Approximate-Current Instanton Analysis: Detecting Vulnerability in the Power Grid

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Introduction

Renewable fluctuations add vulnerability to the power grid.
As renewable generation fluctuates about its forecast, power flows and voltage magnitudes throughout the grid undergo complicated shifts. Most variations are harmless, but even a seemingly benign perturbation can violate network constraints.

The instanton is the most likely generation pattern that violates a constraint.
Of all possible generation patterns, some violate network constraints. Of these, the most likely to occur is the instanton.

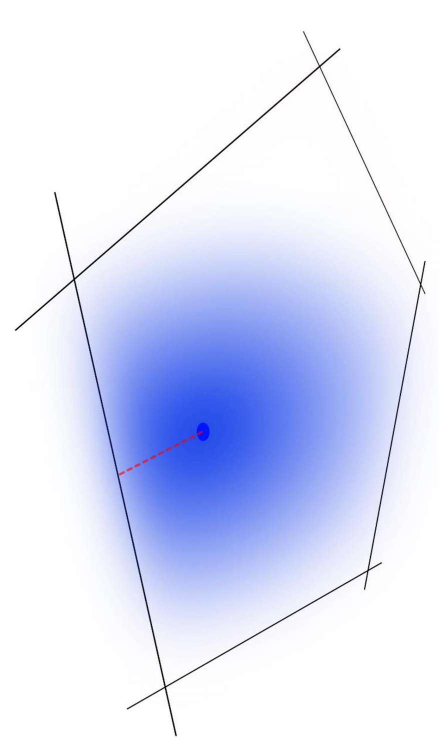
The nonlinear AC instanton problem has no guaranteed solution.
The accuracy of the AC instanton formulation has a price: the problem is non-convex and may not have a solution. This is unacceptable in a real-time environment.

The linear DC instanton problem has questionable accuracy.
When the DC power flow approximation is applied to the instanton problem, an analytic solution may be found by matrix inversion. Because the instanton lies on a network constraint, however, DC assumptions (like flat voltage profile) are suspect.

Accommodating non-flat voltage profiles improves instanton accuracy.
The new method maintains convexity while accommodating non-flat voltage profiles and reactive power injections.

Objectives

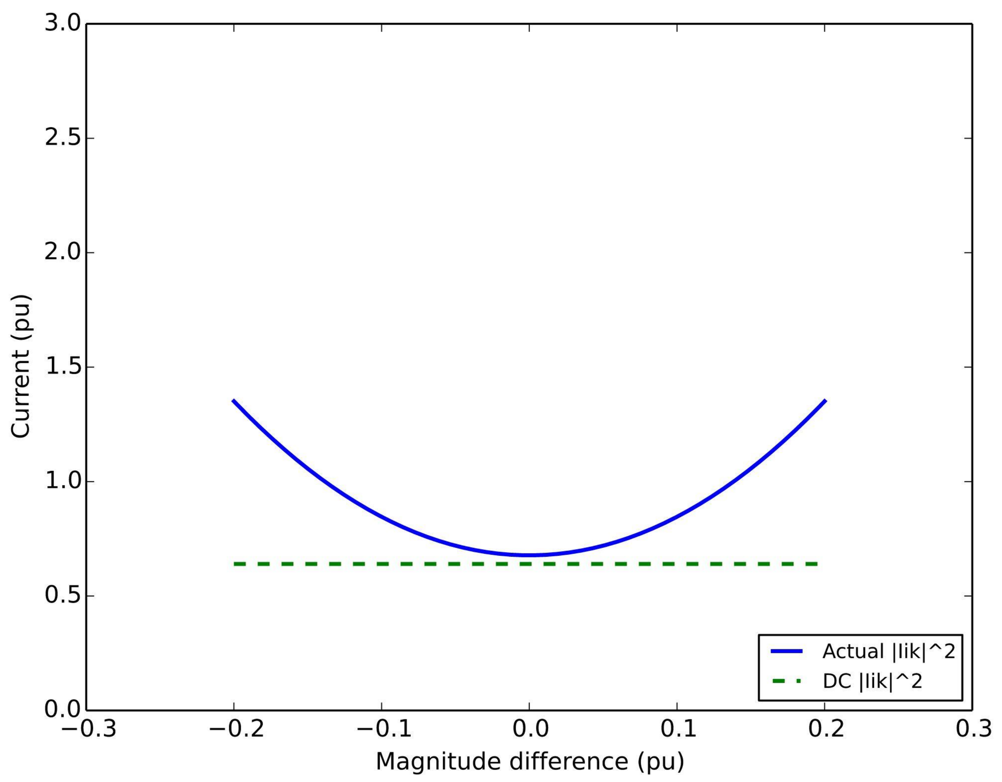
1. To use approximate AC methods to develop an instanton-finding tool with
   a) greater accuracy than DC instanton method,
   b) guaranteed convergence,
   c) rapid solution time (sub-exponential in network size), and
   d) insightful output (ranked list of extreme events, reconfiguration assistance)
2. To characterize differences between DC and approximate AC instanton methods
3. To understand differences between DC and approximate AC power flow methods



Two-dimensional depiction of instanton search. Starting from the forecast (dot), the dashed line shows the most likely shift in generation that would violate a constraint (solid line).

New Method: Current Constraint

Convexified network constraints form a tractable polytope.
Network constraints form a high-order polytope. Because this object may be expressed in terms of renewable generation, our optimization scheme can minimize over each face separately to obtain a ranked list of extreme events.



Comparison of improved current model with DC model

Constrained optimization formalizes the instanton problem.
By definition, the instanton is the most likely renewable generation pattern that violates one or more network constraints. To find the instanton candidate for a single constraint, we minimize deviation from forecast while saturating that constraint.

$$\min \frac{1}{2}(\boldsymbol{Q}_R - \boldsymbol{Q}_R^0)^\top \boldsymbol{\Lambda}(\boldsymbol{Q}_R - \boldsymbol{Q}_R^0)$$

subject to network constraints and saturated constraint

Previous analysis uses DC power flow assumptions: 1) no line resistance, 2) small angle differences, 3) flat voltage profile, 4) no reactive power flows. Retain first two assumptions, drop last two.

Approximate expressions for real and imaginary current:

$$Re(I_{ik}) \approx -B_{ik}[Im(E_i) - Im(E_k)]$$
$$Im(I_{ik}) \approx B_{ik}[Re(E_i) - Re(E_k)]$$

Square each term to obtain approximate current magnitude:

$$|I_{ik}|^2 \approx B_{ik}\left(V_i^2 + V_k^2 - 2V_iV_k\cos\theta_{ik}\right)$$

Solve for angle:

$$\theta_{ik} = \cos^{-1}\left(\frac{V_i^2 + V_k^2 - \left(|I_{ik}^{lim}|x_{ik}\right)^2}{2V_iV_k}\right)$$

Compare with DC constraint: $\theta_{ik} = x_{ik}P_{ik}^{\text{lim}}$

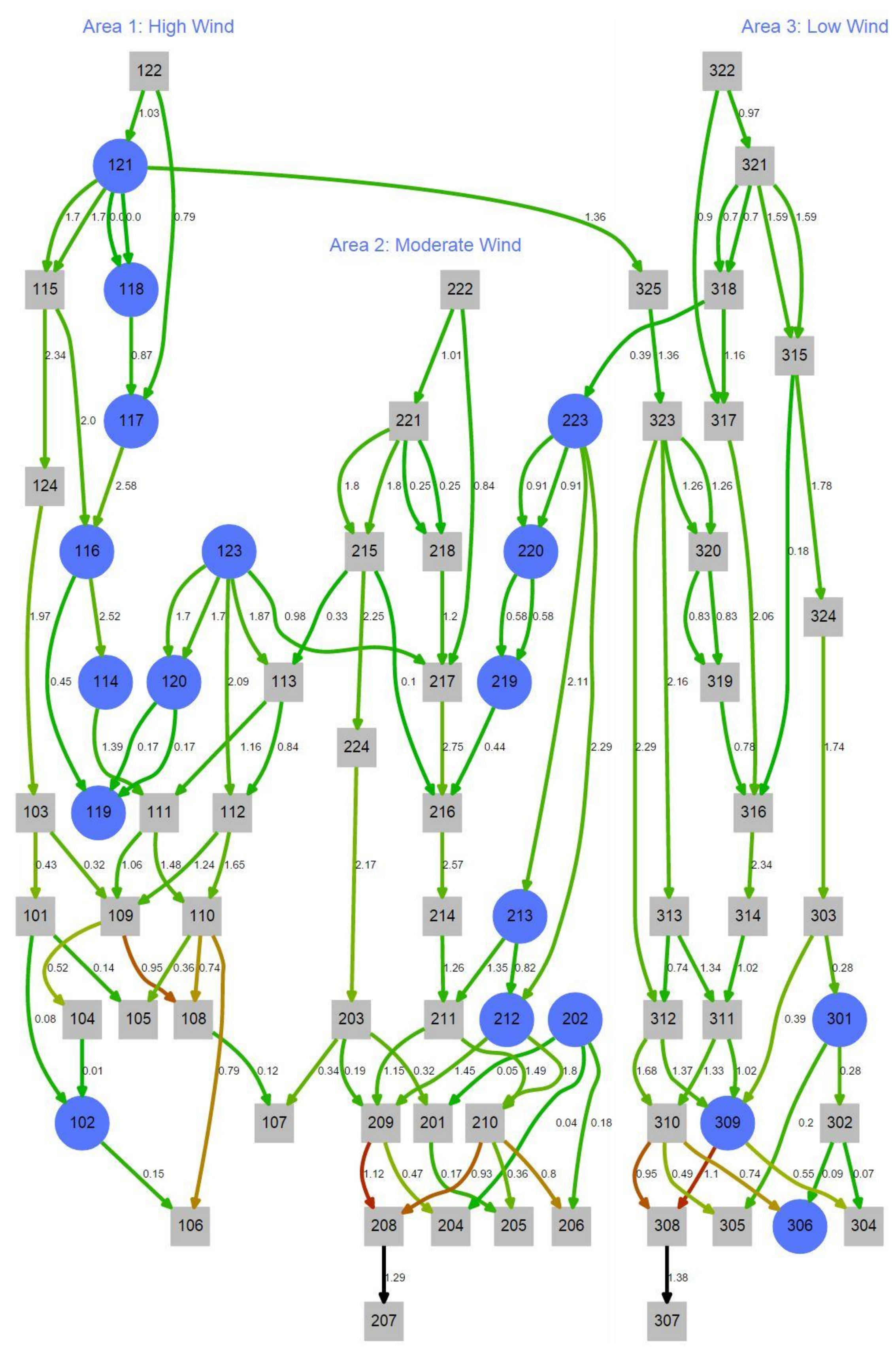
Any line with significant voltage difference is more vulnerable than DC instanton analysis says.

PQ decoupling means we can pre-compute voltage magnitudes from reactive injections. (If voltages were not fixed, we would need to approximate cosine.)

Acknowledgements

- Dr. Hiskens, my adviser
- Dr. Chertkov for hosting me at LANL this past summer
- Dr. Backhaus for discussions and decisions
- Dr. Bent for implementation discussions
- Miles Lubin for introducing me to Julia/JuMP
- Yury Dvorkin for stimulating discussions

Funding from Department of Energy Grid Science Initiative



Implementation on IEEE RTS-96 Network

References

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