Optimal Resilient Distribution Grid Design

Russell Bent

Joint work with Scott Backhaus, Brent Daniel, Harsha Nagarajan, and Emre Yamangil (see poster)

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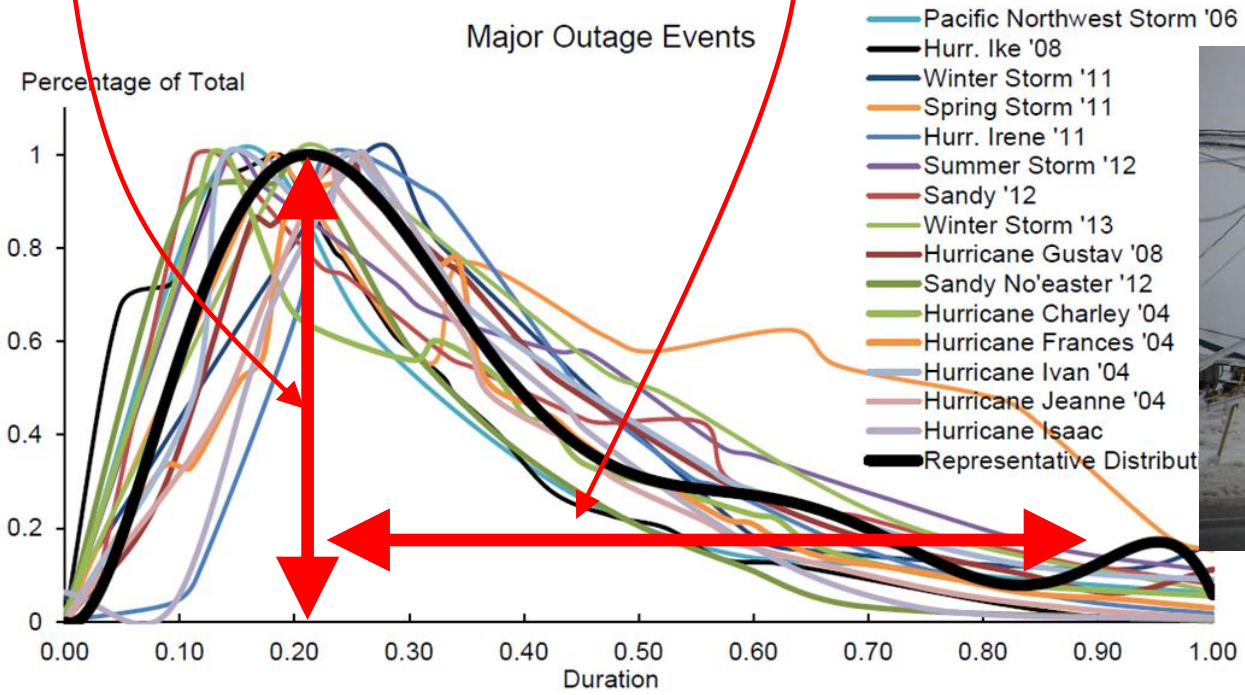
Definition of Resilience

<u>Presidential Policy Directive - Critical Infrastructure Security and Resilience</u>

"The ability to prepare for and adapt to changing conditions and withstand and recover rapidly from disruptions. Resilience includes the ability to <u>withstand</u> and <u>recover</u> from deliberate attacks, accidents, or naturally occurring threats or incidents."



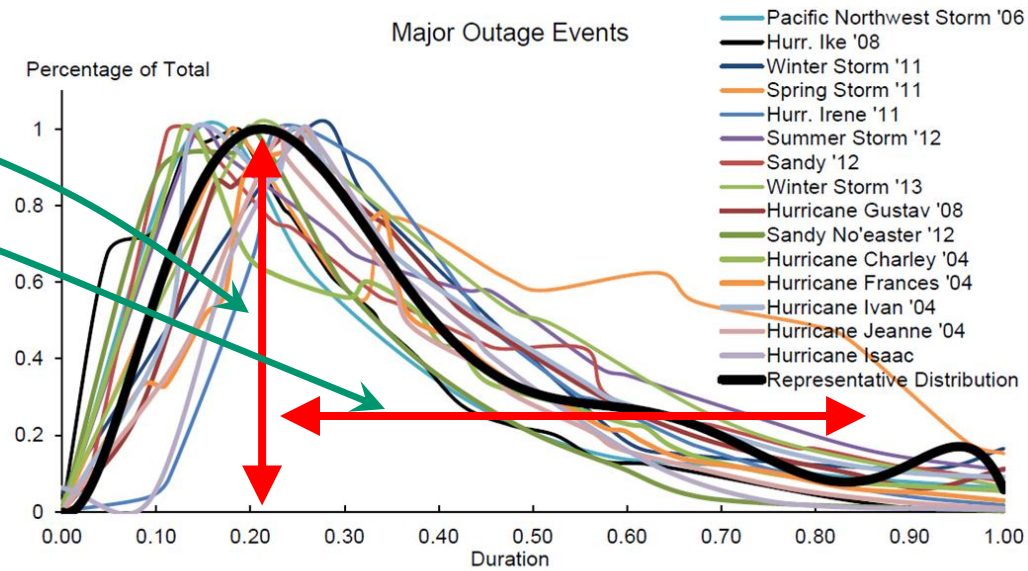
Source: Department of Energy, Office of Electrcity Delivery and Energy Reliability

Problem Overview: Our Goals

<u>Develop new tools, methodologies, and algorithms to enable the design of resilient power distribution systems—utility scale</u>

Hardening/Resilience options

- *Asset hardening*
- *System design*
- *System operations*
- *Repair scheduling*
- *Emergency operations*



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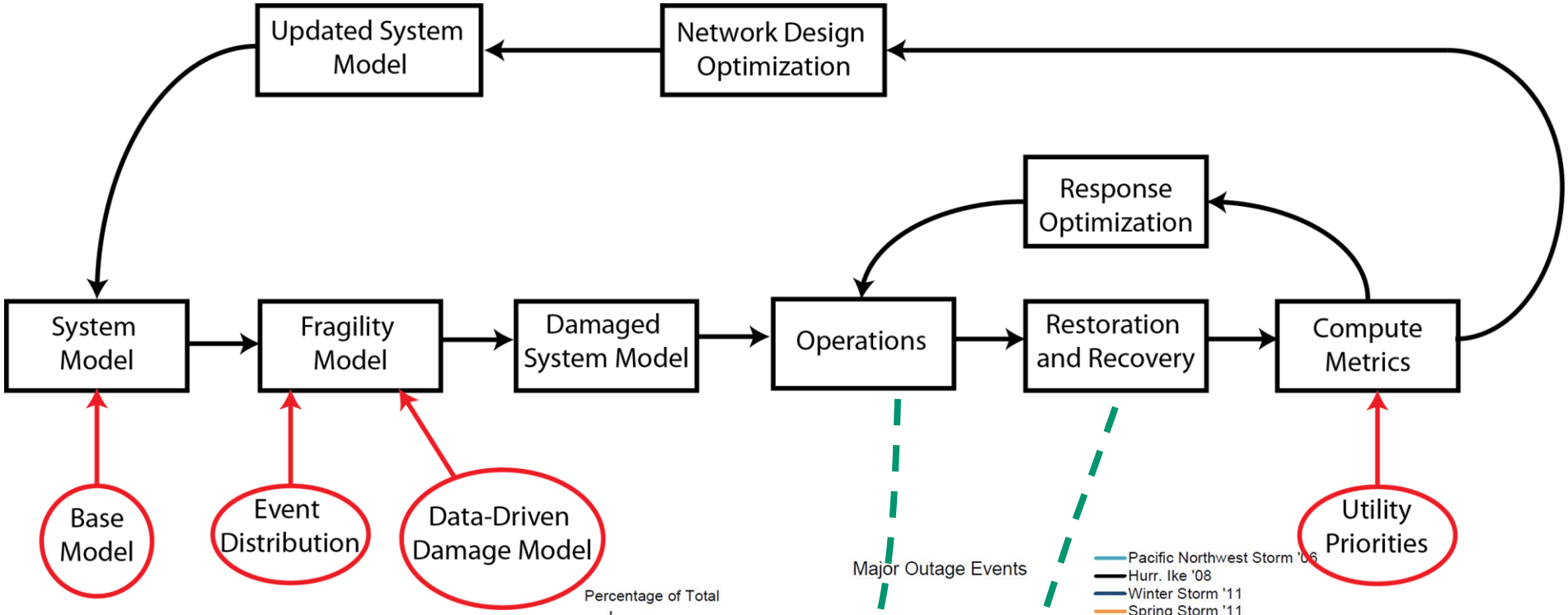
Flexibility for the user

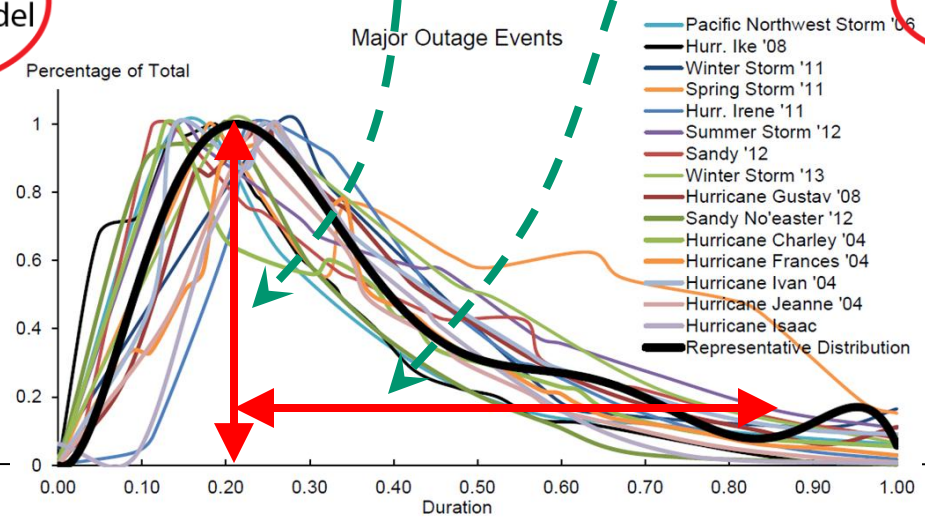
- *User's base network model*
- *User-defined resilience metrics*
- *User suggests upgrades*
- *User-defined costs*
- *User-defined threat and scenarios*

Capabilities

- *Assess current resilience posture*
- *Optimize over user-suggested upgrades to improve resilience considering budget*

Problem Overview





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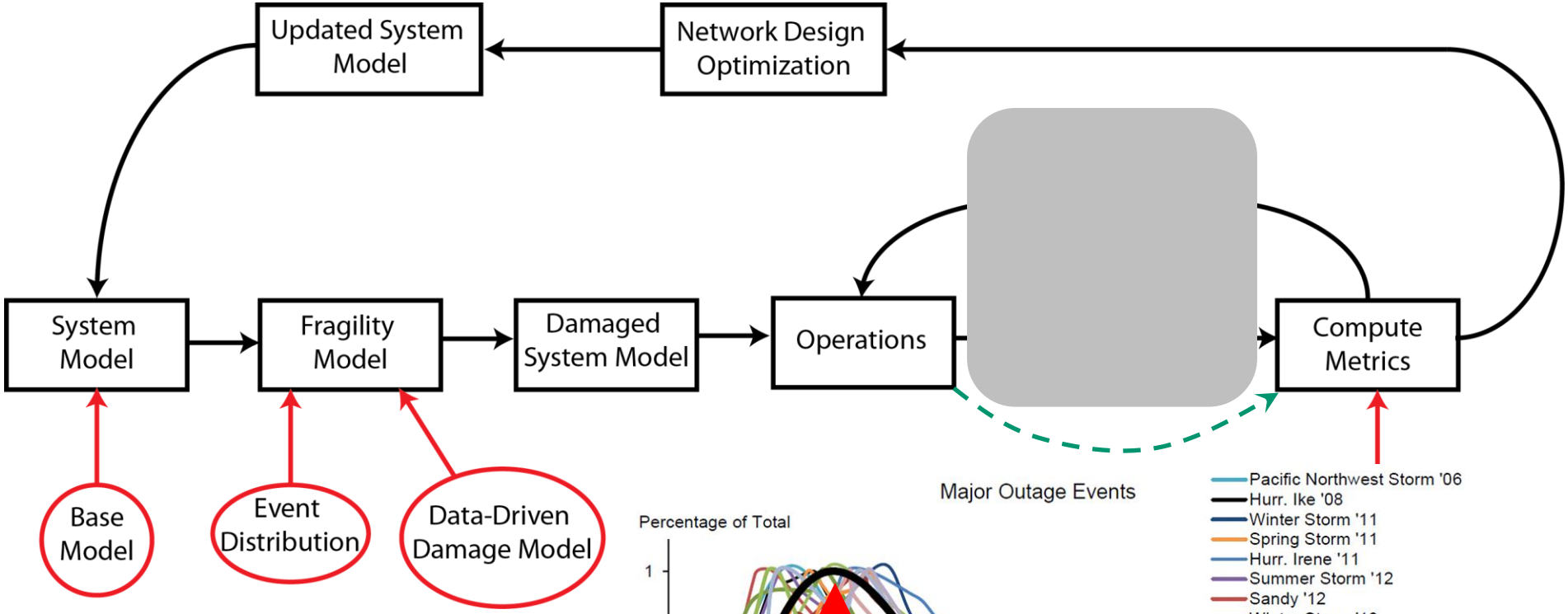
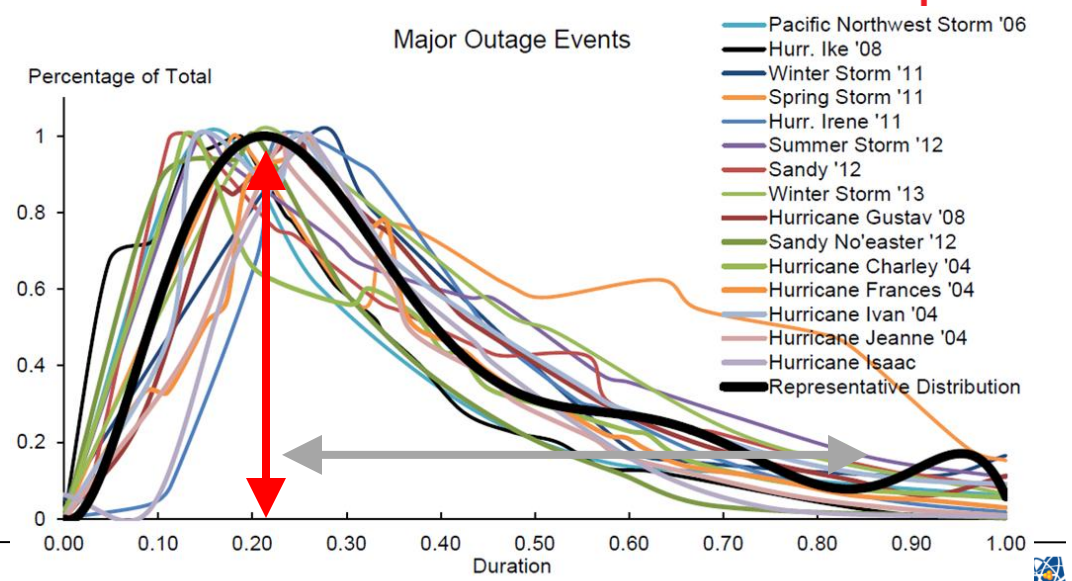
Today's Talk





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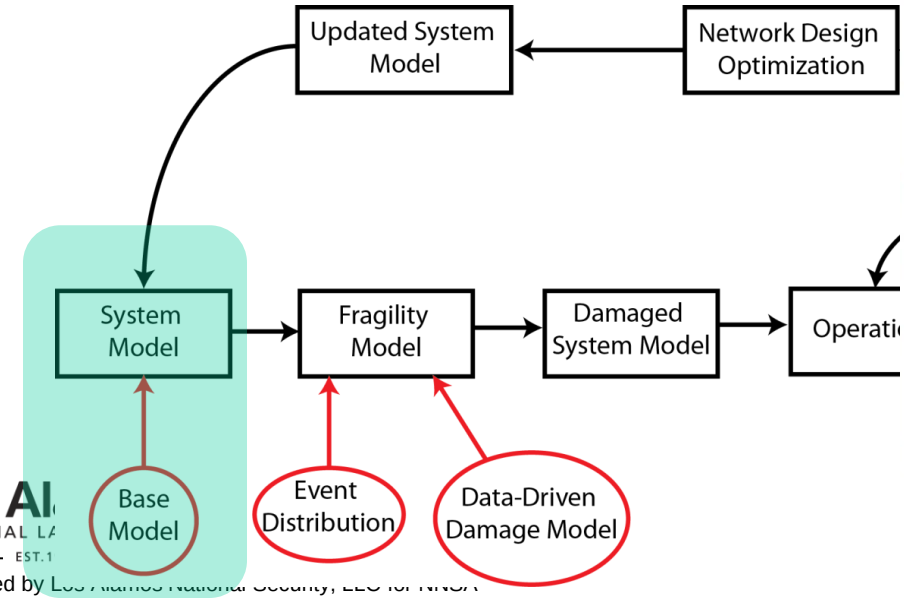
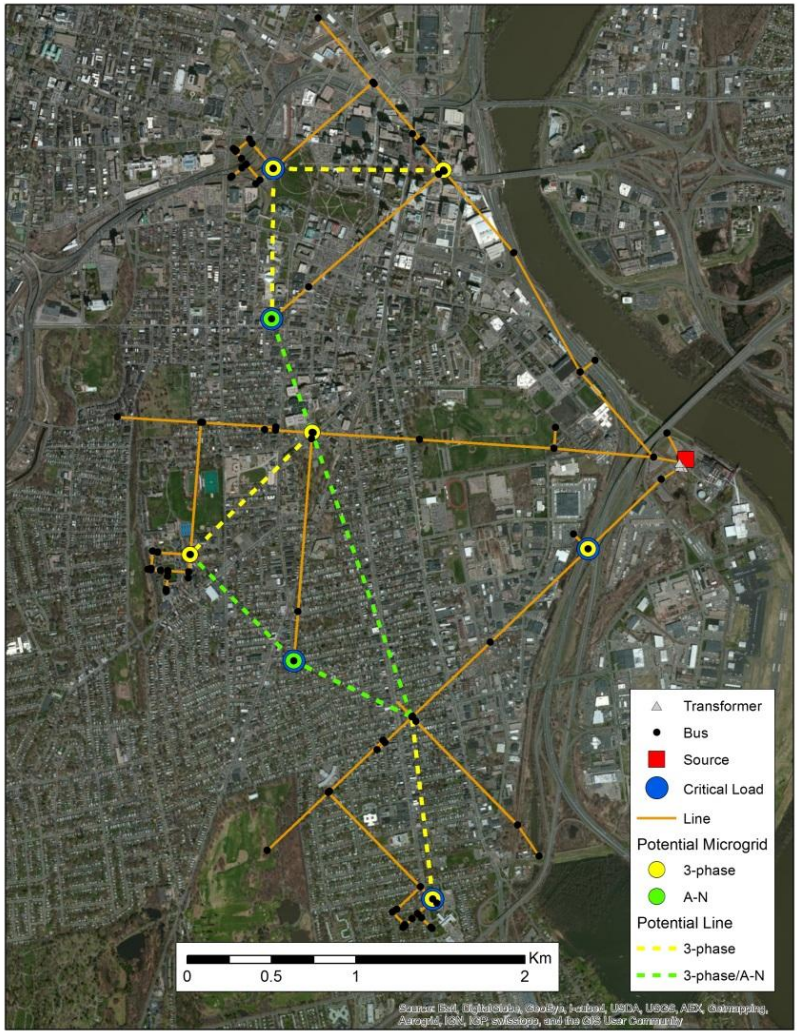
Resilience Design Process Flow—System Model

Flexibility for the user

User's base network model
User-defined resilience metrics, e.g. critical load service
User suggests upgrades
User-defined costs
User-defined threat and scenarios





Resilience Design Process Flow—Direct Impacts

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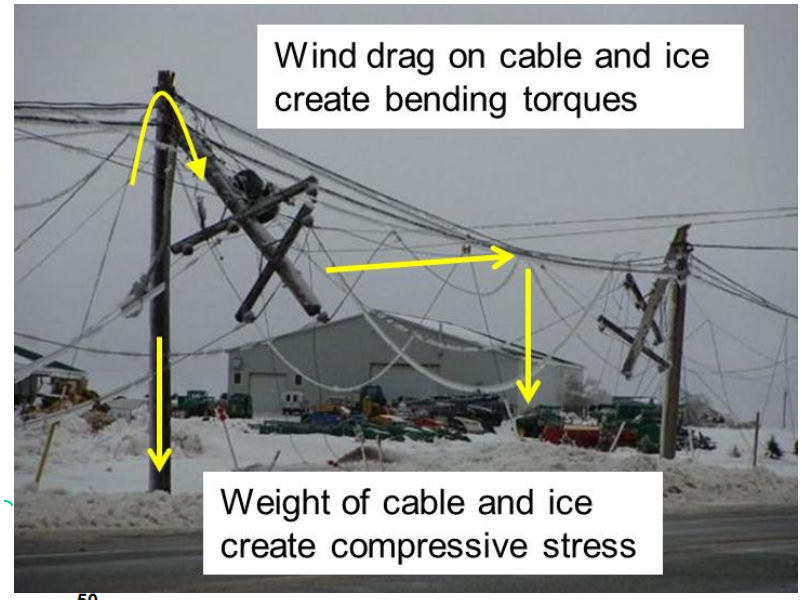
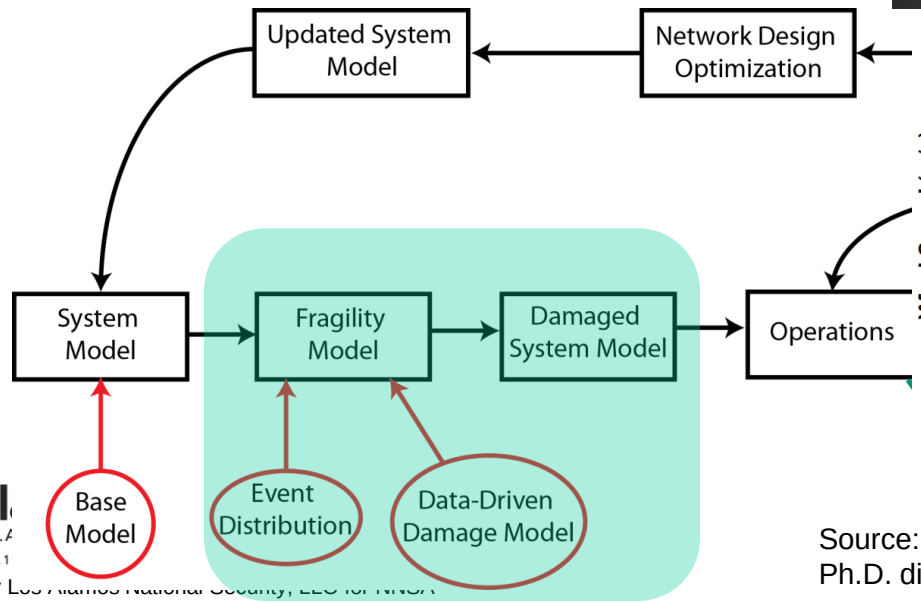
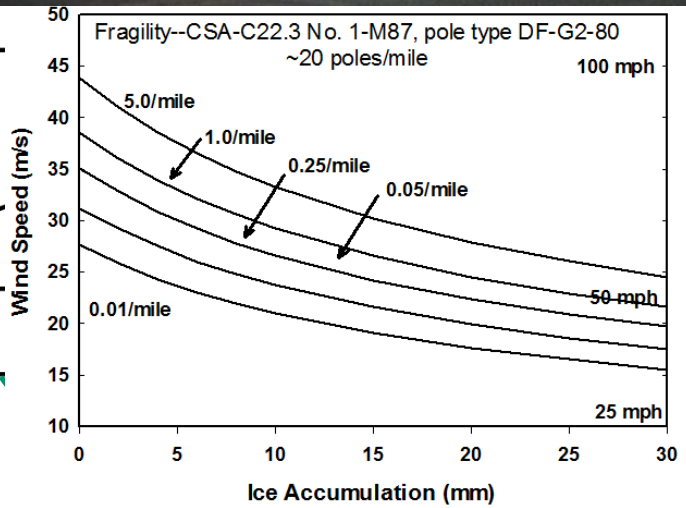
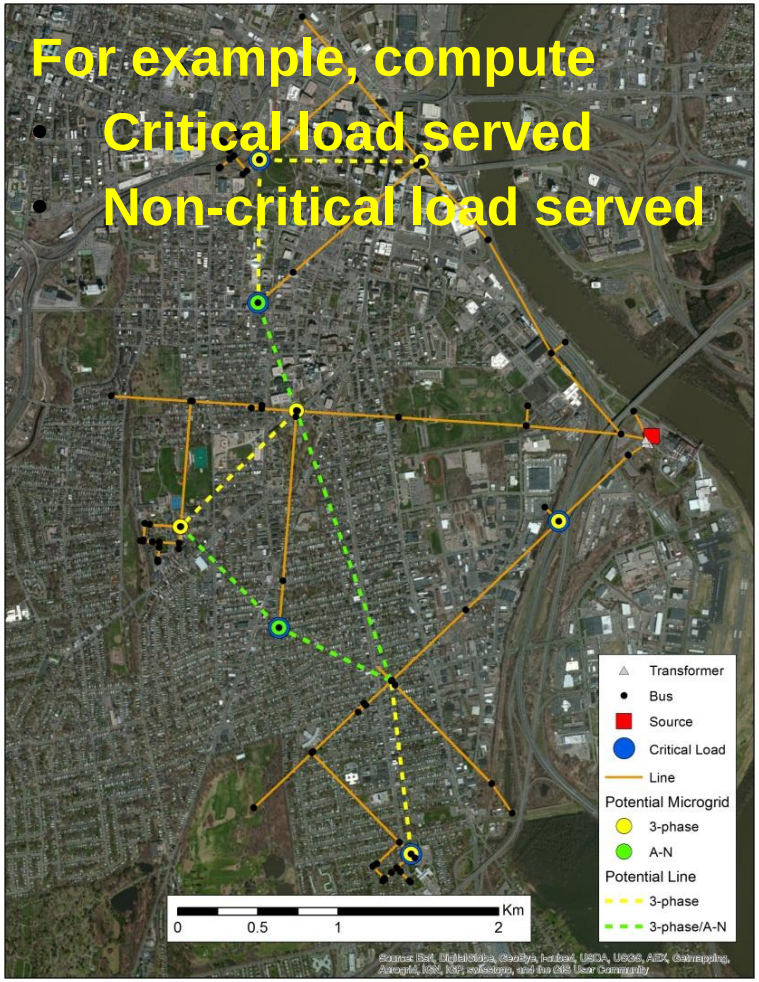






Source: Y. Sa, *Reliability analysis of electric distribution lines* Ph.D. dissertation, McGill University, Montreal, Canada, 2002

Resilience Design Process Flow—Secondary



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Capabilities

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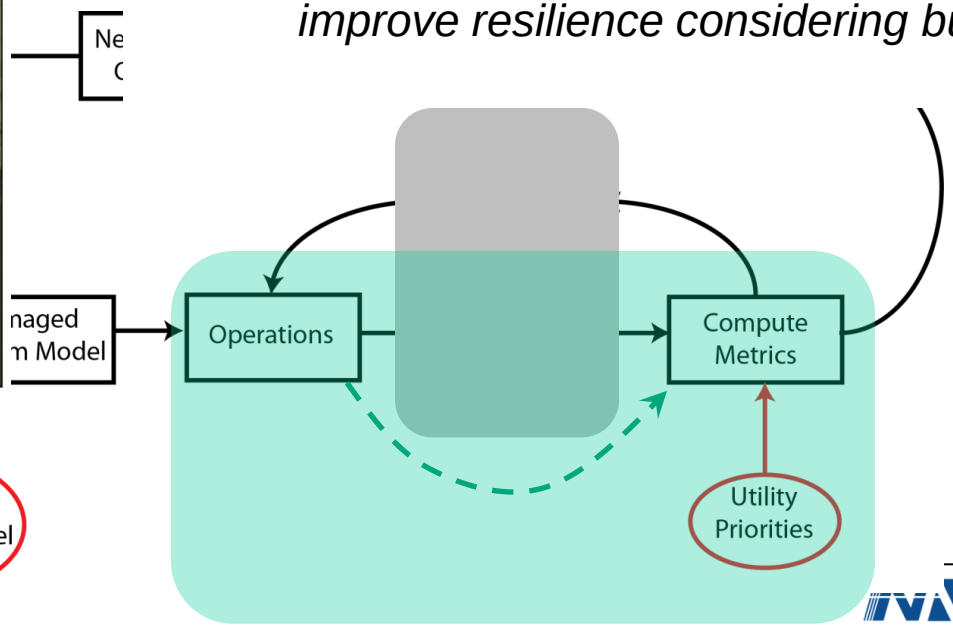


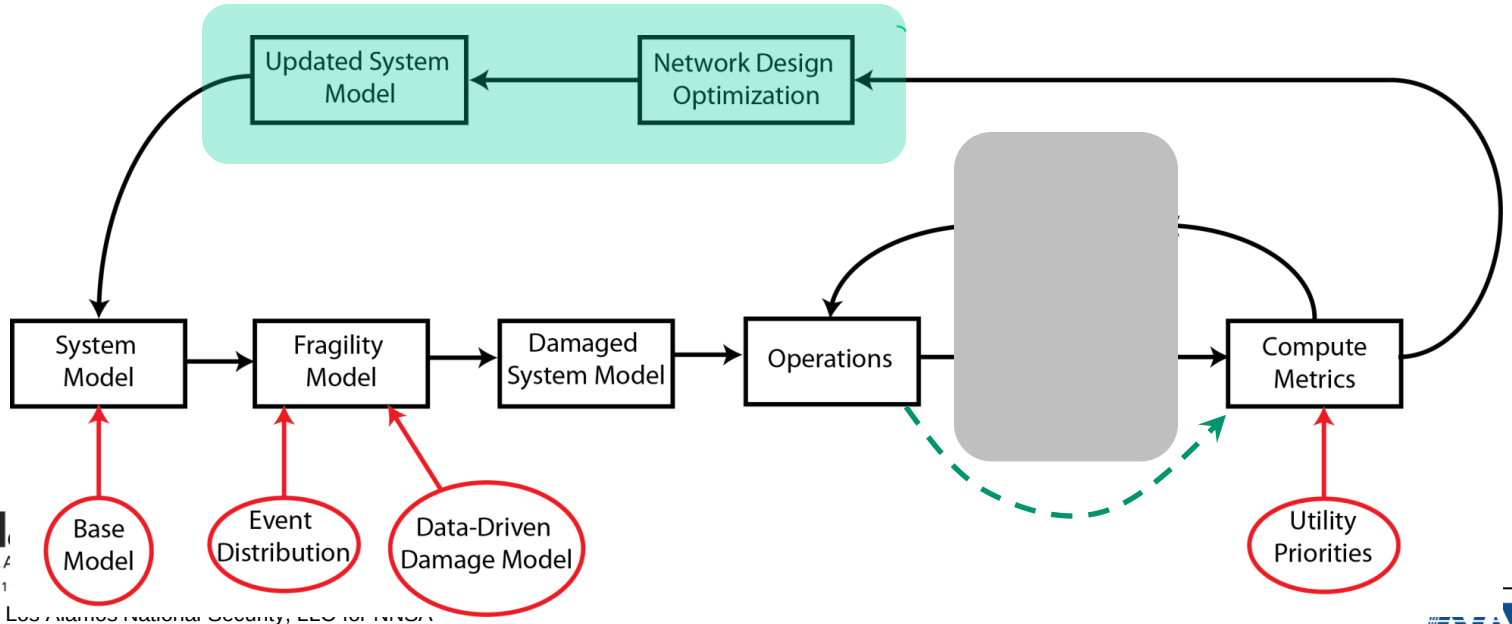
Base Model

Event Distribution

Data-Driven Damage Model

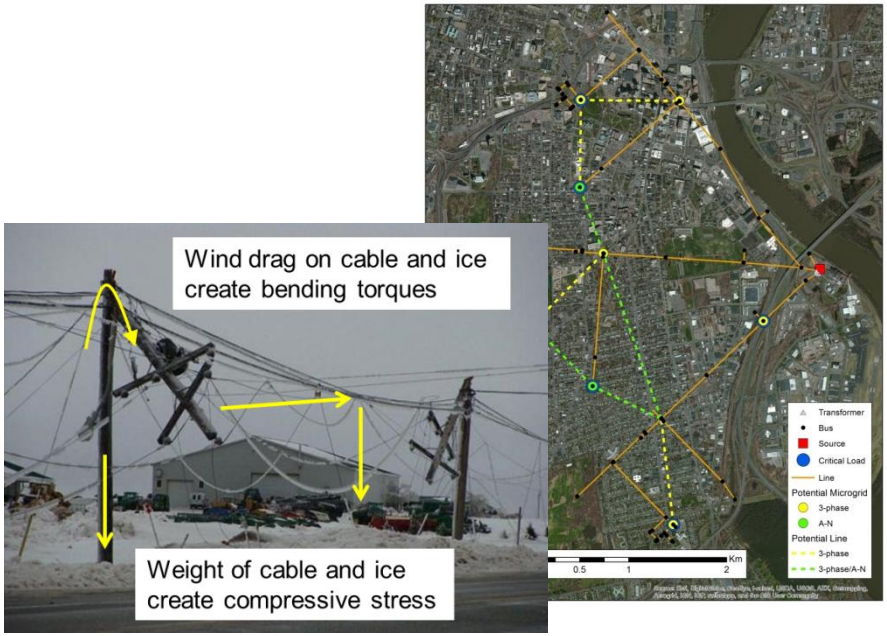
Resilience Design Process Flow—Design Network

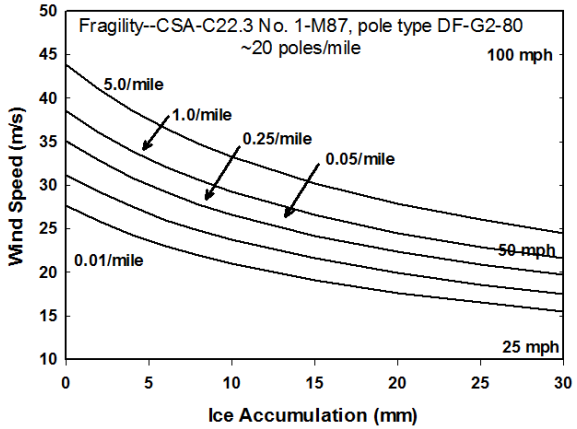
- Hardening/Resilience options
  - *Asset hardening*
  - *System design*
  - *System operations*
  - *Repair scheduling*
  - *Emergency operations*
- Capabilities
  - *Assess current resilience posture*
  - *Optimize over user-suggested upgrade to improve resilience considering budget*



Resiliency Model Details

- **Distribution power system**
  - Power lines, loads, generation
- **Hardening and Resilience Options**
  - Distributed generation
  - 3-phase or 1-phase interties
    - Above ground or underground
  - Add switches to:
    - Reconfigure circuits
    - Shed circuits and/or loads
  - Harden existing components
    - Reduce damage probabilities





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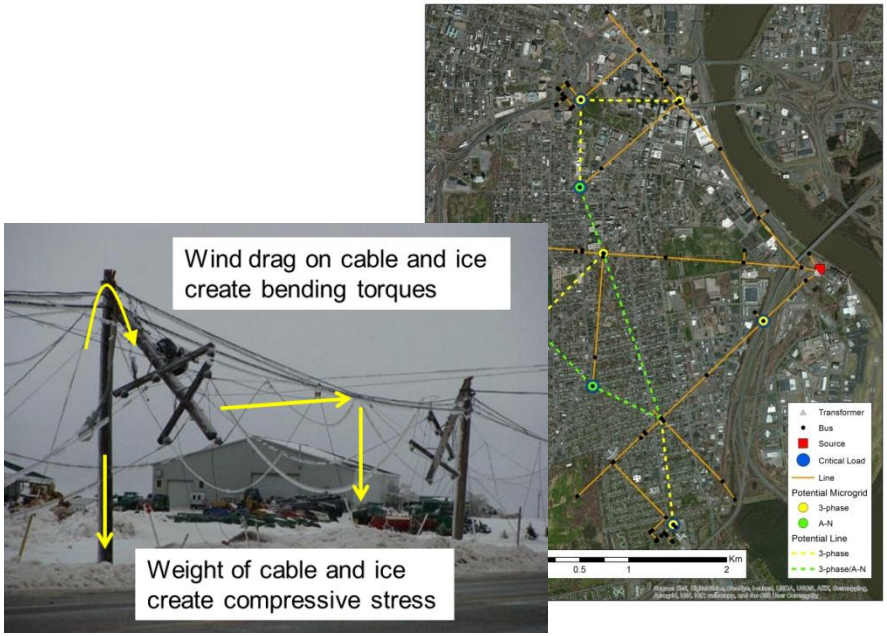
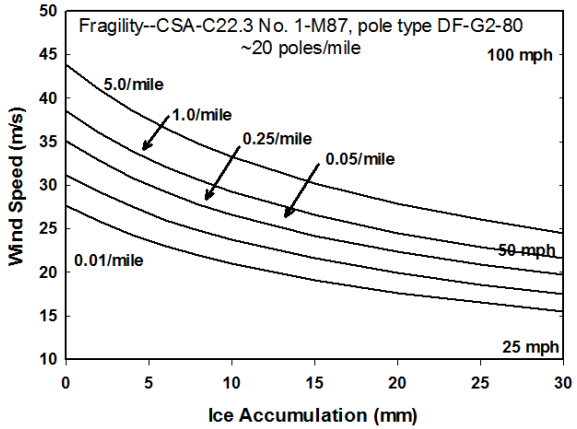
Resiliency Model Details

- **Damage Scenarios**
  - Historical data
  - Probability distribution
- **Operating and Resilience Constraints**
  - Radial operations
  - Load satisfaction
    - Critical and non-critical load
- **Objective**
  - Minimize cost





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Optimization Model

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \le x_{ij}\ , \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

$$-x_{ij0}^s Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{sk} \le \ x_{ij1}^s Q_{ij}^k$$

$$x_{ij0}^s + x_{ij1}^s \le x_{ij}^s$$

$$-(1 - \tau_{ij}^s)\, Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le \ (1 - \tau_{ij}^s)\, Q_{ij}^k$$

$$-\beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$$z_i^k \le M_i^k u_i$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

$$\mathrm{l}_\mathrm{i}^{\mathrm{ks}} = y_i^s d_i^k$$

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k^+}$$

$$\mathrm{g}_\mathrm{i}^{\mathrm{ks}} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0$$

$$0 \le z_i^{ks} \le u_i^s Z_i^k$$

$$\sum_{ij\in s} \left(\overline{x}_{ij}^s + (1 - \overline{\tau}_{ij})\right) \le \ |s| - 1$$

$$\tau_{ij}^s \ge x_{ij}^s + \overline{\tau}_{ij}^s - 1,\ x_{ij}^s \le \overline{x}_{ij}^s$$

$$\sum_{i\in CL,k\in p_i} l_i^{ks} \ge \lambda \sum_{i\in CL,k\in p_i} d_i^k$$

$$\sum_{i\in N\setminus L,k\in p_i} l_i^{ks} \ge \gamma \sum_{i\in N\setminus L,k\in p_i} d_i^k$$

$$x, y, \tau, u, t \in \{0,1\}$$

Key Features

- Least cost design for a set of scenarios
- Three-phase unbalanced real power flows
- Enforces radial operations
- Enforces phase balance tolerance
- Discrete variables for load shedding (per scenario), line switching (per scenario), capital construction (first stage)

- Relaxes unbalanced 3 phase power flows to a multi-commodity flow
- Assumption/Justification: Radial operations + Initial network is voltage feasible, upgrades tend to move loads closer to generation, which improves voltage and lowers line loading.

Optimization Model

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Minimize expansion cost

s.t. $x_{ij}^s \le x_{ij}\ , \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

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Optimization Model

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$$x, y, \tau, u, t \in \{0,1\}$$

Auxiliary variables for linking first and second stage. Useful for decomposition

Optimization Model

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

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$$x, y, \tau, u, t \in \{0,1\}$$

Line capacity constraints. Capacity is 0 when line is unavailable or open.

Optimization Model

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Phase imbalance tolerance

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$x_{ij}^s = t_{ij}^s$ when x is damaged

Links damaged lines with hardening variables

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Optimization Model

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \le x_{ij}\ , \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

$$-x_{ij0}^s Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{sk} \le \ x_{ij1}^s Q_{ij}^k$$

$$x_{ij0}^s + x_{ij1}^s \le x_{ij}^s$$

$$-(1 - \tau_{ij}^s)\, Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le \ (1 - \tau_{ij}^s)\, Q_{ij}^k$$

$$-\beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

$$\boxed{\mathrm{l}_{\mathrm{i}}^{\mathrm{ks}} = y_i^s d_i^k}$$

Load switching

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k^+}$$

$$\mathrm{g}_{\mathrm{i}}^{\mathrm{ks}} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0$$

$$0 \le z_i^{ks} \le u_i^s Z_i^k$$

$$\sum_{ij\in s} \left(\overline{x}_{ij}^s + (1 - \overline{\tau}_{ij})\right) \le \ |s| - 1$$

$$\tau_{ij}^s \ge x_{ij}^s + \overline{\tau}_{ij}^s - 1,\ x_{ij}^s \le \overline{x}_{ij}^s$$

$$\sum_{i\in CL,k\in p_i} l_i^{ks} \ge \lambda \sum_{i\in CL,k\in p_i} d_i^k$$

$$\sum_{i\in N\setminus L,k\in p_i} l_i^{ks} \ge \gamma \sum_{i\in N\setminus L,k\in p_i} d_i^k$$

$$x, y, \tau, u, t \in \{0,1\}$$

Optimization Model

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \le x_{ij}\ , \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

$$-x_{ij0}^s Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{sk} \le \ x_{ij1}^s Q_{ij}^k$$

$$x_{ij0}^s + x_{ij1}^s \le x_{ij}^s$$

$$-(1 - \tau_{ij}^s)\, Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le \ (1 - \tau_{ij}^s)\, Q_{ij}^k$$

$$-\beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

$$\mathrm{l}_{\mathrm{i}}^{\mathrm{ks}} = y_i^s d_i^k$$

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k^+}$$

Power produced

$$\mathrm{g}_{\mathrm{i}}^{\mathrm{ks}} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0$$

$$0 \le z_i^{ks} \le u_i^s Z_i^k$$

$$\sum_{ij\in s} \left(\overline{x}_{ij}^s + (1 - \overline{\tau}_{ij})\right) \le \ |s| - 1$$

$$\tau_{ij}^s \ge x_{ij}^s + \overline{\tau}_{ij}^s - 1,\ x_{ij}^s \le \overline{x}_{ij}^s$$

$$\sum_{i\in CL,k\in p_i} l_i^{ks} \ge \lambda \sum_{i\in CL,k\in p_i} d_i^k$$

$$\sum_{i\in N\setminus L,k\in p_i} l_i^{ks} \ge \gamma \sum_{i\in N\setminus L,k\in p_i} d_i^k$$

$$x, y, \tau, u, t \in \{0,1\}$$

Optimization Model

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \le x_{ij}\ , \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

$$-x_{ij0}^s Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{sk} \le \ x_{ij1}^s Q_{ij}^k$$

$$x_{ij0}^s + x_{ij1}^s \le x_{ij}^s$$

$$-\left(1 - \tau_{ij}^s\right) Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le \ \left(1 - \tau_{ij}^s\right) Q_{ij}^k$$

$$-\beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

$$\mathrm{l}_{\mathrm{i}}^{\mathrm{ks}} = y_i^s d_i^k$$

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k^+}$$

$$\boxed{\mathrm{g}_{\mathrm{i}}^{\mathrm{ks}} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0}$$

Nodal flow balance

$$0 \le z_i^{ks} \le u_i^s Z_i^k$$

$$\sum_{ij\in s} \left(\overline{x}_{ij}^s + \left(1 - \overline{\tau}_{ij}\right)\right) \le \ |s| - 1$$

$$\tau_{ij}^s \ge x_{ij}^s + \overline{\tau}_{ij}^s - 1,\ x_{ij}^s \le \overline{x}_{ij}^s$$

$$\sum_{i\in CL,k\in p_i} l_i^{ks} \ge \lambda \sum_{i\in CL,k\in p_i} d_i^k$$

$$\sum_{i\in N\setminus L,k\in p_i} l_i^{ks} \ge \gamma \sum_{i\in N\setminus L,k\in p_i} d_i^k$$

$$x, y, \tau, u, t \in \{0,1\}$$

Optimization Model

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \le x_{ij}\ , \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

$$-x_{ij0}^s Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{sk} \le \ x_{ij1}^s Q_{ij}^k$$

$$x_{ij0}^s + x_{ij1}^s \le x_{ij}^s$$

$$-(1 - \tau_{ij}^s)\, Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le \ (1 - \tau_{ij}^s)\, Q_{ij}^k$$

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$x_{ij}^s = t_{ij}^s$ when x is damaged

$$\mathrm{l}_{\mathrm{i}}^{\mathrm{ks}} = y_i^s d_i^k$$

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k^+}$$

$$\mathrm{g}_{\mathrm{i}}^{\mathrm{ks}} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0$$

$$0 \le z_i^{ks} \le u_i^s Z_i^k$$

Links generation construction and capacity

$$\sum_{ij\in s} \left(\overline{x}_{ij}^s + (1 - \overline{\tau}_{ij})\right) \le \ |s| - 1$$

$$\tau_{ij}^s \ge x_{ij}^s + \overline{\tau}_{ij}^s - 1,\ x_{ij}^s \le \overline{x}_{ij}^s$$

$$\sum_{i\in CL,k\in p_i} l_i^{ks} \ge \lambda \sum_{i\in CL,k\in p_i} d_i^k$$

$$\sum_{i\in N\setminus L,k\in p_i} l_i^{ks} \ge \gamma \sum_{i\in N\setminus L,k\in p_i} d_i^k$$

$$x, y, \tau, u, t \in \{0,1\}$$

Optimization Model

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \le x_{ij}\ , \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

$$-x_{ij0}^s Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{sk} \le \ x_{ij1}^s Q_{ij}^k$$

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$$-(1 - \tau_{ij}^s)\, Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le \ (1 - \tau_{ij}^s)\, Q_{ij}^k$$

$$-\beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

$$\mathrm{l}_{\mathrm{i}}^{\mathrm{ks}} = y_i^s d_i^k$$

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k^+}$$

$$\mathrm{g}_{\mathrm{i}}^{\mathrm{ks}} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0$$

$$0 \le z_i^{ks} \le u_i^s Z_i^k$$

$$\sum_{ij\in s} \left(\overline{x}_{ij}^s + (1 - \overline{\tau}_{ij})\right) \le \ |s| - 1$$

$$\tau_{ij}^s \ge x_{ij}^s + \overline{\tau}_{ij}^s - 1,\ x_{ij}^s \le \overline{x}_{ij}^s$$

$$\sum_{i\in CL,k\in p_i} l_i^{ks} \ge \lambda \sum_{i\in CL,k\in p_i} d_i^k$$

$$\sum_{i\in N\setminus L,k\in p_i} l_i^{ks} \ge \gamma \sum_{i\in N\setminus L,k\in p_i} d_i^k$$

$$x, y, \tau, u, t \in \{0,1\}$$

Enforces radial operation

Optimization Model

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \le x_{ij}\ , \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

$$-x_{ij0}^s Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{sk} \le \ x_{ij1}^s Q_{ij}^k$$

$$x_{ij0}^s + x_{ij1}^s \le x_{ij}^s$$

$$-(1 - \tau_{ij}^s)\, Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le \ (1 - \tau_{ij}^s)\, Q_{ij}^k$$

$$-\beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

$$\mathrm{l}_{\mathrm{i}}^{\mathrm{ks}} = y_i^s d_i^k$$

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k^+}$$

$$\mathrm{g}_{\mathrm{i}}^{\mathrm{ks}} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0$$

$$0 \le z_i^{ks} \le u_i^s Z_i^k$$

$$\sum_{ij\in s} \left(\overline{x}_{ij}^s + (1 - \overline{\tau}_{ij})\right) \le \ |s| - 1$$

$$\tau_{ij}^s \ge x_{ij}^s + \overline{\tau}_{ij}^s - 1,\ x_{ij}^s \le \overline{x}_{ij}^s$$

$$\sum_{i\in CL,k\in p_i} l_i^{ks} \ge \lambda \sum_{i\in CL,k\in p_i} d_i^k$$

$$\sum_{i\in N\setminus L,k\in p_i} l_i^{ks} \ge \gamma \sum_{i\in N\setminus L,k\in p_i} d_i^k$$

$$x, y, \tau, u, t \in \{0,1\}$$

Resilience criteria—minimum amount of load served

Is generalized to a chance constraint

Algorithm Overview

Exact Algorithms

- **CPLEX 12.6—no parameter tuning**
  - Difficult problem – 50-60k binary variables
- **Decomposition**
  - Benders, Dantzig-Wolfe, Scenario

Heuristics

- **Greedy**
  - Union of single scenario solutions
  - Based on industry algorithms
- **Variable Neighborhood Search**
  - Ruin and Recreate—hybrid of exact methods and local search
  - Iteratively relax variable assignments (ruin)
  - Use exact method to find optimal variable assignments for relaxed variables, given the fixed partial solution (recreate)

Scenario Based Decomposition

$$ResilientDesign(S)$$

$$s \leftarrow chooseScenario(S)$$

$$\sigma \rightarrow solveMIP(s)$$

$$while\ (\sim Feasible(\sigma, S \backslash s))$$

$$s \rightarrow s \cup chooseScenario(S \backslash s)$$

$$\sigma \rightarrow solveMIP(s)$$

Solve over all damage scenarios

Select 1 scenario

Design network for damage scenario 1

Is solution feasible for remaining scenarios

If NOT, add an infeasible scenario to the set under consideration

Find a new solution

Iterate until solution is feasible for all scenarios

Outperformed other decomposition strategies—second stage influences feasibility, not optimality. Continuous investment variables also adds difficulty

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^{*} \leftarrow \sigma', restart \leftarrow false$

Solve the LP relaxation

while $(t < maxTime$ and $i < maxRestarts)$

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^{*}(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \ldots \pi_{|J|} \rangle \in X : |\sigma^{*}(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^{*}(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if $(restart = true)$

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while $(t < maxTime\ and\ j \leq maxIterations)$

$\sigma' \leftarrow Solve(P(\sigma^{*}, J(1, \ldots k))$

if $\left(f(\sigma') < f(\sigma^{*})\right)$

$\sigma^{*} \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$i \leftarrow i + 1, k \leftarrow k - \frac{step}{}$

Intuition: LP relaxation guides the search procedure

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^* \leftarrow \sigma', restart \leftarrow false$

while $(t < maxTime$ and $i < maxRestarts)$

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^*(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \ldots \pi_{|J|} \rangle \in X : |\sigma^*(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^*(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if $(restart = true)$

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while $(t < maxTime\ and\ j \leq maxIterations)$

$\sigma' \leftarrow Solve(P(\sigma^*, J(1, \ldots k))$

if $\left(f(\sigma') < f(\sigma^*)\right)$

$\sigma^* \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$i \leftarrow i + 1, k \leftarrow k - \frac{step}{}$

Count differences between current best solution and relaxation

Intuition: *n* is a parameter used to control the size of the neighborhood. Larger differences between the LP relaxation and the incumbent solution indicate that a larger neighborhood should be considered.

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^{*} \leftarrow \sigma', restart \leftarrow false$

while $(t < maxTime$ and $i < maxRestarts)$

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^{*}(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \ldots \pi_{|J|} \rangle \in X : |\sigma^{*}(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^{*}(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if $(restart = true)$

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while $(t < maxTime\ and\ j \leq maxIterations)$

$\sigma' \leftarrow Solve(P(\sigma^{*}, J(1, \ldots k))$

if $\left(f(\sigma') < f(\sigma^{*})\right)$

$\sigma^{*} \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$i \leftarrow i + 1, k \leftarrow k - \frac{step}{}$

Order variables by difference from LP relaxation

Intuition: Variables whose assignments differ from the LP relaxation have more potential to improve the incumbent solution.

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^{*} \leftarrow \sigma', restart \leftarrow false$

while $(t < maxTime$ and $i < maxRestarts)$

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^{*}(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \ldots \pi_{|J|} \rangle \in X : |\sigma^{*}(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^{*}(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if $(restart = true)$

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while $(t < maxTime\ and\ j \leq maxIterations)$

$\sigma' \leftarrow Solve(P(\sigma^{*}, J(1, \ldots k))$

if $\left(f(\sigma') < f(\sigma^{*})\right)$

$\sigma^{*} \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$i \leftarrow i + 1, k \leftarrow k - \frac{step}{}$

Compute best solution in neighborhood J(1…k)

Intuition: Ruin and recreate

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^{*} \leftarrow \sigma', restart \leftarrow false$

while $(t < maxTime$ and $i < maxRestarts)$

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^{*}(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \ldots \pi_{|J|} \rangle \in X : |\sigma^{*}(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^{*}(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if $(restart = true)$

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while $(t < maxTime\ and\ j \leq maxIterations)$

$\sigma' \leftarrow Solve(P(\sigma^{*}, J(1, \ldots k))$

if $\left(f(\sigma') < f(\sigma^{*})\right)$

$\sigma^{*} \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$i \leftarrow i + 1, k \leftarrow k - \frac{step}{}$

Update best solution

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^{*} \leftarrow \sigma', restart \leftarrow false$

while $(t < maxTime$ and $i < maxRestarts)$

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^{*}(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \ldots \pi_{|J|} \rangle \in X : |\sigma^{*}(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^{*}(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if $(restart = true)$

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while $(t < maxTime\ and\ j \leq maxIterations)$

$\sigma' \leftarrow Solve(P(\sigma^{*}, J(1, \ldots k))$

if $\left(f(\sigma') < f(\sigma^{*})\right)$

$\sigma^{*} \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$i \leftarrow i + 1, k \leftarrow k - step$

Intuition: When a better solution is not found, increase the size of the neighborhood

Increase neighborhood size

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^{*} \leftarrow \sigma', restart \leftarrow false$

while $(t < maxTime$ and $i < maxRestarts)$

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^{*}(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \ldots \pi_{|J|} \rangle \in X : |\sigma^{*}(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^{*}(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if $(restart = true)$

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while $(t < maxTime\ and\ j \leq maxIterations)$

$\sigma' \leftarrow Solve(P(\sigma^{*}, J(1, \ldots k))$

if $\left(f(\sigma') < f(\sigma^{*})\right)$

$\sigma^{*} \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$i \leftarrow i + 1, k \leftarrow k - \frac{step}{}$

Shuffle ordering after restart

Intuition: Consider different sets of variables to relax

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^{*} \leftarrow \sigma', restart \leftarrow false$

while $(t < maxTime$ and $i < maxRestarts)$

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^{*}(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \ldots \pi_{|J|} \rangle \in X : |\sigma^{*}(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^{*}(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if $(restart = true)$

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while $(t < maxTime\ and\ j \leq maxIterations)$

$\sigma' \leftarrow Solve(P(\sigma^{*}, J(1, \ldots k))$

if $\left(f(\sigma') < f(\sigma^{*})\right)$

$\sigma^{*} \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$i \leftarrow i + 1, k \leftarrow k - \frac{step}{}$

Adjust neighborhood parameters

Intuition: Neighborhood size is based on differences between LP relaxation and incumbent solution.

Variable Neighborhood Search with Decomposition

$$ResilientDesign(S)$$

$$s \leftarrow chooseScenario(S)$$

$$\sigma \rightarrow solveVNS(s)$$

$$while\ (\sim Feasible(\sigma, S \backslash s))$$

$$s \rightarrow s \cup chooseScenario(S \backslash s)$$

$$\sigma \rightarrow solveVNS(s)$$

Solve over all damage scenarios

Select 1 scenario

Design network for damage scenario 1

Is solution feasible for remaining scenarios

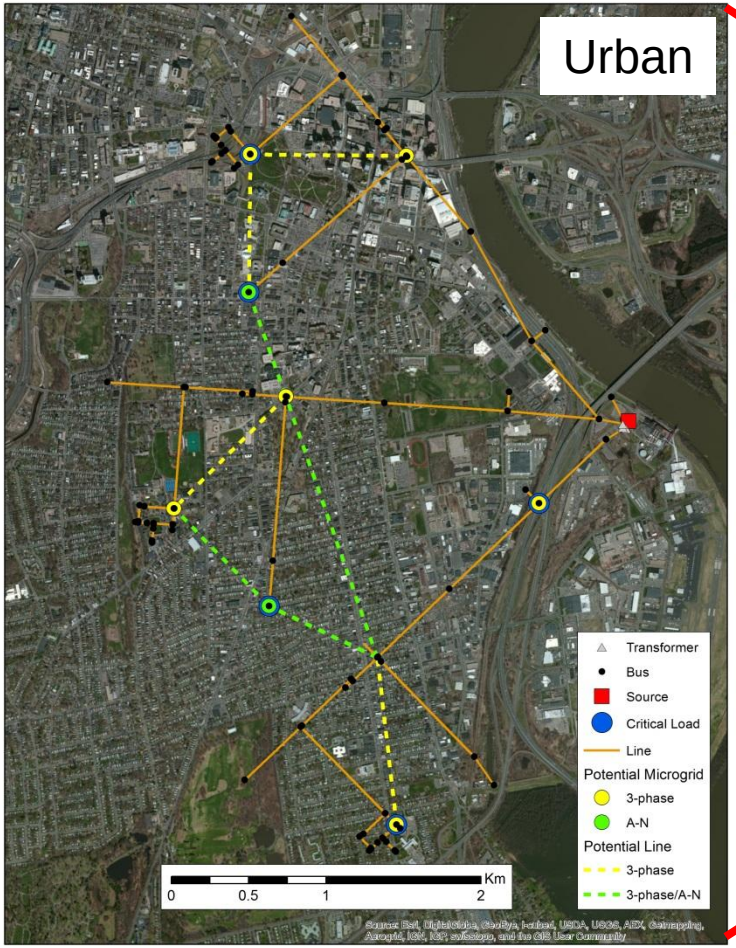
If NOT, add an infeasible scenario to the set under consideration

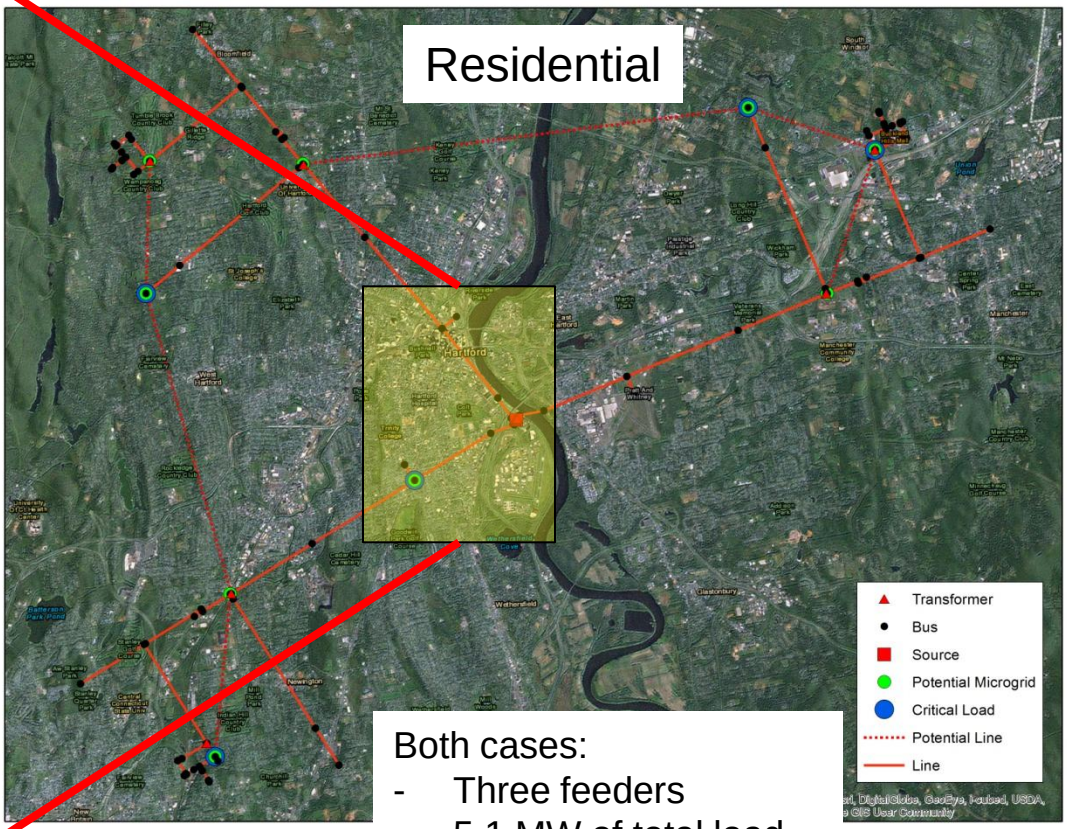
Find a new solution

Iterate until solution is feasible for all scenarios

Test Cases

Two base-model configurations—“Dense Urban” and “Sparse Residential”
Range of damage intensity—Light damage to Heavy damage
Different trade off between 1) distributed generation 2) new interties 3) hardening
Based on IEEE 34 (100 Scenarios, 109 nodes, 148 edges)





Both cases:
- Three feeders
- 5.1 MW of total load
- 2.1 MW of critical load

Assumptions

Distributed generators provide firm generation, e.g. natural gas CHP

Circuits or sections of circuits configured as trees

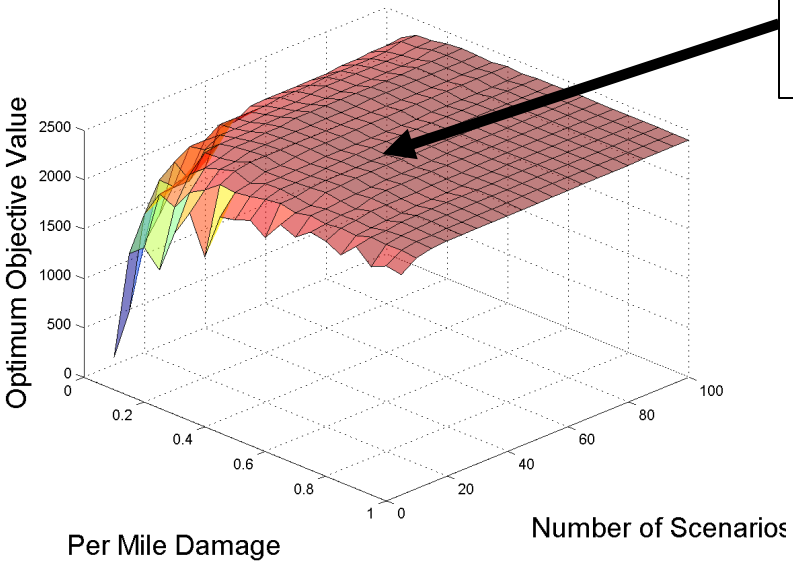
Loads and/or generators stay on the phases where they were installed

Costs…… (can be modified based on user specifications)

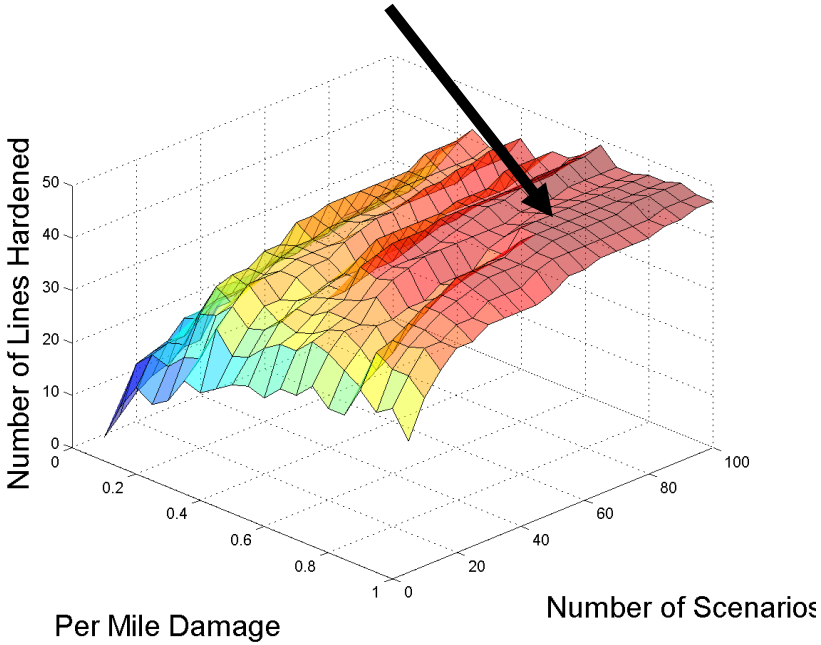
Device	Type	Cost Range	Suggested cost	Source
$c_{i,j}$	overhead 3-phase	\$60k-\$150k/mile	\$95k/mile	State of
$c_{i,j}$	overhead 1-phase	\$40k-\$75k/mile	\$55k/mile	Virginia Study on
$c_{i,j}$	underground 3-phase	\$40k-\$1,500k/mile	\$500k/mile	Underground Circuits
$c_{i,j}$	underground 1-phase	\$40k-\$1,500k/mile	\$100k/mile	-
$\kappa_{i,j}$	automatic, 3-phase, overhead	–	\$15k	Tom Bialek (SDG&E)
$\kappa_{i,j}$	automatic, 3-phase, underground	–	\$30k	-
$\kappa_{i,j}$	manual, 3-phase, overhead	–	\$7.5k	-
$\kappa_{i,j}$	manual, 3-phase, underground	–	\$20k	-
$\kappa_{i,j}$	automatic, 1-phase, overhead	–	\$10k	-
$\kappa_{i,j}$	automatic, 1-phase, underground	–	\$25k	-
$\kappa_{i,j}$	manual, 1-phase, overhead	–	\$5	-
$\kappa_{i,j}$	manual, 1-phase, underground	–	\$15k	-
$\zeta_{i,j}$	Natural gas CHP variable	–	\$1,500k/MW	EIA 2025 Study
$\zeta_{i,j}$	Natural gas CHP fixed	–	\$500k	

100 Scenarios are sufficient (empirically)

Solution quality doesn't change much



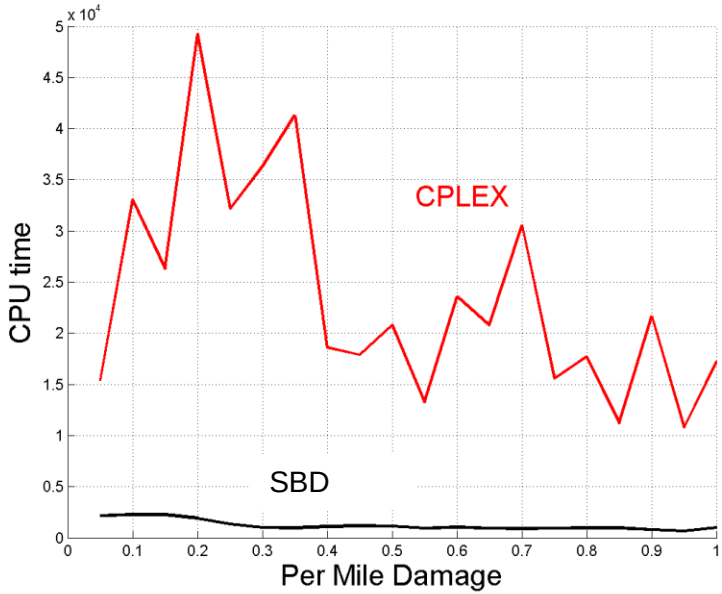
Solution changes slightly

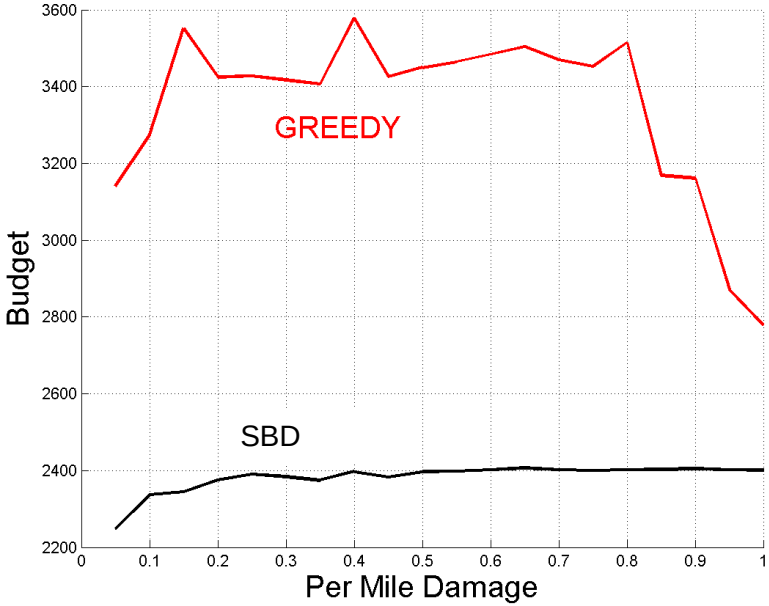


Residential Problem

Algorithm Comparisons

Residential Problem

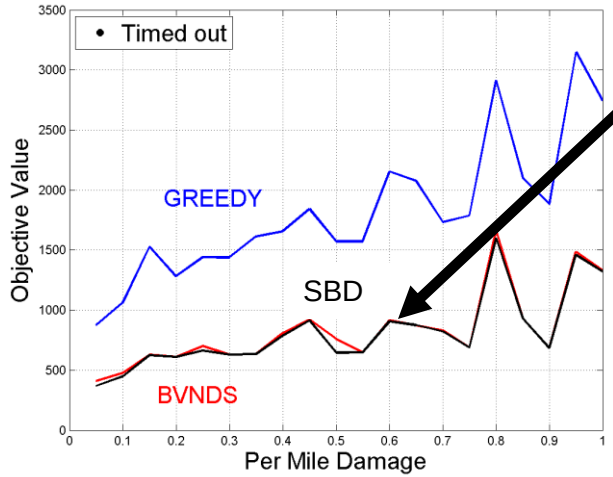


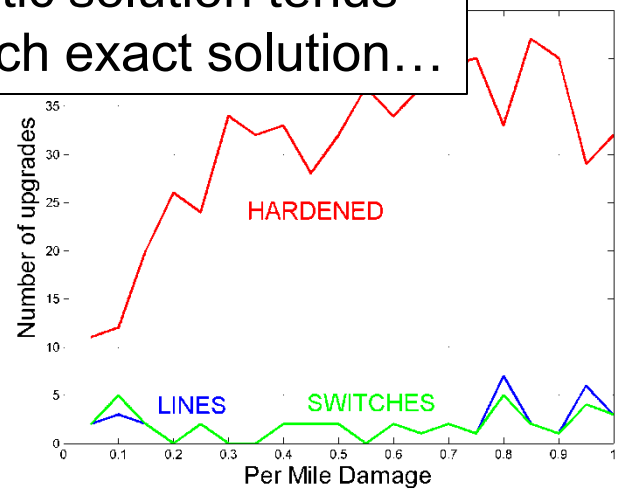


Gaps widen as problems become larger

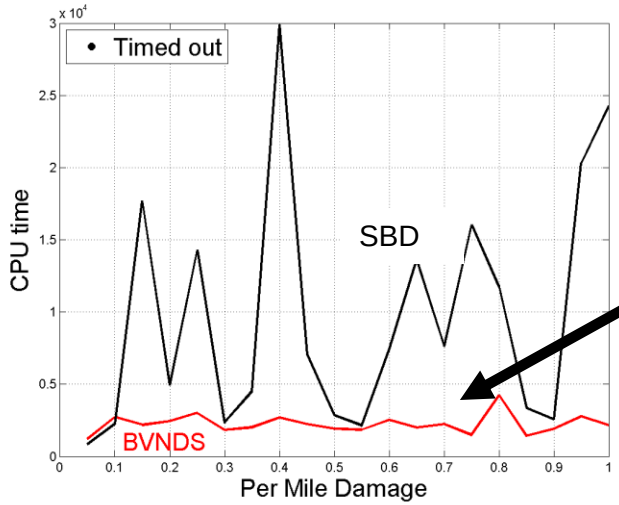
Exact vs. Heuristics

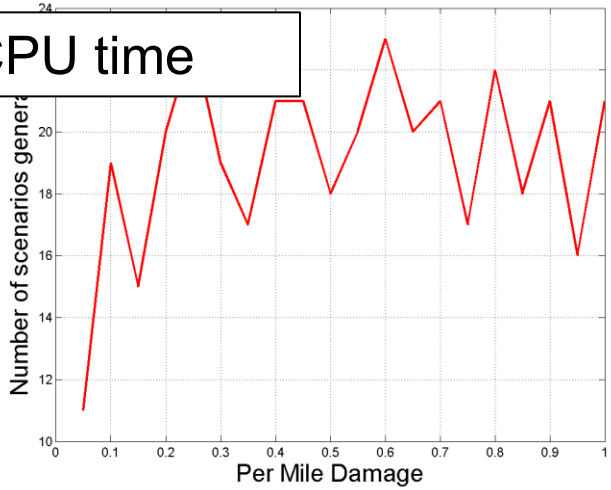
Heuristic solution tends to match exact solution…





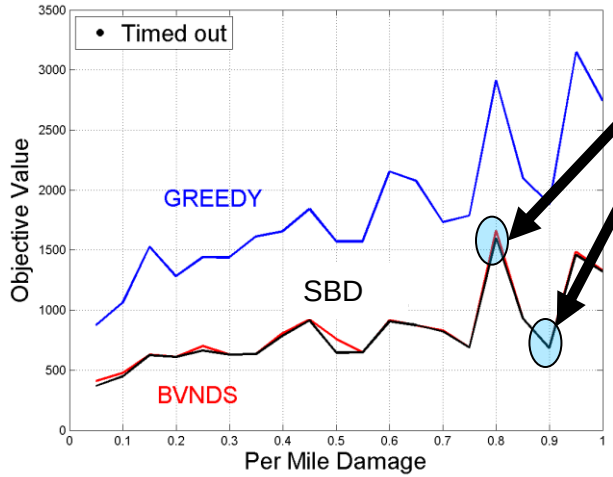
… with less CPU time

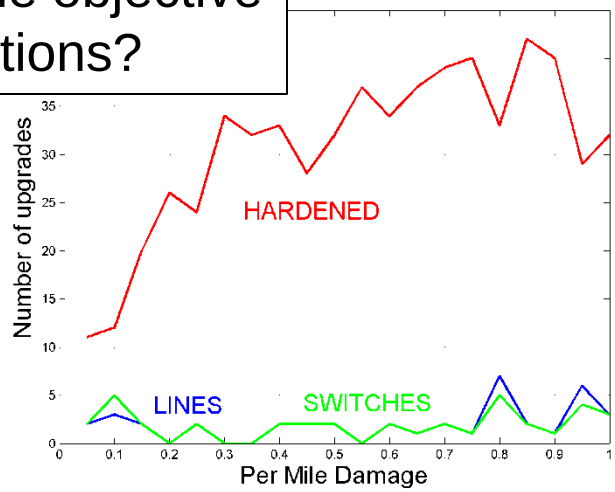


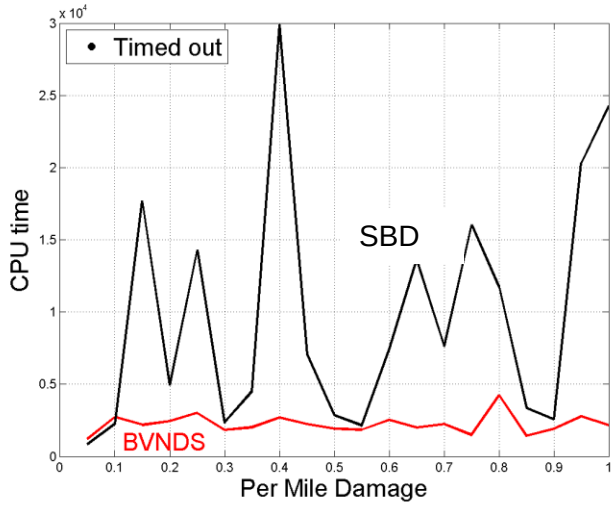


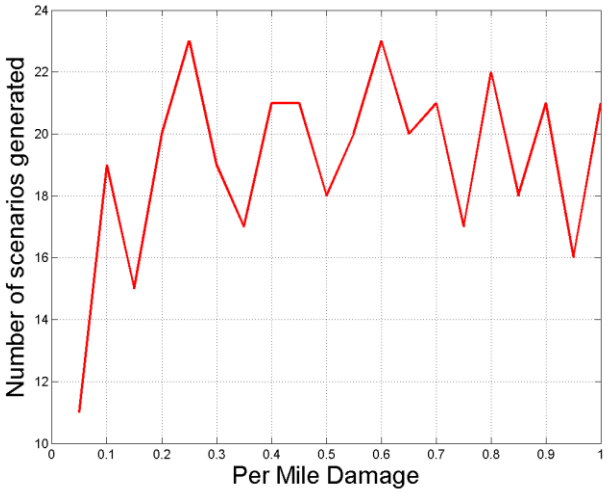
Exact vs. Heuristics

Why the objective fluctuations?



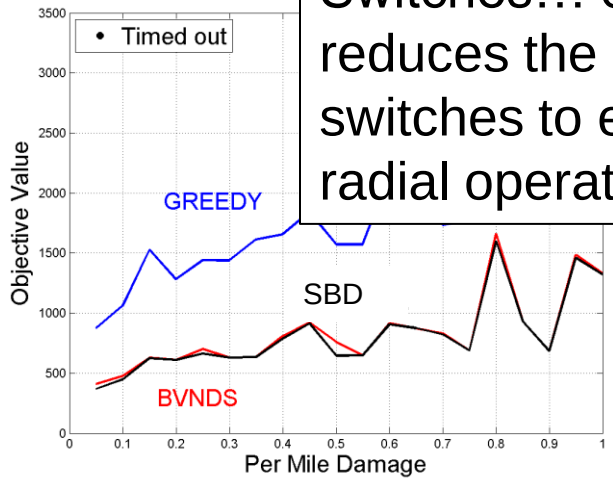


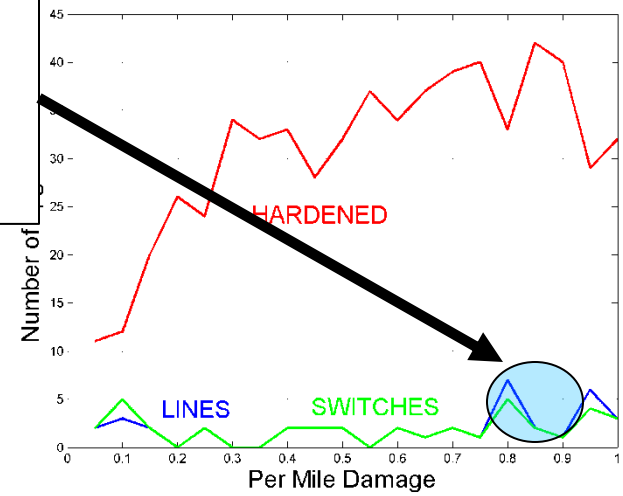


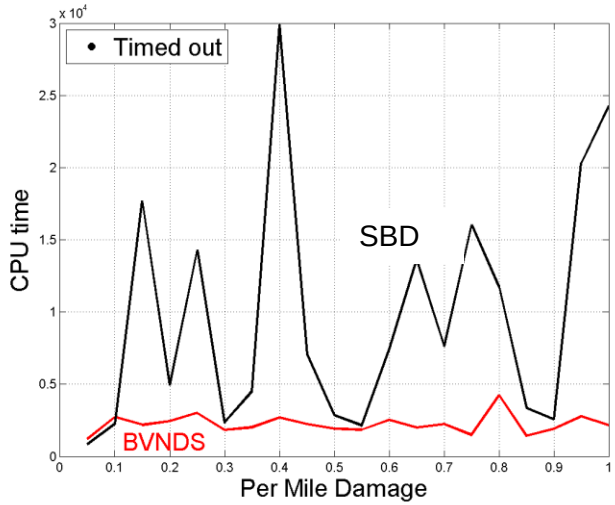


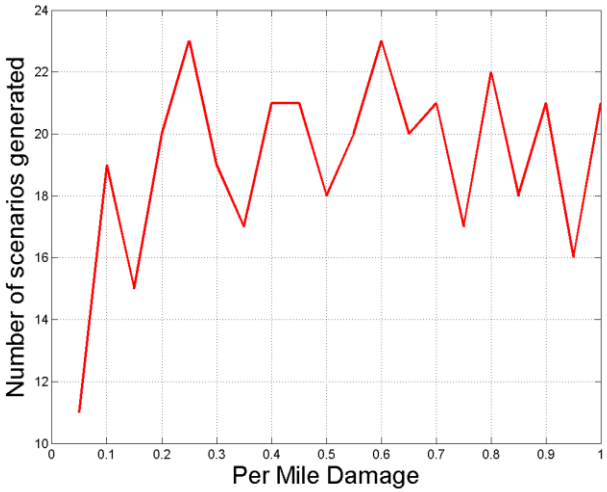
Exact vs. Heuristics

Switches… damage reduces the need for switches to enforce radial operations



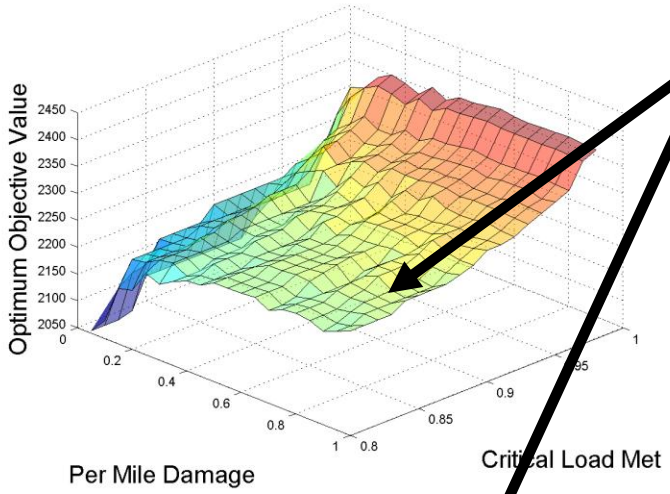


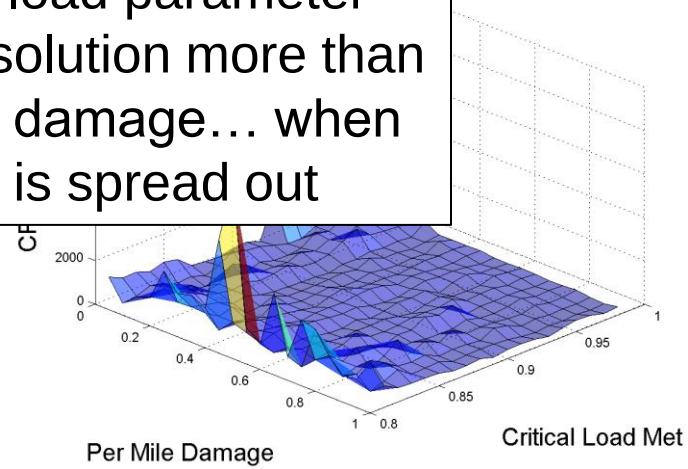


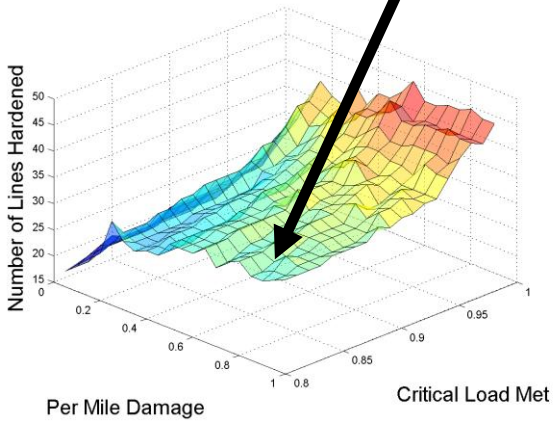


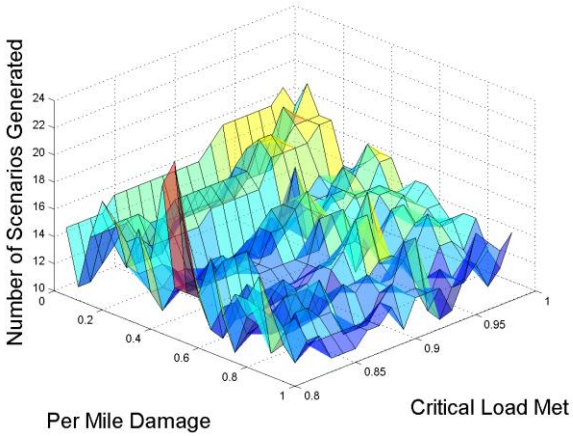
Resilience Criteria: Residential

Critical load parameter drives solution more than system damage… when system is spread out



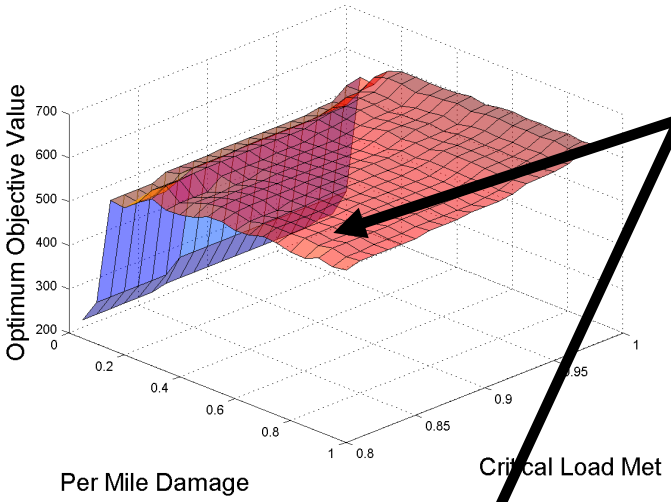


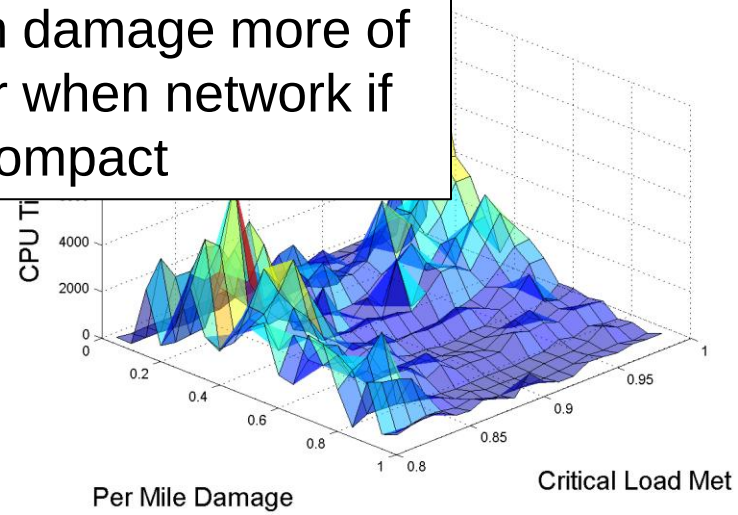


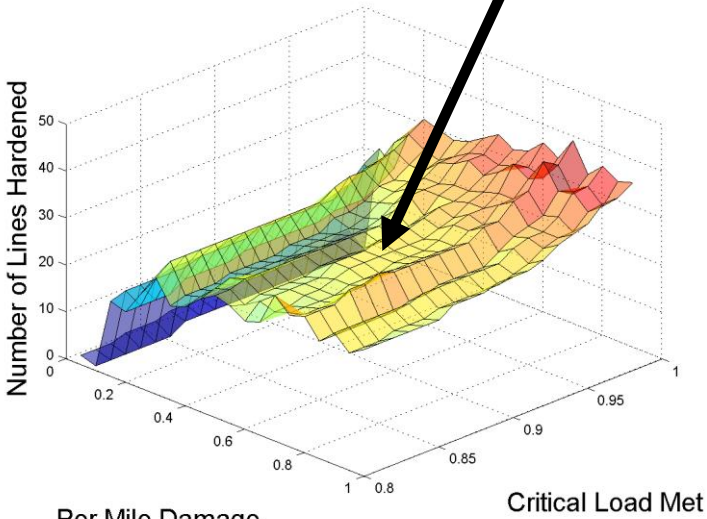


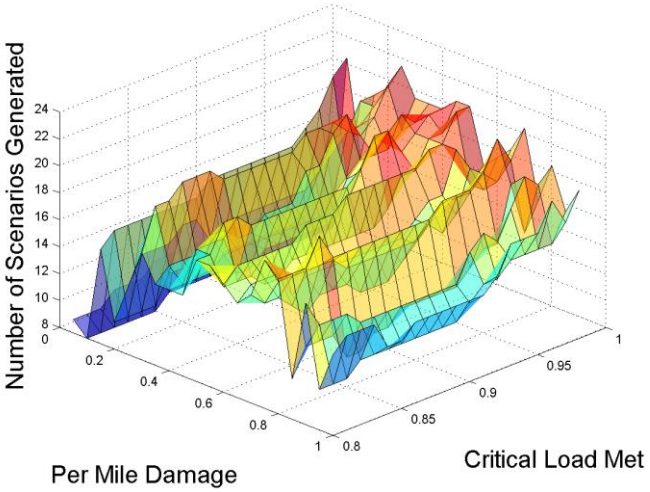
Resilience Criteria: Urban

System damage more of a driver when network if more compact



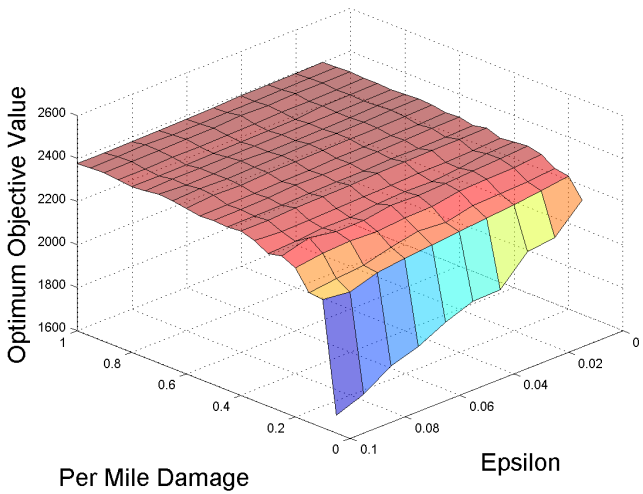


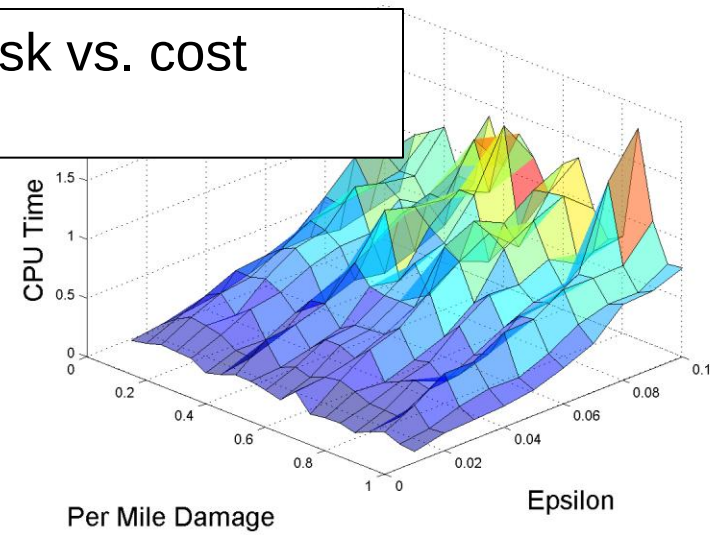


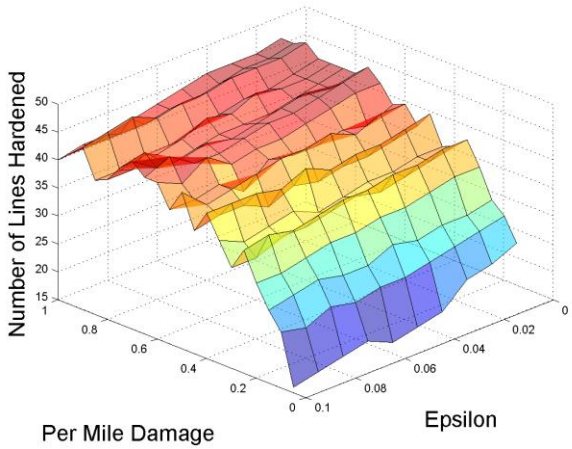


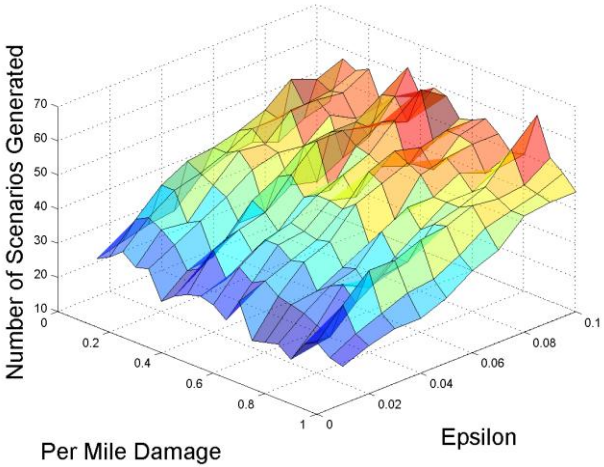
Chance Constraints

Assess risk vs. cost tradeoff

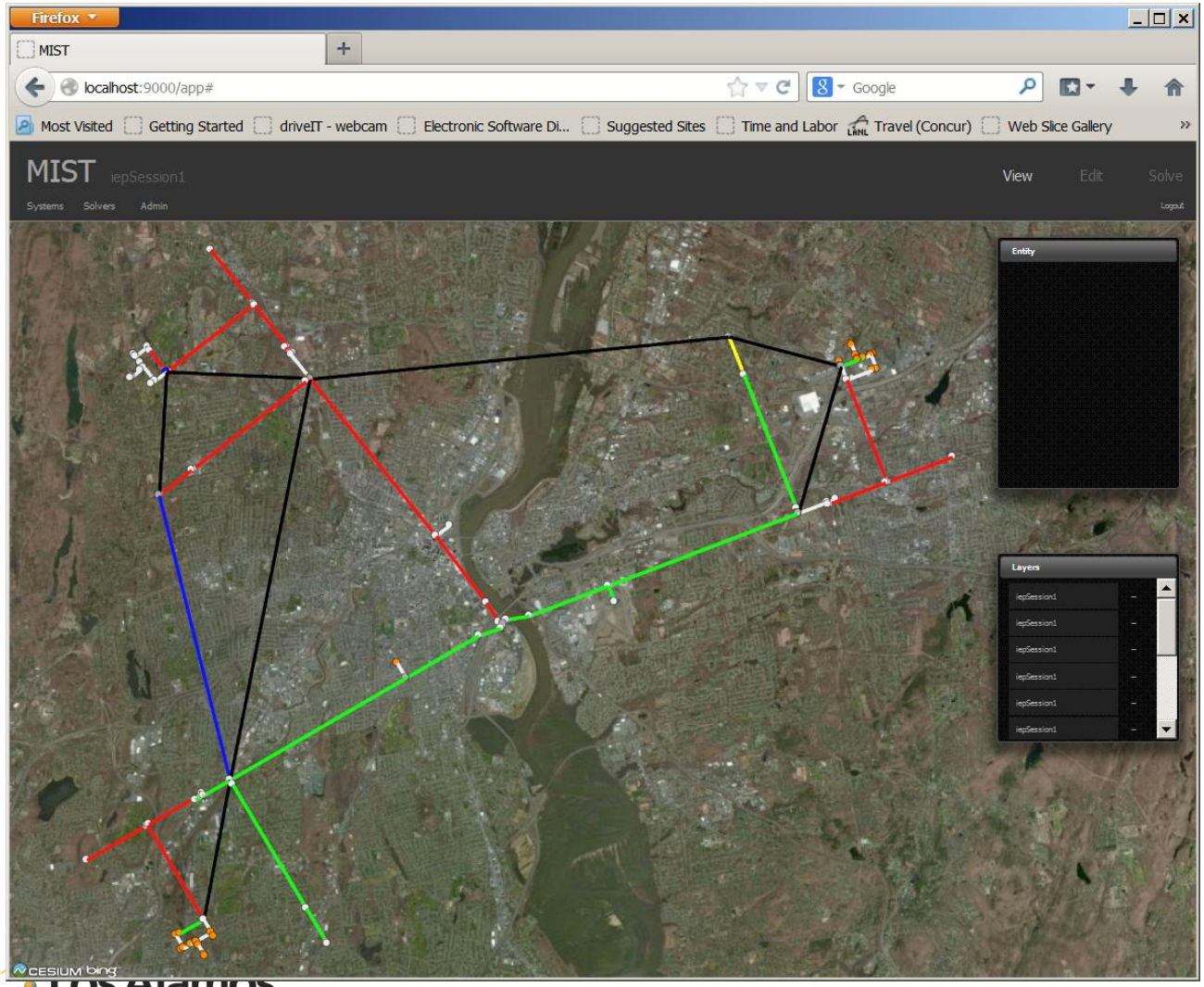








System interaction via MIST



- Critical Load
- Generation
- Damaged Lines
- Hardened Lines
- New Lines
- Unbuilt New Lines
- Switches

Future Work

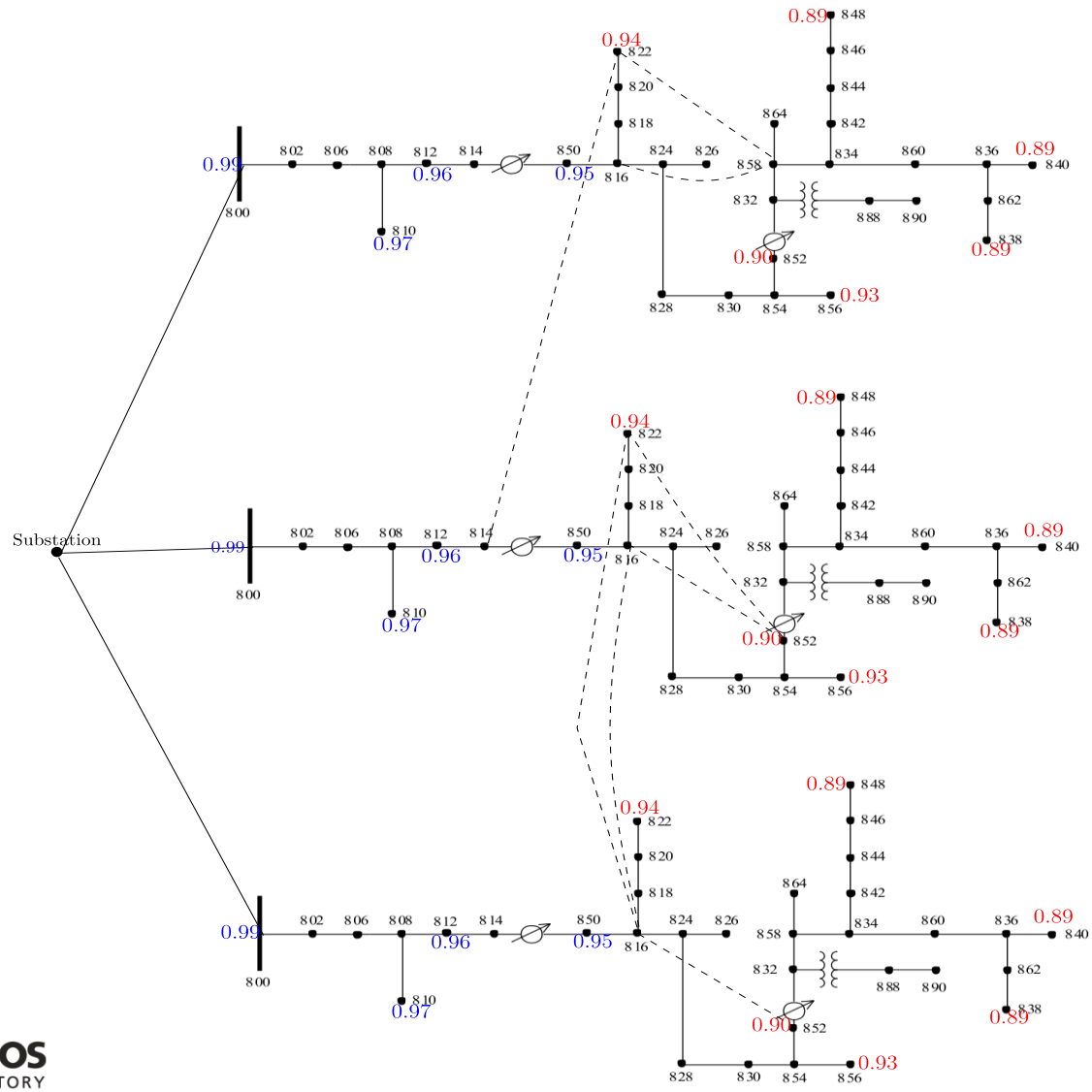
Reference

- E. Yamangil, R. Bent, S. Backhaus. *Optimal Resilient Distribution Grid Design Under Stochastic Events*, AAAI 2015

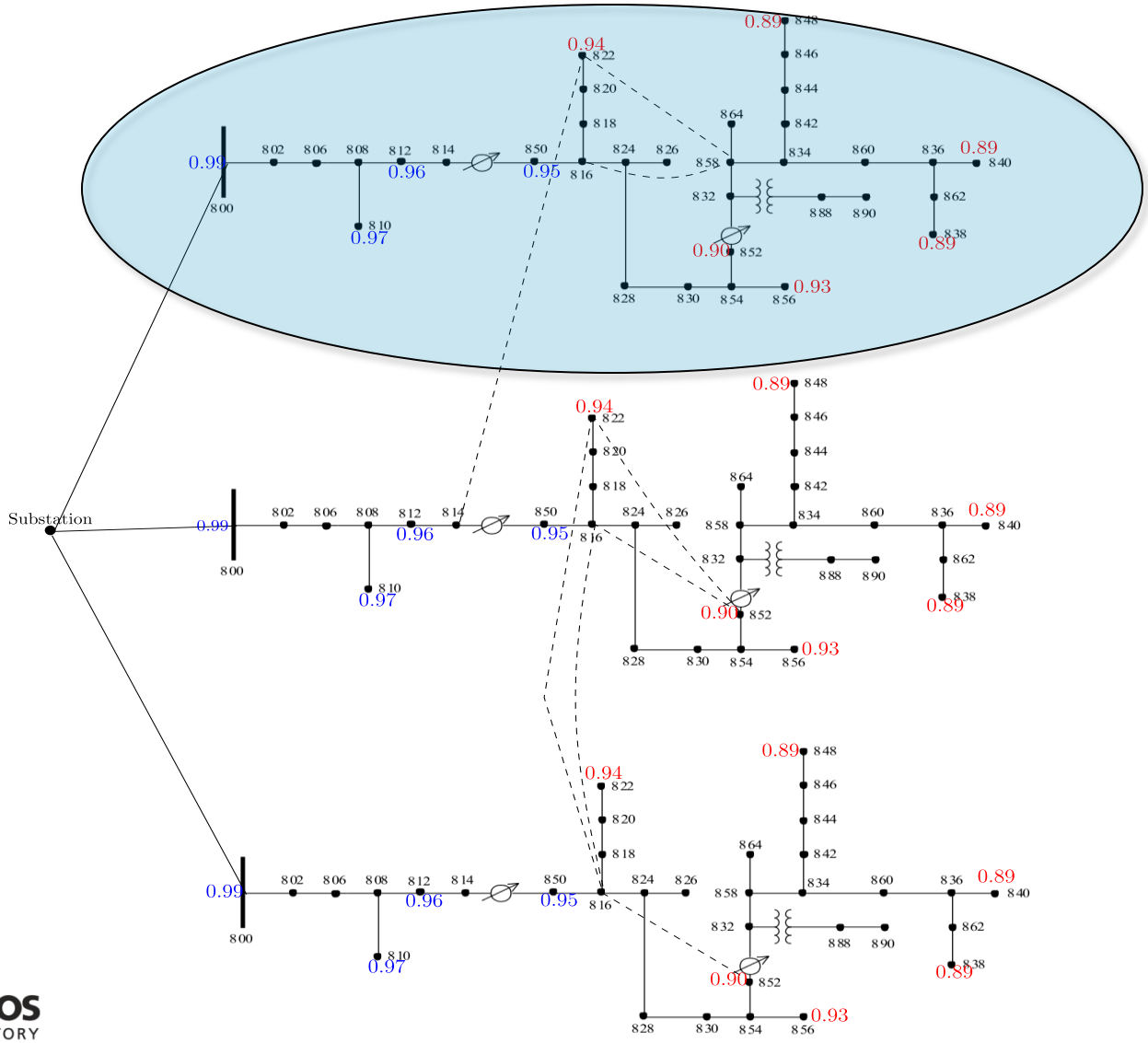
Generalizations

- Multi-Commodity Flow Relaxation
  - Voltage and Reactive power are ignored.
  - Initial network is voltage feasible, upgrades tend to move loads closer to generation, which improves voltage and lowers line flows
  - May not always hold
    - No good/L-shaped cuts
    - DistFlow formulation – derive a linearization of 3 phase formulation
      - Gan and Low 2014
- Larger networks
  - Graph-based decompositions

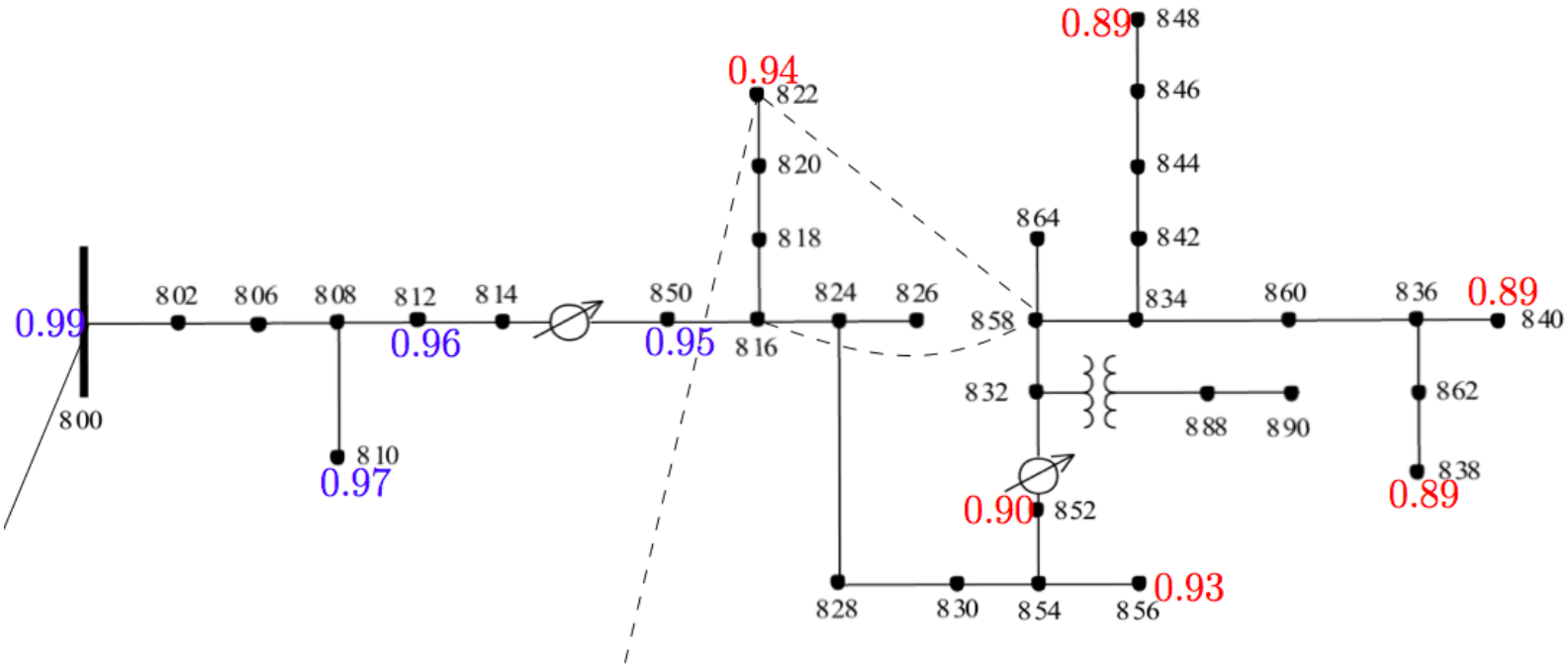
Algorithm Enhancements (example)



Algorithm Enhancements (example)



Algorithm Enhancements (example)



Algorithm Enhancements

A. Cutting plane algorithm

1. Solve the MILP based on the decomposition approaches. Let T_1 be a radial network satisfying a scenario.
2. Solve the scenario base topology (T_1) using a power flow solver (i.e. GridLab-D) to obtain the voltage and reactive power profile.
3. If the voltage profile and line limits are within the prescribed bounds, the MILP solutions satisfies the 3-phase AC power flow equations. Else, augment the MILP with the following "no good" cut:

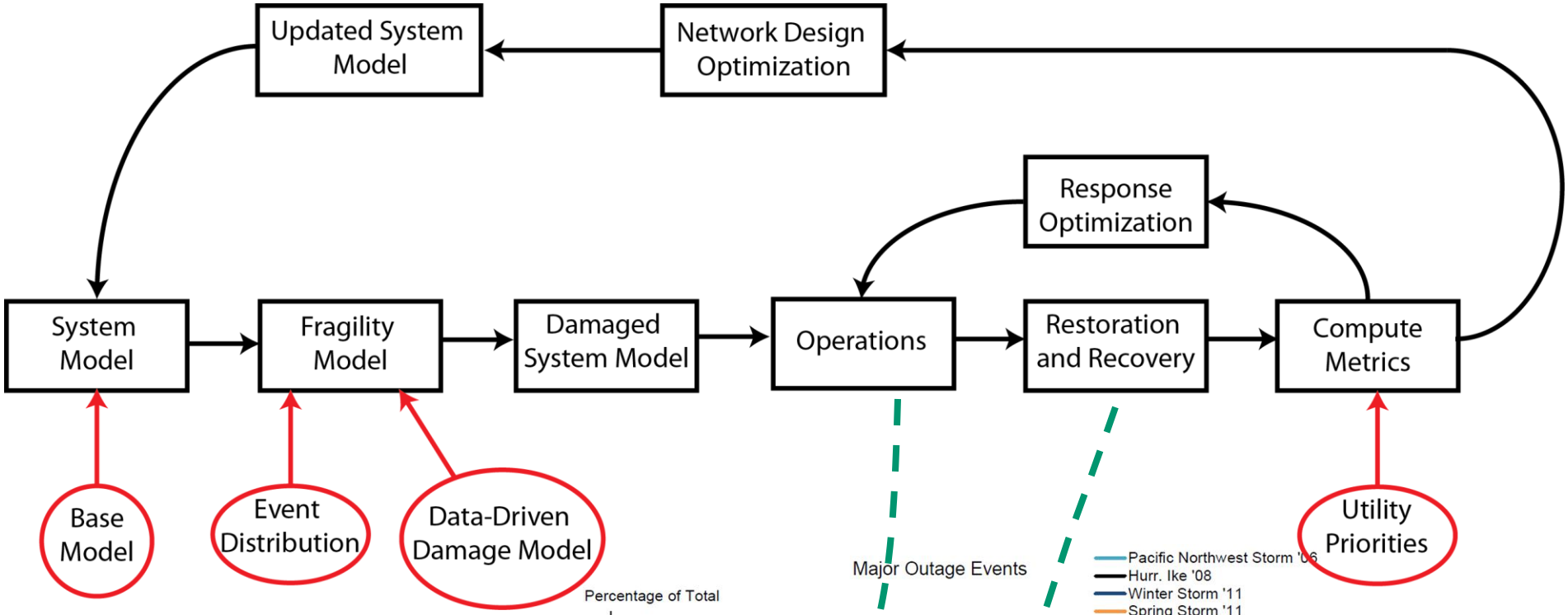
$$\sum_{e \in T_1} x_e \leq (|T_1| - 1)$$

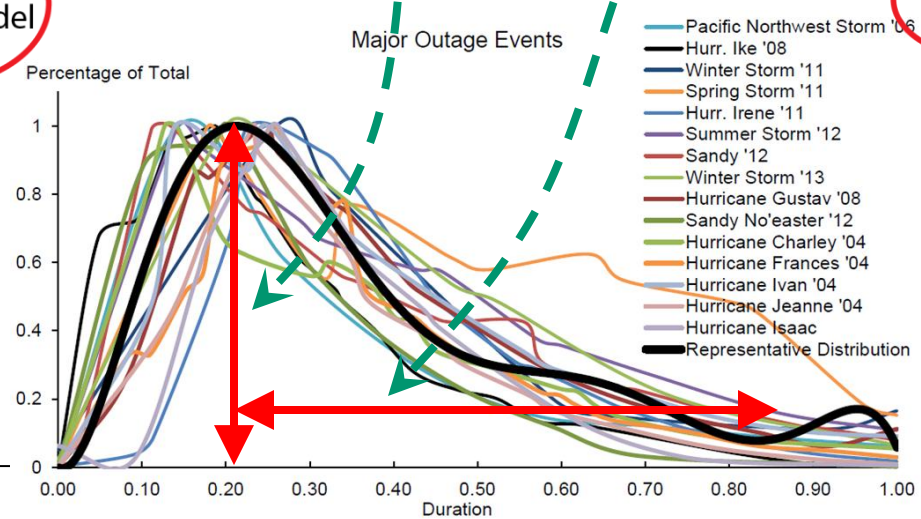
where $|T_1|$ represents the number of edges in T_1

Disclaimer: Stronger cuts can be derived when the details of the underlying power flow equations are known

B. Strengthening the power flow model

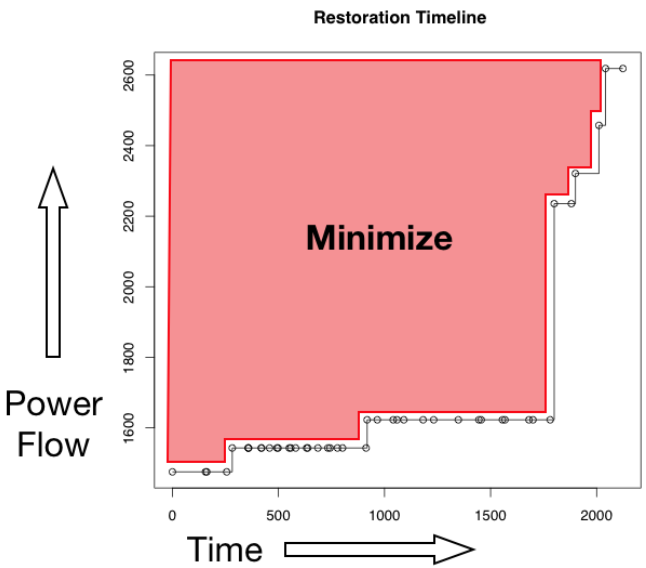
Long Term: Incorporate restoration





Source: Department of Energy, Office of Electrcity Delivery and Energy Reliability

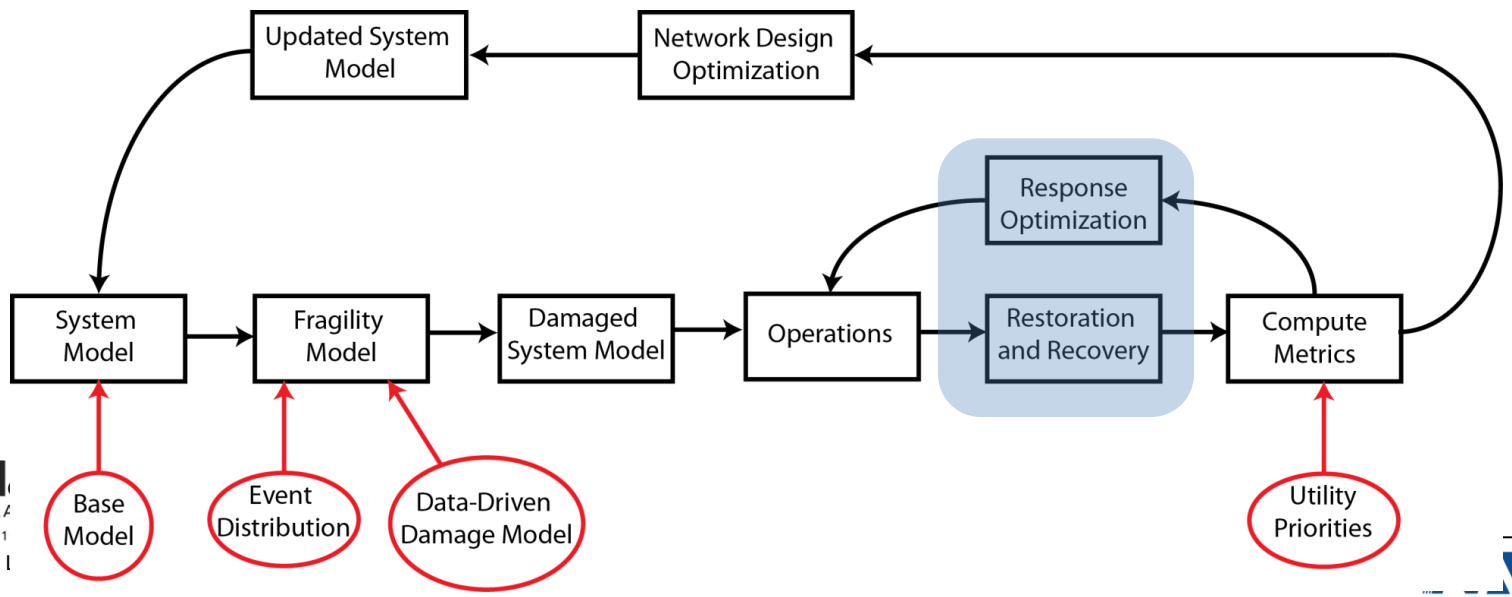
Restoration



Example: Minimize the size and duration of a black out.

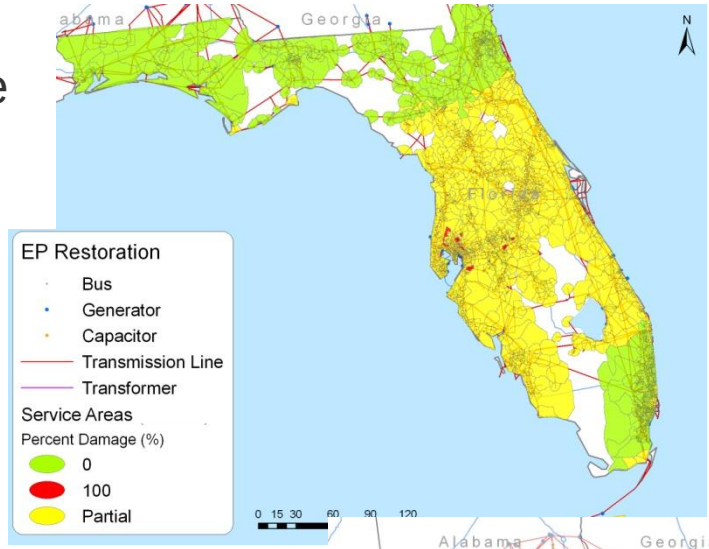
Combine grid operation requirements (restore power as quickly as possible) with transportation requirements (routing crews on a potentially damaged road network)

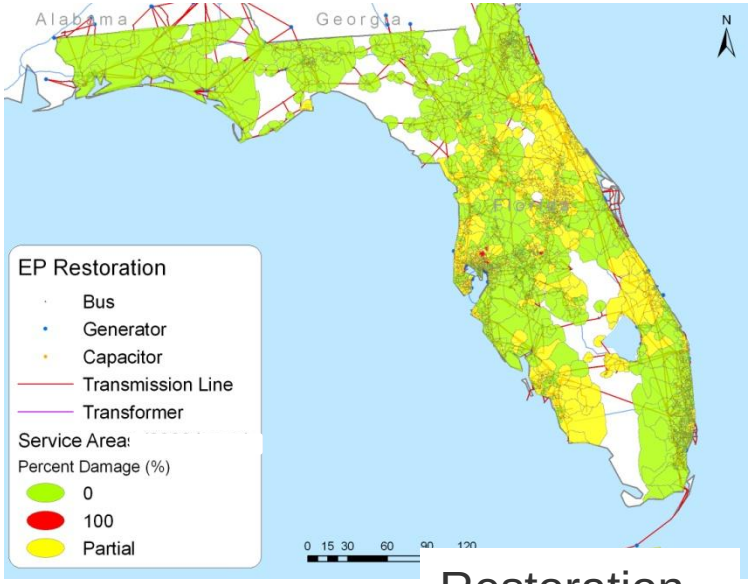
P. van Hentenryck, C. Coffrin, and R. Bent *Vehicle Routing for the Last Mile of Power System Restoration*. 17th Power Systems Computation Conference (PSCC 2011), August 2011, Stockholm, Sweden.



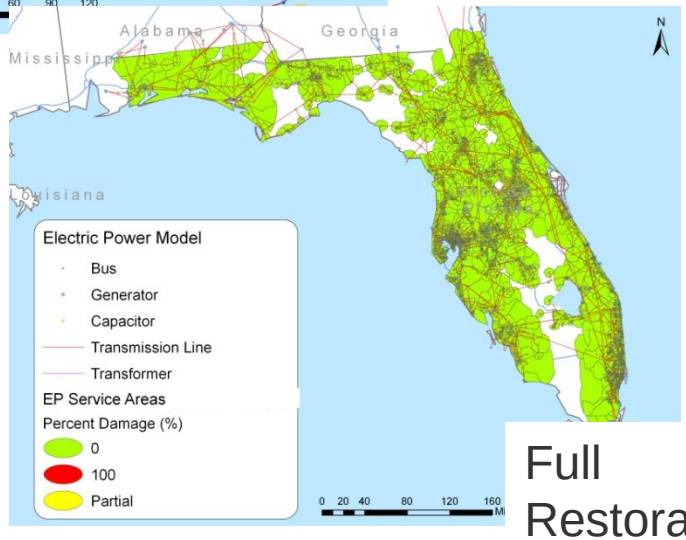
Restoration (only) Example—Applied To Transmission Setting

Initial Outage Area





Restoration After 2 Weeks

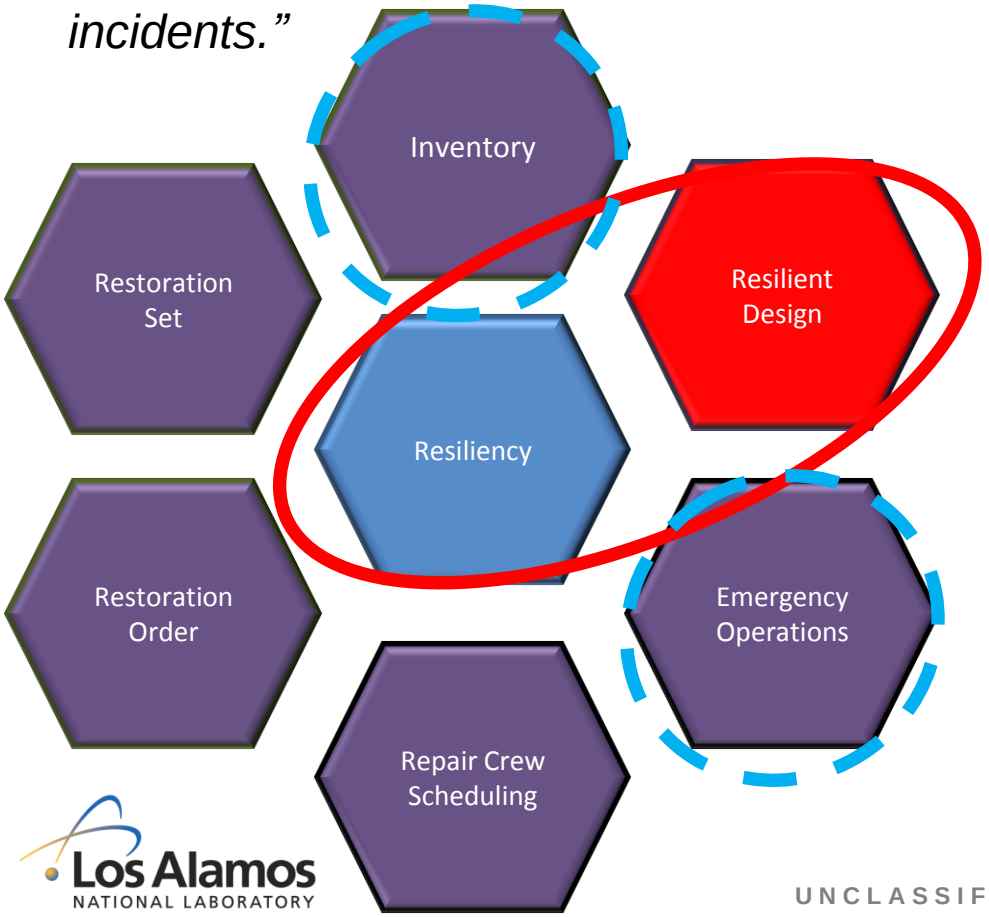


Full Restoration

Beyond the End Goal—Resiliency Tool Suite

Presidential Policy Directive - Critical Infrastructure Security and Resilience

*"The ability to **prepare for** and adapt to changing conditions and **withstand** and **recover** rapidly from disruptions. Resilience includes the ability to **withstand** and **recover** from deliberate attacks, accidents, or naturally occurring threats or incidents."*



Decision support tool for critical infrastructure disaster planning and response, composed of interconnected modules

Today—Resilient deign to **withstand** initial blow

End Goal— + System restoration to capture **recovery** from initial blow

Beyond the End Goal— + Inventory and Emergency operation to **prepare** for events