Constrained Mean Field Control for Large Populations of Plug-in Electric Vehicles

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Introduction

Vehicles obtaining some or all of their energy from the electricity grid, as Plug-in (hybrid) Electric Vehicles (PEVs), may achieve significant market penetration over the next few years. This raises the question of how to optimally fulfill the corresponding energy requirement, by regulating the collective charging profile of large populations of PEVs, while guaranteeing the interest and the privacy of the users.

Here we address this task using a mean field game theoretical approach [1,2]. We consider PEVs as heterogeneous agents, with different charging constraints (plug-in times and deadlines), that minimize their own charging cost and are weakly coupled via a common electricity price.

Problem Formulation

Consider a charging horizon of T time steps. Let $\mathbf{u}_n = [u_{n,1}, \ldots, u_{n,T}]^\top$ be the vector of the energy required by vehicle $n \in \{1, \ldots, N\}$, which must belong to the personalized constraint set

$$\mathcal{U}_n := \left\{ \mathbf{u}_n \in \mathbb{R}^T \mid \sum_t u_{n,t} = \gamma_n, \;\; 0 \le u_{n,t} \le M_{n,t} \right\},$$

and $p\left(u_t^{\text{avg}}\right) := a(u_t^{\text{avg}} + d_t)$, $a > 0$, be an affine price function that depends on the total energy demand at time t. Each agent minimizes its charging cost by solving

$$\mathbf{u}_n^\star\left(\{\mathbf{u}_i\}_{i\neq n}^N\right) := \arg\min_{\mathbf{u}_n} \sum_t p\left(u_t^{\text{avg}}\right) u_{n,t} \qquad (1)$$
$$\text{s.t.} \quad \mathbf{u}_n \in \mathcal{U}_n$$

which leads to the aggregate behavior

$$\mathbf{u}^{\text{avg}} = \frac{1}{N}\sum_{n=1}^N \mathbf{u}_n.$$

As shown in Figure 1 (left) if uncontrolled the total energy demand may present undesirable peaks.

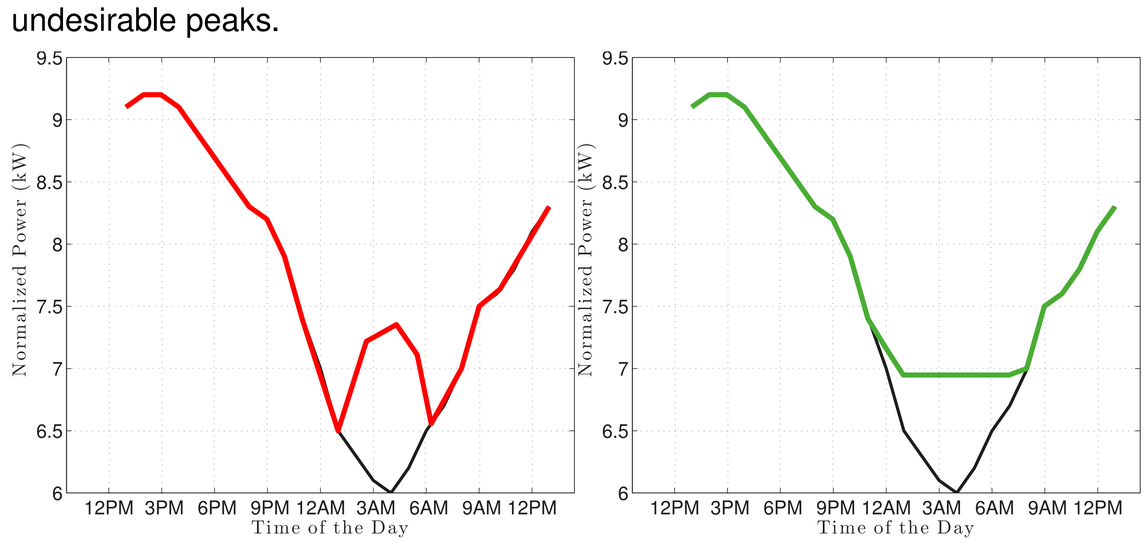

Figure 1 : Total energy demand divided by the number of vehicles N, according to different charging policies. The black line is the normalized base load demand d_t.

Nash Equilibrium

A set of strategies $\{\bar{\mathbf{u}}_n\}_{n=1}^N$ is stable if no agent has interest in deviating from its strategy given what the others are doing. Formally it is an ε-Nash equilibrium if

$$J(\bar{\mathbf{u}}_n | \{\bar{\mathbf{u}}_i\}_{i\neq n}^N) \le J(\mathbf{u}_n | \{\bar{\mathbf{u}}_i\}_{i\neq n}^N) + \varepsilon \quad \forall \mathbf{u}_n \in \mathcal{U}_n.$$

It was proven in [3], that the Nash equilibrium of problem (1), *in the absence of upper bounds*, is valley-filling, see Figure 1 (right). Hence the Nash equilibrium is both valley-filling and socially fair.

To steer the population to such desirable equilibrium we consider a quadratic relaxation of the original problem using the modified cost

$$J_\delta(\mathbf{u}^{\text{avg}}) := \sum_t p\left(u_t^{\text{avg}}\right) u_{n,t} + \delta(u_{n,t} - u_t^{\text{avg}})^2$$

where the parameter $\delta > 0$ has to be as small as possible. Note that, in the limit of infinite population size, the average $\mathbf{u}^{\text{avg}}$ can be though of as an exogenous signal $\mathbf{z}$ that must satisfy

$$\mathcal{A}(\mathbf{z}) := \frac{1}{N}\sum_{n=1}^N \mathbf{u}_n^\star(\mathbf{z}) := \frac{1}{N}\sum_{n=1}^N \arg\min_{\mathbf{u}_n \in \mathcal{U}_n} J_\delta(\mathbf{z}) = \mathbf{z}.$$

$\bar{\mathbf{z}}$ fixed point of the aggregation mapping $\mathcal{A}(\cdot)$

$\Downarrow$

$\{\mathbf{u}_n^\star(\bar{\mathbf{z}})\}_{n=1}^N$ is an ε-Nash equilibrium, with $\varepsilon \sim \mathcal{O}(1/N)$

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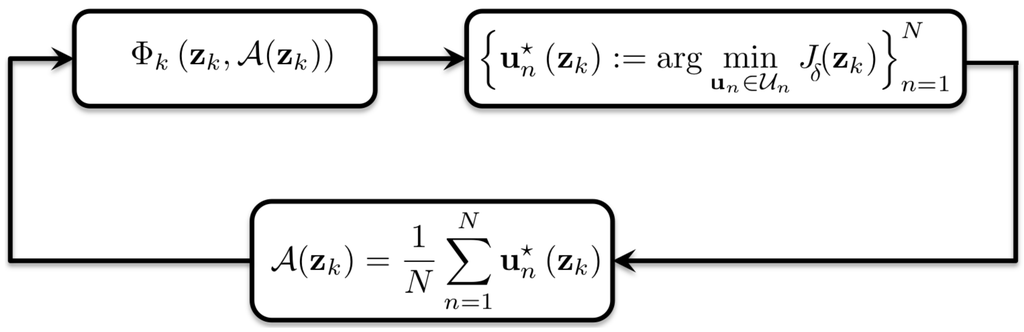
A Mean Field Control Algorithm

In order to find the fixed point of the aggregation mapping we consider an iterative scheme between

- a *central operator* that broadcasts at each iteration k the price signal $p(\mathbf{z}_k)$,
$$\mathbf{z}_k := \Phi_{k-1}(\mathbf{z}_{k-1}, \mathcal{A}(\mathbf{z}_{k-1}));$$
- the *agents* that respond by computing $\mathbf{u}_n^\star(\mathbf{z}_k)$;
- an *aggregator* that computes $\mathcal{A}(\mathbf{z}_k)$ and send it back to the central operator.



The iteration-dependent feedback mapping $\Phi_k(\cdot,\cdot)$ should be selected such that the algorithm converges to a fixed point of the aggregation mapping. To this end, we consider two fixed point iteration mappings

Picard-Banach [3] : $\Phi^{\text{P-B}}(\mathbf{z}_k, \mathcal{A}(\mathbf{z}_k)) := \mathcal{A}(\mathbf{z}_k)$

Mann [4] : $\Phi_k^{\text{M}}(\mathbf{z}_k, \mathcal{A}(\mathbf{z}_k)) := (1-\alpha_k)\mathbf{z}_k + \alpha_k \mathcal{A}(\mathbf{z}_k), \quad \alpha_k \propto 1/k$

The following convergence guarantees hold

	$\Phi^{\text{P-B}}$	Φ^{M}
$\delta > a/2$	✓	✓
$\delta > 0$		✓

Simulation

The use of the Mann iteration instead of the Picard-Banach allows one to find the fixed point for arbitrarily small values of δ, hence allowing to recover the Nash-equilibrium of the original Problem (1), [4]

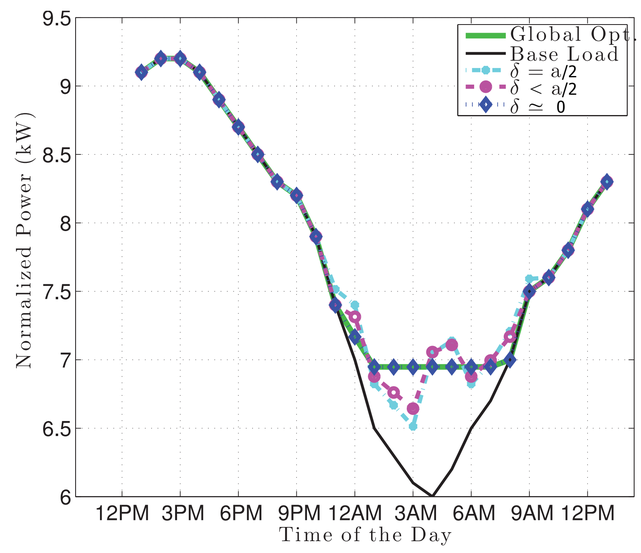


Figure 2 : Normalized total energy consumption relative to the Nash-equilibrium for different values of δ.

Generalization

The above results can be generalized to the broader class of quadratic, convex constrained, mean field games where each agent computes

$$\mathbf{x}_n^\star(\mathbf{z}) := \arg\min_{\mathbf{x}} \; \mathbf{x}^\top Q\mathbf{x} + (\mathbf{x}-\mathbf{z})^\top \Delta(\mathbf{x}-\mathbf{z}) + 2(C\mathbf{z}+c)^\top \mathbf{x}$$
$$\text{s.t.} \quad \mathbf{x} \in \mathcal{X}_n$$

In [5], it is proven that, under different conditions on the matrices Q, Δ, C, different feedback mappings $\Phi(\mathbf{z}, \mathcal{A}(\mathbf{z}))$ can be used to steer the population to an ε-Nash equilibrium, with $\varepsilon \sim \mathcal{O}(1/N)$.

References

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