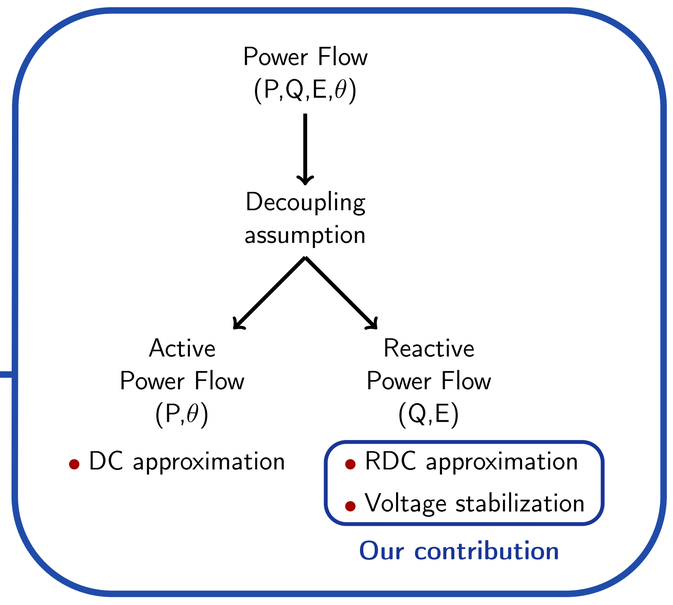
On Reactive Power Flow and Voltage Stability in Microgrids

Basilio Gentile and Florian Dörfler
Automatic Control Laboratory, ETH Zürich



ABSTRACT

As of today, solutions to the reactive power flow can be found only via numerical methods, yet an analytical understanding would be beneficial to the rigorous design of future electric grids. Firstly, for sufficiently high reference voltages, we guarantee the existence of a high-voltage solution for the reactive power flow and provide its approximate analytical expression (RDC approximation). We validate the accuracy of our approximation through numerical simulation of the IEEE 37 test case. Secondly, we consider a recently proposed droop control for voltage stabilization in a microgrid equipped with inverters. We prove the existence and the exponential stability of a high-voltage fixed point of the closed-loop dynamics. We provide an approximate expression for this fixed point.

TWO-NODE GRID

Figure 1 shows a grid composed by a source (or PV bus) and a load (or PQ bus), connected by a power line. The reactive power flow at the load reads

$$Q_L = V_L^2\ell - V_L V_S \cos(\theta_L - \theta_S)\ell - V_L V_S \sin(\theta_L - \theta_S) g \simeq V_L^2 \ell - V_L V_S \ell \,. \qquad (1)$$

The last approximation derives from the fact that during regular power system operation the angles difference is negligible.

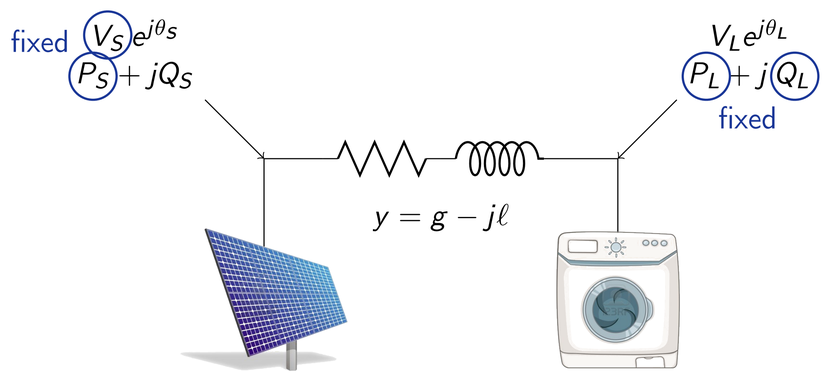


Figure 1: two-node grid

In the two-node case (1) becomes a second order equation with solution

$$V_{L,1,2} = V_S\left(\frac{1}{2} \pm \frac{1}{2}\sqrt{1 + \frac{4Q_L}{\ell V_S^2}}\right) \qquad \text{if} \quad V_S \geq \sqrt{-\frac{4Q_L}{\ell}}\,.$$

If $4Q_L/(\ell V_S^2) \ll 1$ the first-order Taylor expansion of the square root yields

$$V_{L,1} \simeq V_S + \frac{Q_L}{\ell V_S}, \qquad V_{L,2} \simeq -\frac{Q_L}{\ell V_S}\,.$$

The solution $V_{L,1}$ is the desired one, as it corresponds to a high-voltage low-current configuration for the network, resulting in low power losses. We can interpret the solution as being an odd-exponent power expansion in the source voltage V_S, with the first term neglected being of the order of $1/V_S^3$.

RDC APPROXIMATION

In order to generalize the concepts introduced with the two-node grid, we model a power network in synchronous steady-state as a connected graph (see Figure 2). The nodes of the graph are partitioned into sources (or PV buses, red circles in Figure 2) and loads (or PQ buses, blue squares in Figure 2); the edges represent the power lines.
By applying the same decoupling assumption introduced for the two-node case, the reactive power flow reads

$$Q_L = \mathrm{diag}(V_L)\begin{bmatrix} L_{LL} & L_{LS}\end{bmatrix}\begin{bmatrix} V_L \\ V_S\end{bmatrix}\,. \qquad (2)$$

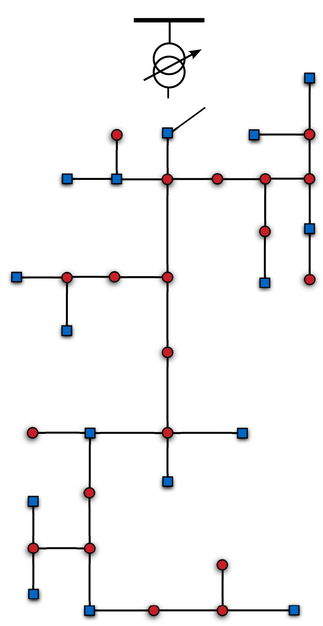


Figure 2: IEEE 37 grid

It is not possible to derive an explicit analytic solution for the system (2) of quadratic equations in the variables V_L, yet the practical importance of the reactive power flow motivates the interest in extending the results of the two-node case to the system (2). To this end the preliminary concepts of source baseline voltage V_N and source voltage spread η must be introduced:

$$V_N := \min_i\{V_i,\, i \in S\} \qquad \eta:\ V_S = (\mathbb{1} + \eta)V_N.$$

Theorem (RDC approximation).

If $E_N > \sqrt{\dfrac{4\|L_{LL}^{-1}\| \cdot \|Q_L\|}{\min_i\{|1 + (L_{LL}^{-1}L_{LS}\eta)_i|^2\}}}$ the reactive power flow admits the solution

$$V_L = V_L^{RDC} + \mathcal{O}\left(\frac{1}{V_N^3}\right)$$

$$\text{where} \quad V_L^{RDC} := V_N(\mathbb{1} - L_{LL}^{-1}L_{LS}\eta) + \frac{1}{V_N}\left(L_{LL}^{-1}\mathrm{diag}^{-1}(\mathbb{1} - L_{LL}^{-1}L_{LS}\eta)Q_L\right).$$

It is worth noticing that the threshold above which the existence of a solution is guaranteed is a generalization of the threshold found for the two-node grid; in the same way, the RDC approximation V_L^{RDC} generalizes the Taylor-expansion proposed in the two-node case.

NUMERICAL ACCURACY

We studied the IEEE 37 test case (Figure 2) and compared the RDC approximation to the numerical solution of the system (2), for different values of the baseline voltage V_N. Figure 3 reports the relative approximation error: we can notice that at the network operating voltage of 4.8 kV the relative error is of the order of 10^{-6}.
Moreover, one can measure from the plot that the approximation error tends to zero as $1/V_N^3$, as predicted by the above Theorem.

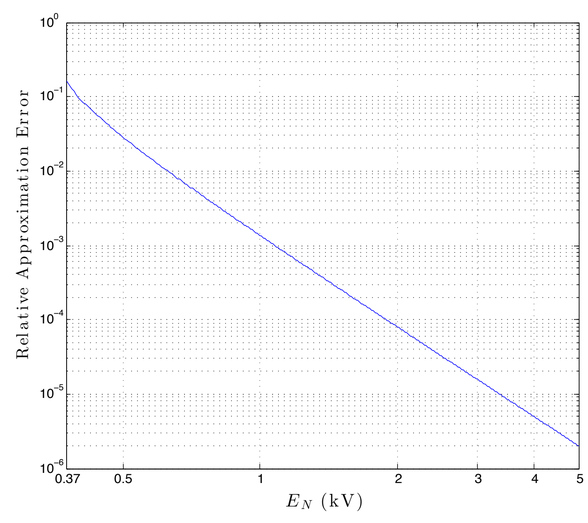


Figure 3: the relative approximation error

APPLICATION TO DROOP CONTROL

The RDC approximation can be used to study the problem of voltage stabilization in an inverter-based microgrid. The sources of the grid represent inverters which are regulated by [2]:

$$\dot{V}_i = -C_i V_i(V_i - V_i^R) - Q_i\,, \qquad i \in S\,,$$

where V_i^R is the reference voltage for inverter i and C_i is a fixed parameter. Coupling this control law with the reactive power flow (1) yields

$$\begin{bmatrix} \mathbf{0} \\ \dot{V}_S \end{bmatrix} = \begin{bmatrix} Q_L \\ C\,\mathrm{diag}(V_S)(V_R - V_S)\end{bmatrix} - \mathrm{diag}(V)LV\,.$$

By using the RDC approximation it is possible to show that such differential-algebraic system possesses a high-voltage locally exponentially stable fixed point and to provide an approximate expression of the fixed point.

REFERENCES

[1] B. Gentile, J. W. Simpson-Porco, F. Dörfler, S. Zampieri, and F. Bullo, "On reactive power flow and voltage stability in microgrids," in *American Control Conference*, Portland, OR, USA, June 2014.

[2] J. W. Simpson-Porco, F. Dörfler, and F. Bullo, "Voltage stabilization in microgrids via quadratic droop control," in IEEE Conf. on Decision and Control, Florence, Italy, Dec. 2013, pp. 7582–7589.

Basilio Gentile: gentileb@control.ee.ethz.ch
Florian Dörfler: dorfler@control.ee.ethz.ch

Automatic Control Laboratory
Institut für Automatik
http://control.ee.ethz.ch