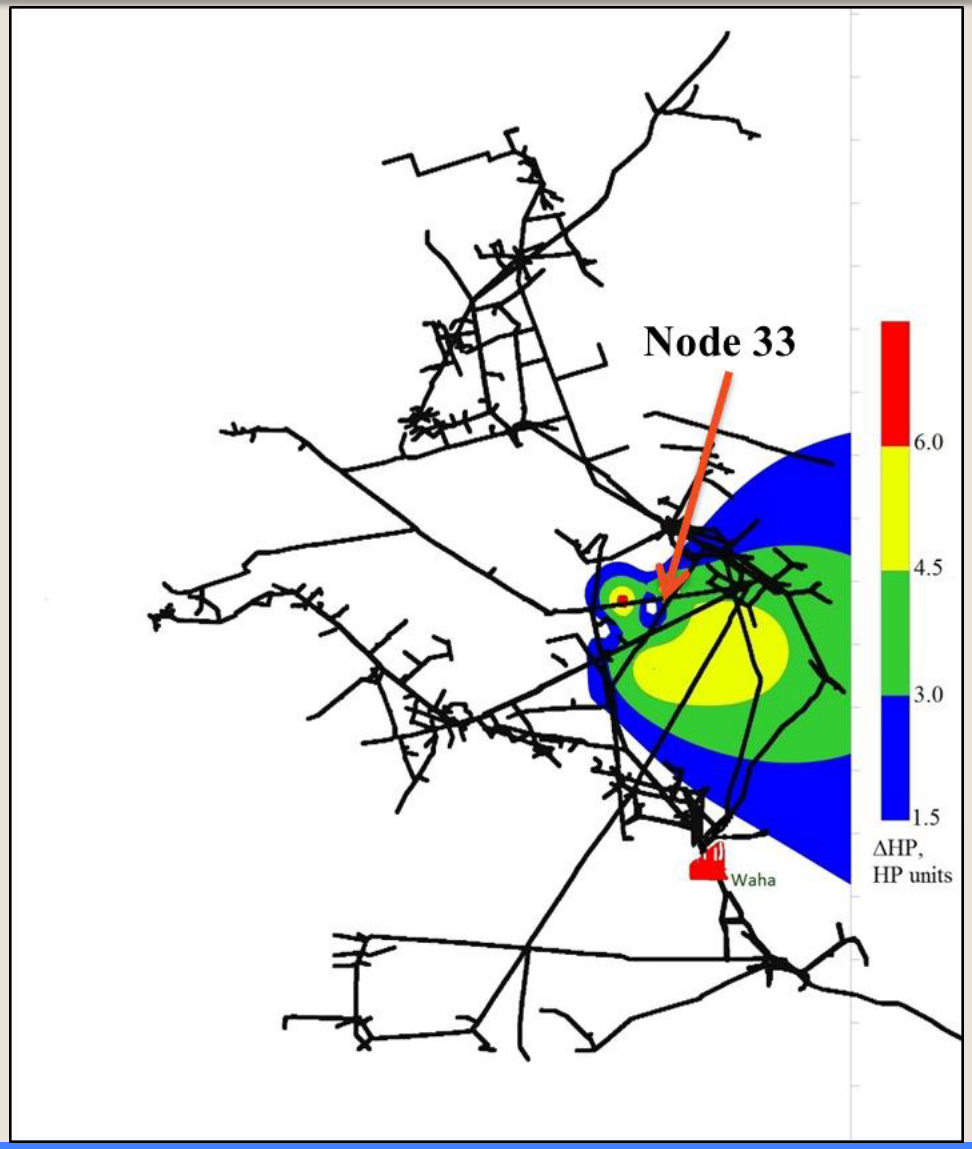
1. Construct minimum spanning tree (we just use Kruskal's method)
2. Network equivalencing to isolate subgraphs with loops
3. Use previous screening tools to assess these subgraphs

Another Approach: Spanning Trees



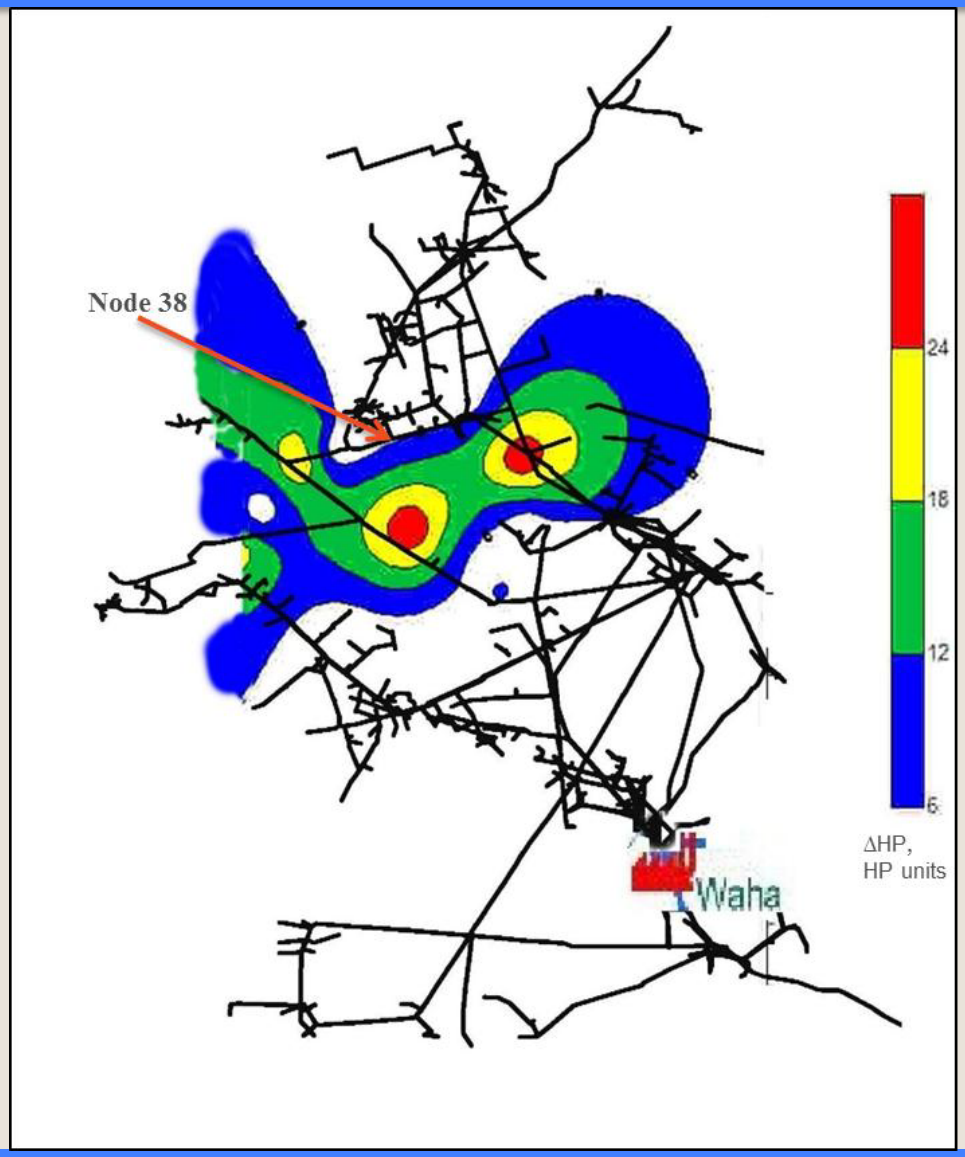
- Elements in the spanning tree are shown in bold;
- Subgraphs of interest are highlighted (there are more possible subgraphs to consider…possibly unwieldy….

The Reach of Topological Inefficiency



- Pressure is held constant at node 33.
- Multiple topological inefficiences contribute to increased horsepower requirements

The Reach of Topological Inefficiency



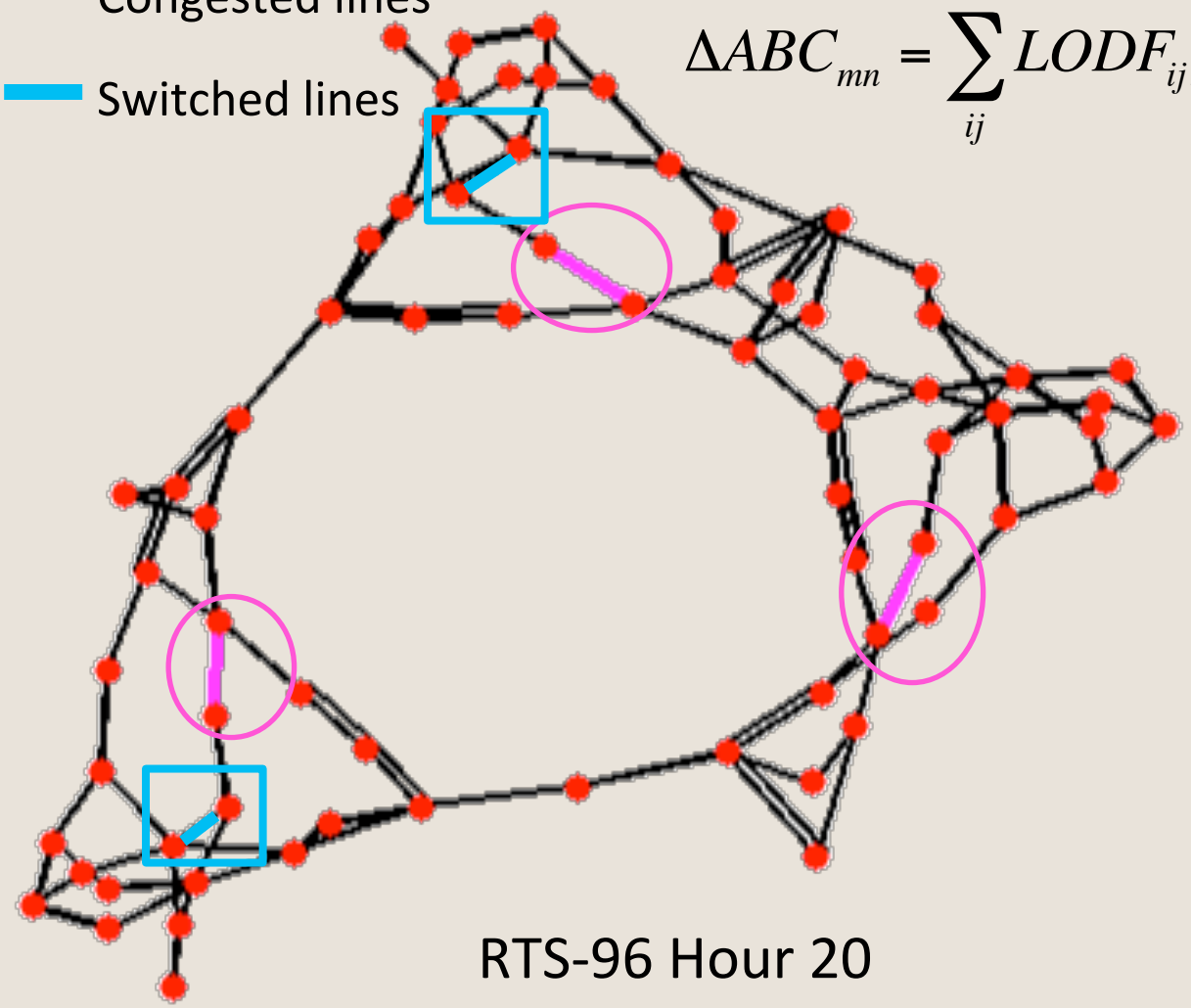
- Here, pressure is held constant at node 38.
- Note that holding pressure constant at the demand node does not itself induce any paradoxical behavior (this is probably a fluke though we aren't sure).

So What?

Congested lines

Switched lines

$$\Delta ABC_{mn} = \sum_{ij} LODF_{ij,mn} \times \left(P^*_{mn}\, \mu_{ij} - P_{mn} \mu_{ij} \right)$$

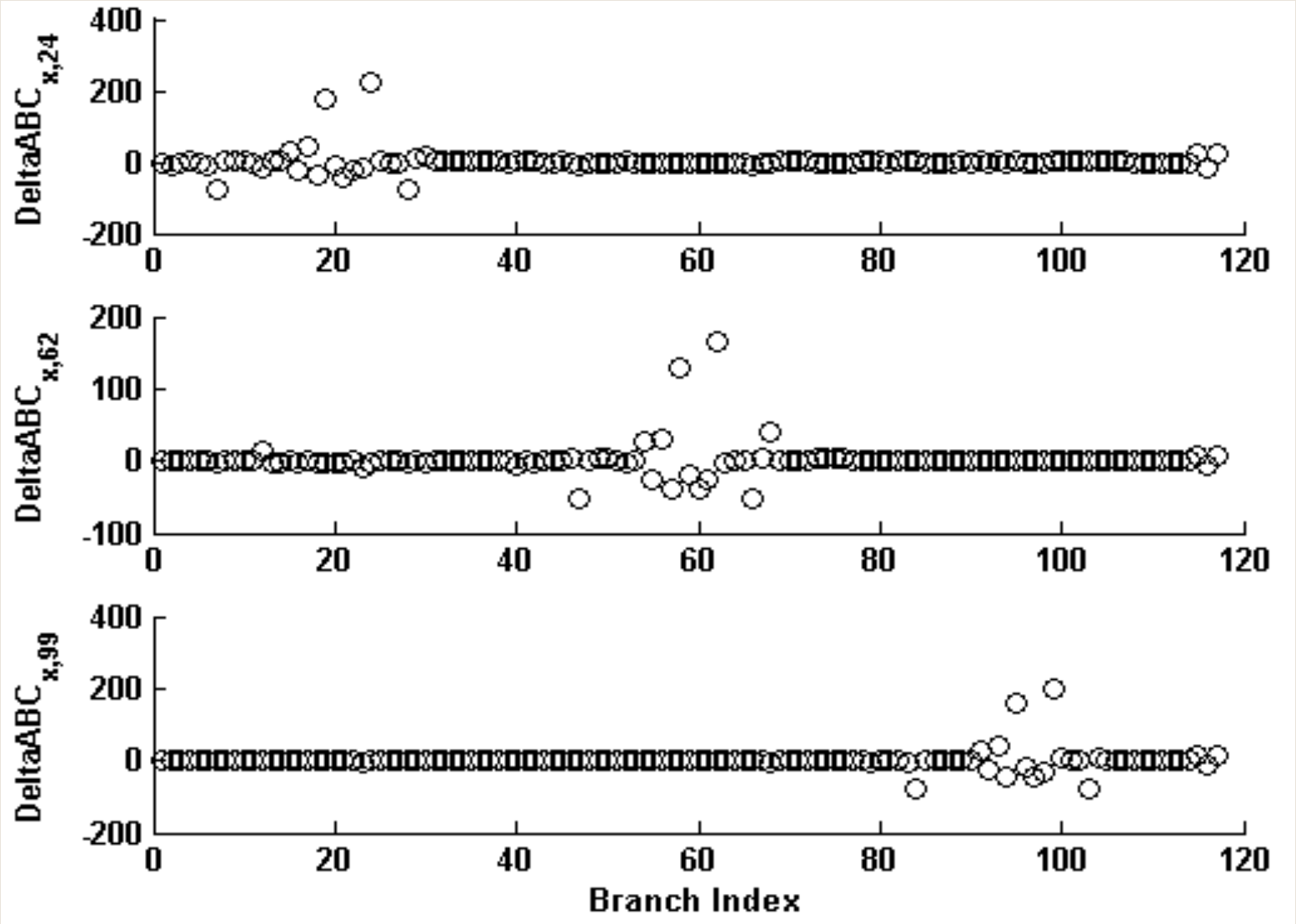


RTS-96 Hour 20

Measures re-allocation of flow from overloaded to under-utilized branches

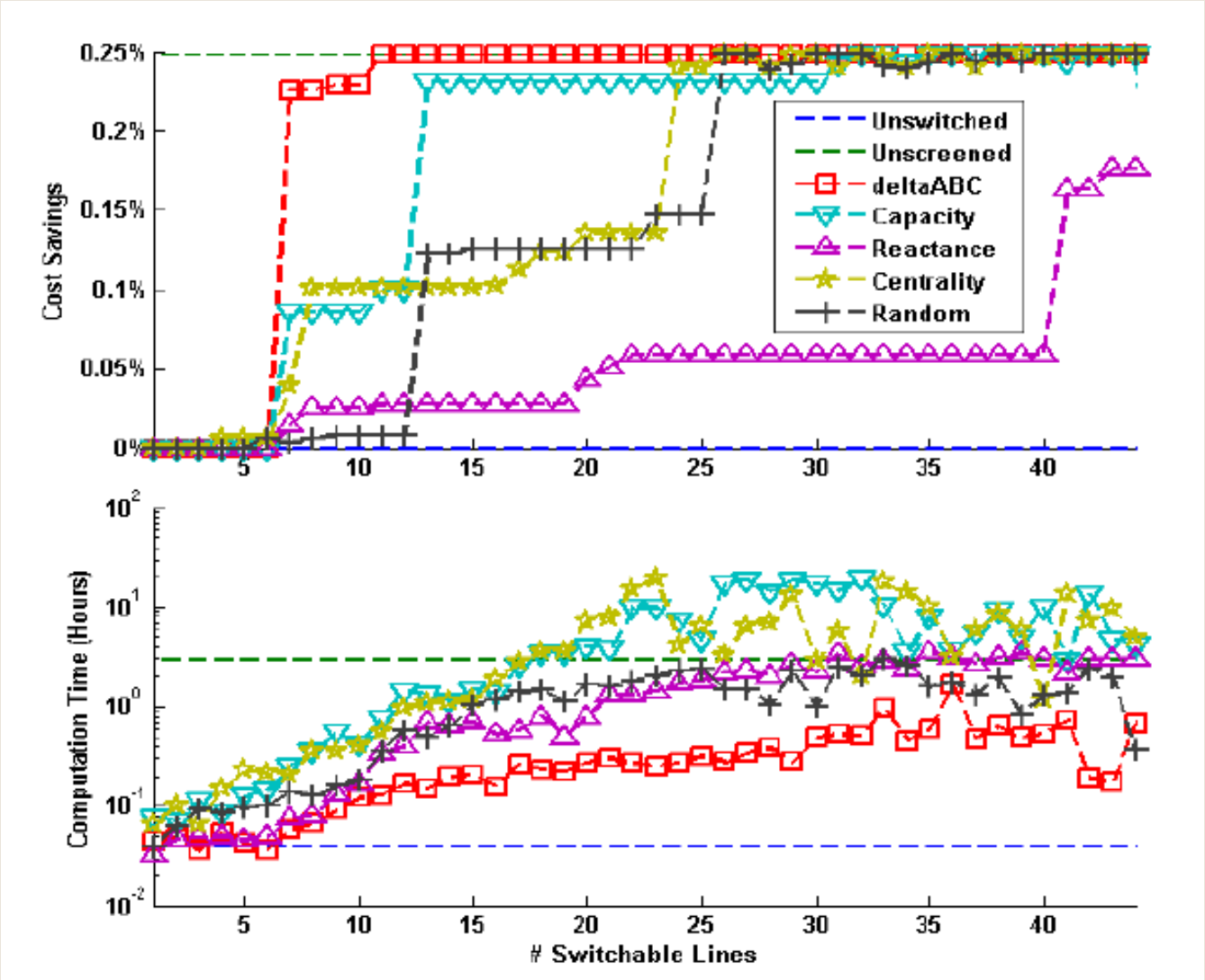
Source: Barrows, Blumsack and Bent, *Proc. HICSS-46*, 2013

Screen for RTS-96 System



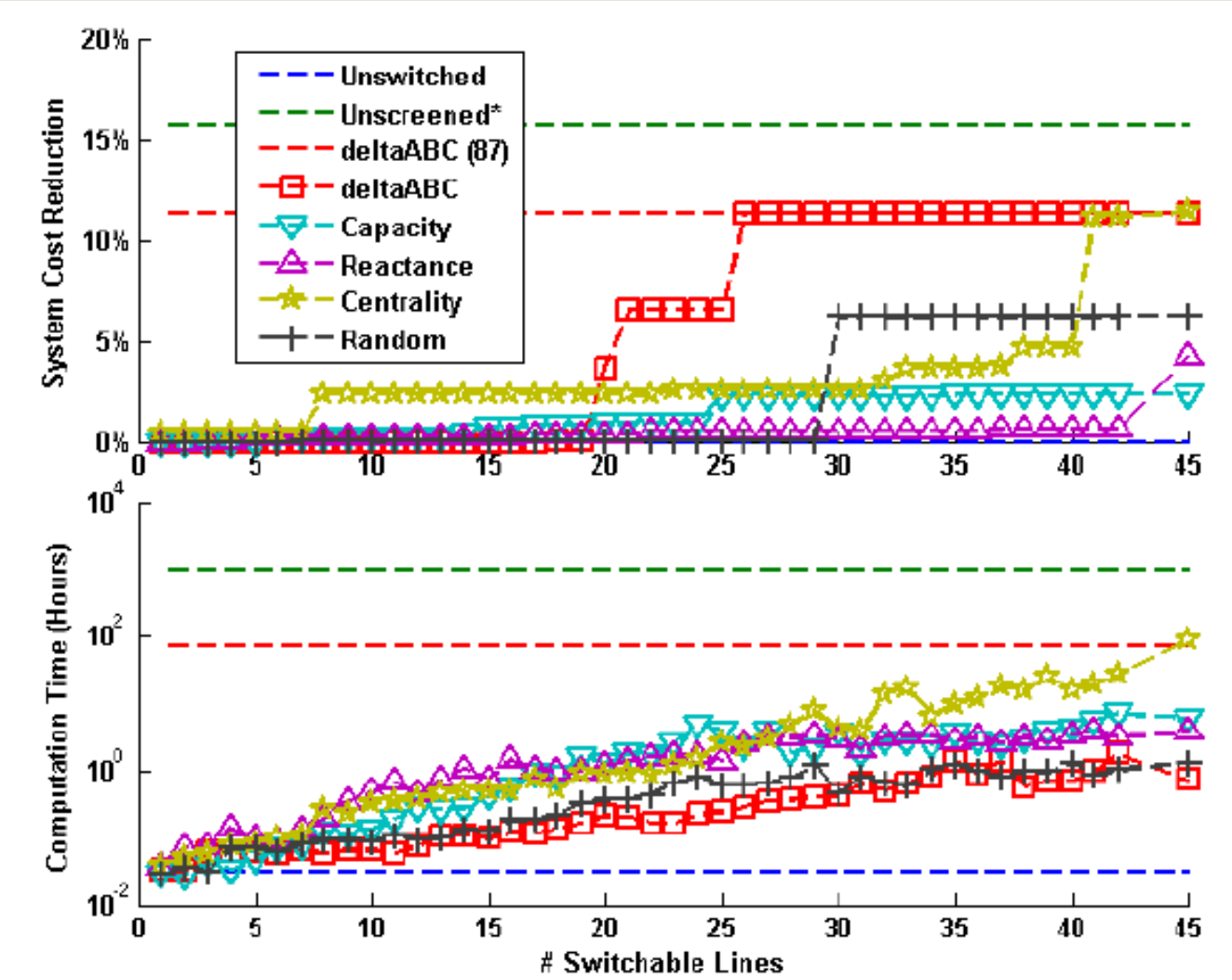
Source: Barrows, Blumsack and Bent, *Proc. HICSS-46*, 2013

RTS-96, Hour 14



Source: Barrows, Blumsack and Bent, *Proc. HICSS-46*, 2013

IEEE 118 Bus System



Source: Barrows, Blumsack and Bent, *Proc. HICSS-46*, 2013

Prospects for Topology Control

- Discrete topology control is a hard optimization problem. But there are probably clever ways to shrink the size of the problem.
- Subgraph screening is one possible way, but algorithms (mine, anyway) need improvement.
  - Some are fast but run the risk of false negatives (and false positives? we don't really know)
  - Others work well…but just aren't that efficient.

Thank You!

Seth Blumsack

blumsack@psu.edu