Lyapunov Functions Family Approach to Transient Stability Assessment

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Motivations

1. Emerging technologies are changing the way power grids operate
2. The existing planning and operation computational techniques have to be reassessed
3. Variations in the grids (e.g. mechanical torques, topology) result in different operating conditions

Problems

We consider two stability assessment problems of power grids:

Transient stability: *Estimate the region of attraction of the stable equilibrium point* $\delta^* = [\delta_1^*, ..., \delta_n^*, 0, ..., 0]^T$, *i.e. the set of initial conditions* $\{\delta_k(0), \dot{\delta}_k(0)\}_{k=1}^n$ *starting from which the system* (1)-(2) *converges to* δ^*.

Robust transient stability: *Certify the stability of the system* (1)-(2), *where the mechanical torques* P_k *are varying such that the stable equilibrium point* δ^* *is in the polytope* Θ *defined by the inequalities* $|\delta_{kj}^*| \leq \Delta_{kj}$.

Contributions

1. Introduced the LFF approach to certify the transient stability of structure-preserving multimachine power grids
2. This approach is applicable to lossy power grids, which is impossible by the standard energy methods
3. Presented optimization and LMI-based techniques to explicitly construct the stability certificates and to adapt the Lyapunov functions to initial states
4. Posed a new control problem of stability assessment for systems with unknown equilibria
5. Applied the LFF approach to uncertain power grids with unknown equilibrium points and provided robust stability certificates

Future Directions

1. Extend the LFF stability assessment approach to more realistic models of generators, loads, and transmission network:

   - Structure-preserving models with reactive powers: extending the nonlinearity F
   - Higher-order models of generators with voltage dynamics, and higher-order loads

2. Design optimization algorithms to improve computational efficiency

Structure Preserving Dynamical Swing Equations

Consider the structure-preserving model of power grids described by the swing equations:

$$(\mathcal{G}) \quad m_k\ddot{\delta}_k + d_k\dot{\delta}_k = P_{m_k} - V_k^2 G_k - \sum_{j\in\mathcal{N}_k} V_k V_j B_{kj}\sin(\delta_k - \delta_j), \qquad k = 1, ..., m \tag{1}$$

$$(\mathcal{L}) \quad d_k\dot{\delta}_k = -P_{d_k}^0 - \sum_{j\in\mathcal{N}_k} V_k V_j B_{kj}\sin(\delta_k - \delta_j), \qquad k = m+1, ..., n \tag{2}$$

The operating condition is characterized by the angle differences $\delta_{kj}^* = \delta_k^* - \delta_j^*$ satisfying:

$$\sum_{j\in\mathcal{N}_k} V_k V_j B_{kj} \sin\delta_{kj}^* = P_k, k = 1, \ldots, n \tag{3}$$

LFF Approach

Nonlinearity separation and bounding

$$\dot{x} = Ax - BF(Cx) \tag{4}$$

$x = [\delta_1, \ldots, \delta_m, \dot{\delta}_1, \ldots, \dot{\delta}_m, \delta_{m+1}, \ldots, \delta_n]^T$
$F(Cx) = [(\sin\delta_{kj} - \sin\delta_{kj}^*)]_{\{k,j\}\in\mathcal{E}}^T$

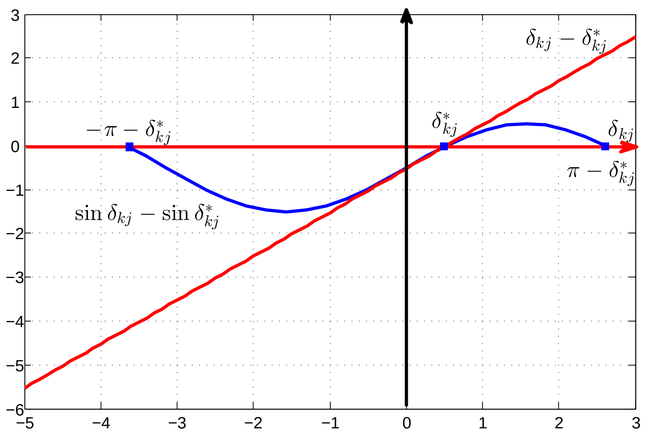

FIG. 1: Linear bounding of the nonlinear couplings

Lyapunov Functions Family

$$V(x) = \frac{1}{2}x^T Qx - \sum K_{k,j}(\cos\delta_{kj} + \delta_{kj}\sin\delta_{kj}^*)$$

where Q, K, H are solutions of the LMIs:

$$\begin{bmatrix} A^TQ + QA & R \\ R^T & -2H \end{bmatrix} \leq 0, \tag{5}$$

with $R = QB - C^TH - (KCA)^T$.
Then, $\dot{V}(x) \leq 0$ for any state x stays within the polytope $\mathcal{P} := \{x : |\delta_{kj} + \delta_{kj}^*| \leq \pi\}$.

Adaptation to Initial States
Let ϵ be a positive constant.
Step 1: Find $Q^{(1)}, K^{(1)}, H^{(1)}$ by solving (5).
Step n: Find $Q^{(n)}, K^{(n)}, H^{(n)}$ by solving the following LMIs:

$$\begin{bmatrix} A^TQ^{(n)} + Q^{(n)}A & R^{(n)} \\ (R^{(n)})^T & -2H^{(n)} \end{bmatrix} \leq 0,$$

$$V^{(n)}(x_0) \leq V_{min}^{(n-1)} - \epsilon$$

Once infeasible, ϵ is replaced by $\epsilon/2$.

Results

Let $V_{\min} := \min_{x\in\partial\mathcal{P}^{out}} V(x)$. *Then, the set* $\mathcal{R} := \{x \in \mathcal{P} : V(x) < V_{\min}\}$ *is invariant, and an estimate of stability region of the SEP* δ^*.

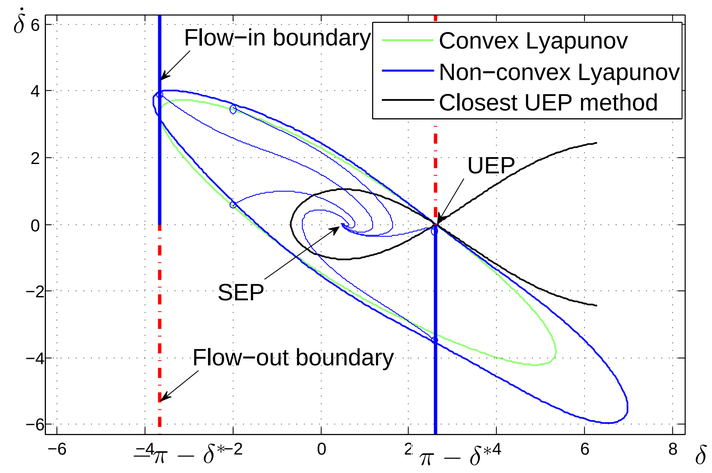

FIG. 2: Stability region estimates are intersections of the Lyapunov function sublevel sets and $\mathcal{P}$

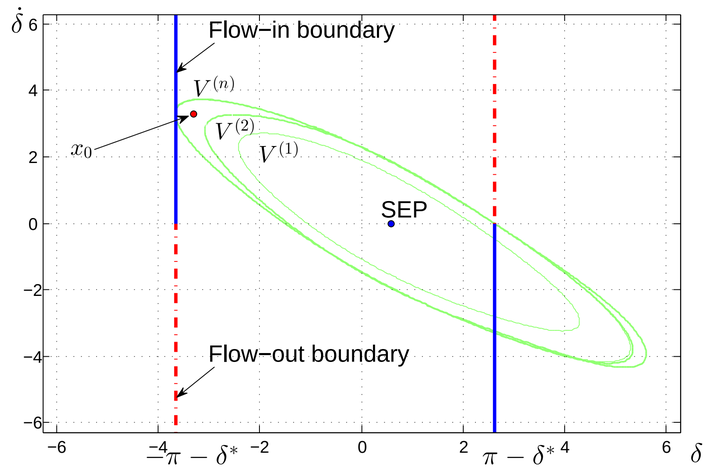

FIG. 3: Adaptation of the Lyapunov functions to x_0

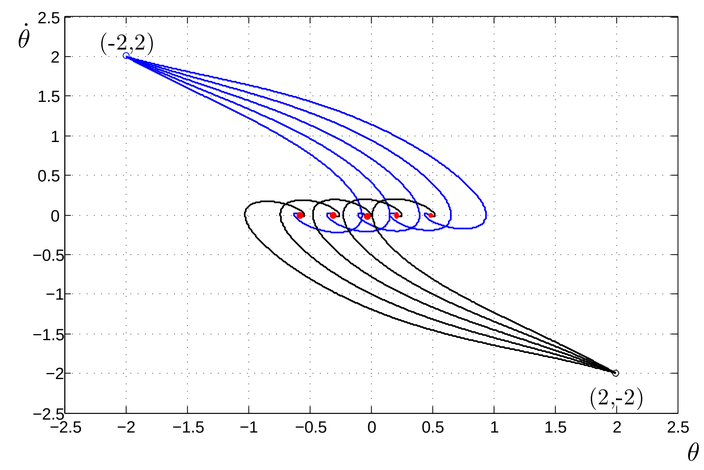

FIG. 4: Robust stability of $\{\delta_0 = -2, \dot{\delta}_0 = 2\}$ when δ^* is in the set $-\pi/6 - 0.05 \leq \delta^* \leq \pi/6 - 0.05$

References

[1] T. L. Vu, K. Turitsyn. Lyapunov Functions Family Approach to Transient Stability Assessment. *IEEE Transactions on Power Systems*, in revision.

[2] T. L. Vu, K. Turitsyn. Synchronization Stability of Lossy and Uncertain Power Grids. *2015 American Control Conference*, accepted.

[3] T. L. Vu, K. Turitsyn. Geometry-based Estimation of Stability Region for a Class of Structure Preserving Power Systems. *2015 PES General Meeting*, in review.

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