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Security Constrained Optimal Power Flow with Distributionally Robust Chance Constraints

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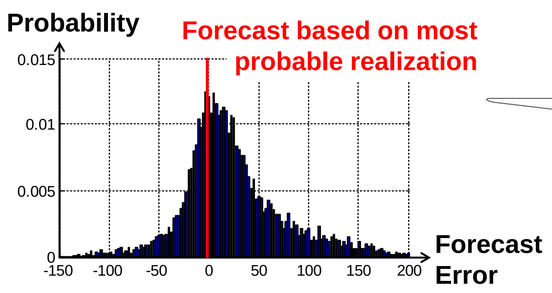
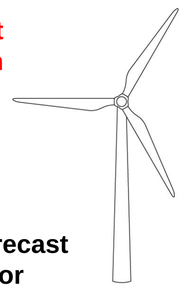
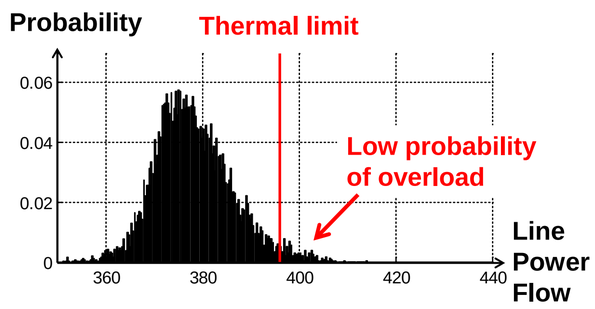
PROBLEM: Uncertain power injections → uncertain power flows

Uncertainty from
- Renewables
- Market liberalization (intra-day trading)





GOAL: Keep the system operation N-1 secure despite uncertainty



Use *chance constrained optimal power flow* to keep the *probability of line overload* below acceptable values

1. CHANCE CONSTRAINED OPTIMAL POWER FLOW

- Chance constraints reflect the probability of constraint violation
- Optimal power flow formulation based on DC power flow
- If a violation occurs, additional remedial actions are required from the system operator in real-time

- **CHANCE CONSTRAINT FOR POST-CONTINGENCY LINE FLOW**

Scheduled power flow + change due to outage — Change due to RES fluctuations — Desired confidence level

$$\mathbb{P}\left(A^i P^i_{inj} + D^i \delta P_{inj} \leq P^{max}_{ij}\right) \geq 1 - \varepsilon$$

P_{inj}	Scheduled in-feeds	δP_{inj}	Uncertain in-feeds
A^i	System topology	μ	Mean vector
D^i	Influence of RES fluctuations	Σ	Covariance matrix
P^{max}_{ij}	Line flow limit	ε	Accepted violation prob.

2. ANALYTIC REFORMULATION OF THE CHANCE CONSTRAINTS

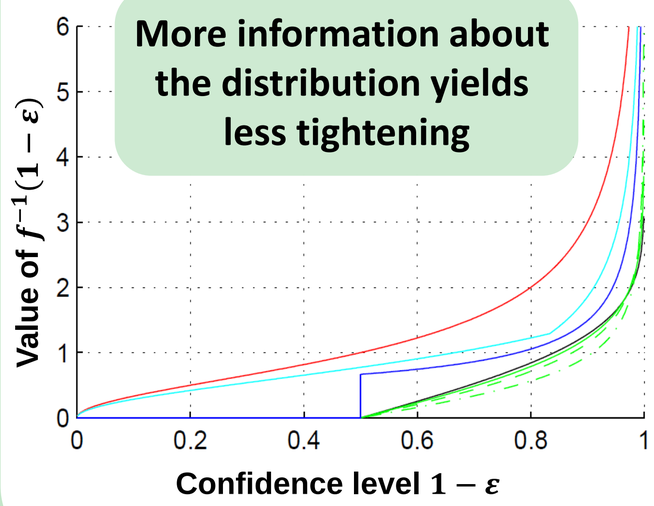
+ Influence of each uncertainty source is traceable
+ Deterministic solution (not dependent on the choice of samples)
– Only applicable when D^i is constant (linear program) or linearly in the decision variables (SOCP)

- **REFORMULATED CONSTRAINT**

Deterministic constraint — Stochastic tightening of constraint

$$A^i P^i_{inj} \leq P^{max}_{ij} - f^{-1}(1-\varepsilon)\left\|D^i \Sigma^{1/2}\right\|_2 - D^i \mu$$

3. $f^{-1}(1-\varepsilon)$ DEPENDS ON (UNKNOWN) DISTRIBUTION OF δP_{inj}

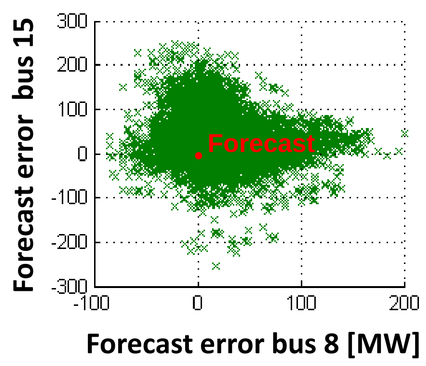


Exact reformulation if distribution is known and elliptical:
- normal distribution
- t distribution

Distributionally robust constraints if distribution is partially known:
- Chebyshev (known μ & Σ)
- Unimodal with known μ & Σ
- Symmetric, unimodal with known μ & Σ

CASE STUDY: IEEE RTS 96 WITH UNCERTAIN IN-FEEDS

- Two uncertain in-feeds (bus 8, 15)
- μ, Σ based on samples of historical data from APG
- **Not normally distributed!**
- Accepted violation probability: $\varepsilon = 0.075$
- Different assumptions on probability distribution of δP_{inj}

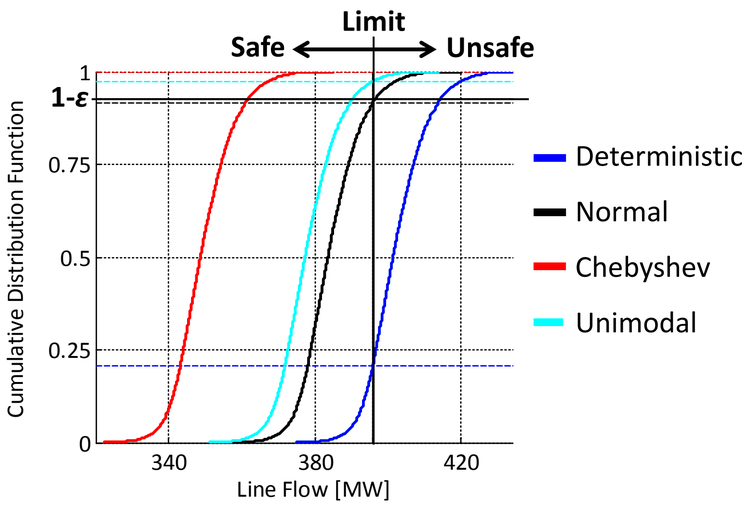


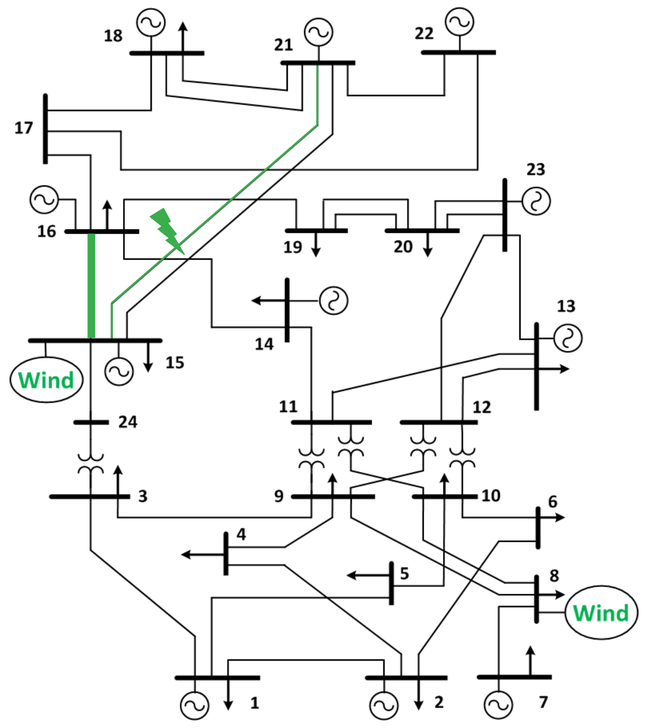
CASE STUDY RESULTS

POWER FLOW ON LINE 15 – 16

Active constraint: Power flow on line 15-16 after outage of line 15 – 21

Cumulative distribution function:

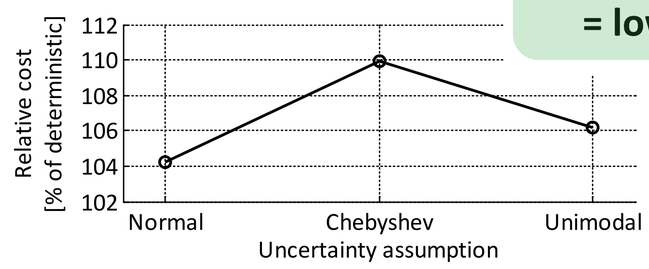




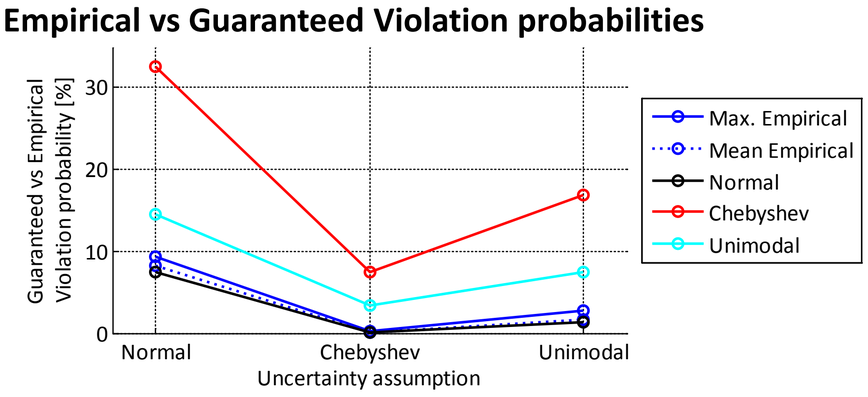
COST AND VIOLATIONS

Objective function cost

More information = lower cost



Empirical vs Guaranteed Violation probabilities



Normal distribution is a «good guess», but provides no probabilistic guarantees

Chebyshev provides probabilistic guarantees, but is very conservative

Unimodal provides probabilistic guarantees, and is less conservative

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