Structure Learning and Statistical Estimation in Radial Distribution Networks

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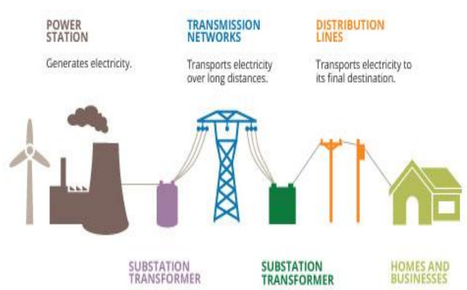
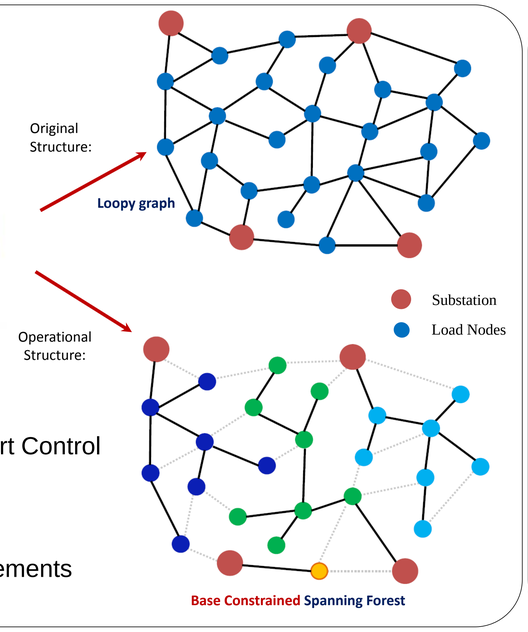
MOTIVATION





Distribution Grid

- ❑ Final tier in electricity transfer
- ❑ Influences: Demand Response/ Smart Control
- ❑ Insufficient measurements

Data available: *(No line data)*

- ❑ Node voltage and/or phasor measurements

SYSTEM MODEL

AC power flow:

$$P_a + \hat{\imath} Q_a = \sum_{(a,b)\text{ is edge}} \frac{\varepsilon_a e^{\hat{\imath}\theta_a}(\varepsilon_a e^{-\hat{\imath}\theta_a} - \varepsilon_b e^{-\hat{\imath}\theta_b})}{(R_{ab} - \hat{\imath} X_{ab})}$$

***** LC lossless power flow *****

$$V_a = \varepsilon_a - 1 \approx 0,\ \theta_a \approx 0$$

$$P = (M^T B M)\theta + (M^T G M)V = H_B\theta + H_G V$$

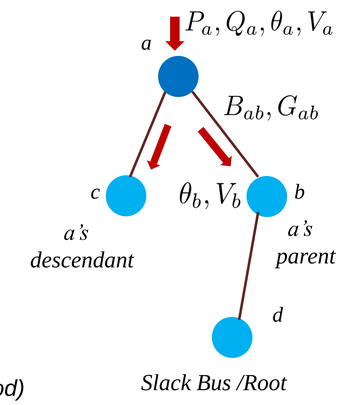
$$Q = -(M^T G M)\theta + (M^T B M)V = -H_G\theta + H_B V$$

$$\theta = H_{1/X}^{-1}P - H_{1/R}^{-1}Q$$
$$V = H_{1/R}^{-1}P + H_{1/X}^{-1}Q$$

Additive & Separable *(Good)*
Has weighted Laplacians *(Better)*

$\Sigma_V = \mathbb{E}[VV^T]$ ➢ *Second Moment*

$\Omega_V = \mathbb{E}[(V-\mu_V)(V-\mu_V)^T]$ ➢ *Covariance*



TREES ARE GOOD!!

$$M = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \qquad M^{-1} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & -1 & 1 \end{pmatrix}$$

Reduced Incidence Matrix: *INVERSE IS 0 - 1*

$$H_B = \begin{pmatrix} B_{ab}+B_{ac} & -B_{ab} & -B_{ac} \\ -B_{ab} & B_{ab}+B_{bd} & -B_{bd} \\ -B_{ac} & 0 & B_{ac} \end{pmatrix}$$

Reduced Weighted Laplacian Matrix

$H_B^{-1}(a,b) = \sum_{(i,j)\in E_a \bigcap E_b} \frac{1}{B_{ij}}$ ***TRACTABLE*** INVERSE

sum of inverse susceptance of lines common to paths from a & b to slack bus

SUMMARY OF RESULTS

Considered Learning Problems:

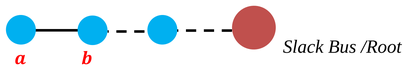
- ❑ ***Structure Learning***: current operational state
- ❑ Estimation of ***Load Statistics*** (mean/correlations)
- ❑ Learning with ***Missing Data*** (limited observations)

Potential Uses

- ➢ Failure detection
- ➢ Flow optimization
- ➢ Non-intrusive control
- ➢ Privacy quantification

STATISTICAL TRENDS IN TREES

❑ If node ***b*** is parent of node ***a***,



- $\Sigma_V(a,a) > \Sigma_V(b,b)$
- $\Omega_V(a,a) > \Omega_V(b,b)$
- *Highest second-moment at **leaf node***

- $\mathbb{E}[(V_a - V_b)^2] = \sum_{k_1,k_2 \in D_a^{\mathcal{T}_i}} R_{ab}^2 \Sigma_P(k_1,k_2) + 2R_{ab}X_{ab}\Sigma_{PQ}(k_1,k_2) + X_{ab}^2\Sigma_Q(k_1,k_2)$
- *Mean sq. difference contributed only by **descendant nodes***

- $b = \arg\min_{k \notin D_a^{\mathcal{T}_i}} \mathbb{E}[(V_a - \mu_{V_a}) - (V_k - \mu_{V_k})]^2$
- *Minimum mean sq. centered difference among non-descendants at **parent***

STRUCTURE LEARNING ALGORITHMS

❑ **ASSUMPTIONS considered**

➢ $\Sigma_P(a,b) > 0, \Sigma_Q(a,b) > 0, \Sigma_{PQ}(a,b) > 0$

➢ $\Omega_{QP}(a,a) \geq 0$
$\Omega_P(a,b), \Omega_Q(a,b), \Omega_{QP}(a,b) \quad = 0 \forall a \neq b$

*** All load nodes are energy consumers*

*** Fluctuations at different nodes are uncorrelated, at same node are aligned*

❑ **ALGORITHM:**

- Pick undiscovered node ***b*** with largest Σ_V OR Ω_V
- For every discovered node ***a***, join ***b*** with ***a*** if

$$\mathbb{E}[(V_a - V_b)^2] = \sum_{k_1,k_2 \in D_a^{\mathcal{T}_i}} R_{ab}^2 \Sigma_P(k_1,k_2) + 2R_{ab}X_{ab}\Sigma_{PQ}(k_1,k_2) + X_{ab}^2\Sigma_Q(k_1,k_2)$$

If $\Sigma_P, \Sigma_Q, \Sigma_{PQ}$ are known

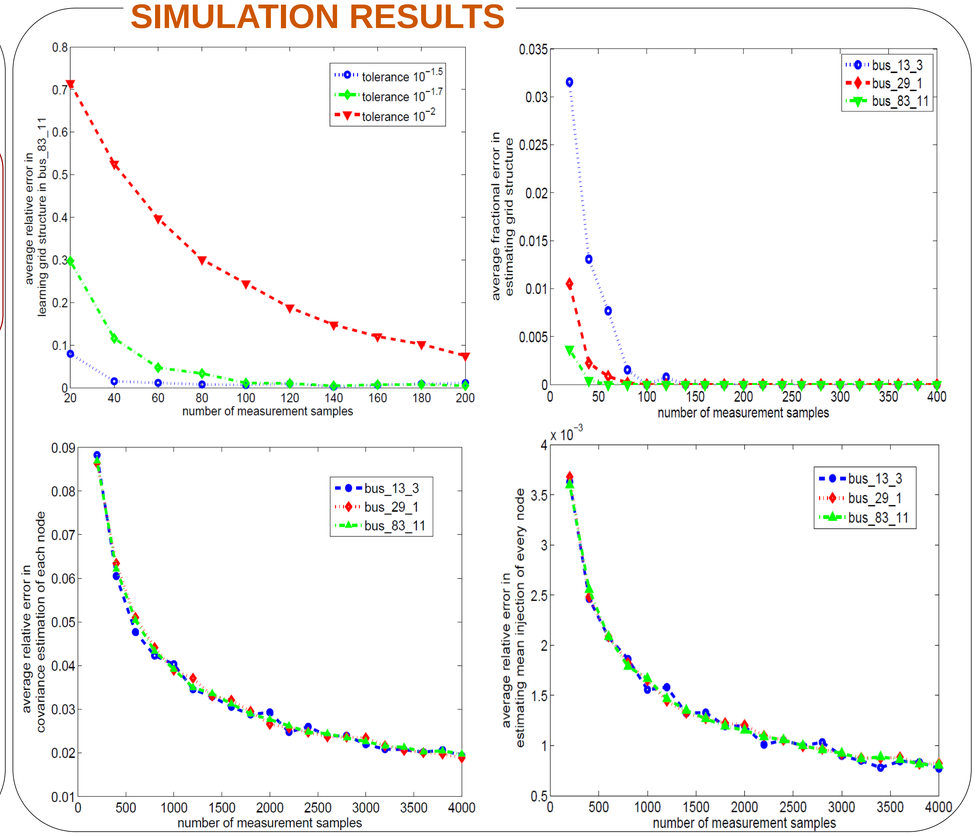
OR

$$b = \arg\min_{k \notin D_a^{\mathcal{T}_i}} \mathbb{E}[(V_a - \mu_{V_a}) - (V_k - \mu_{V_k})]^2$$

If $\Omega_P, \Omega_Q, \Omega_{PQ}$ are diagonal

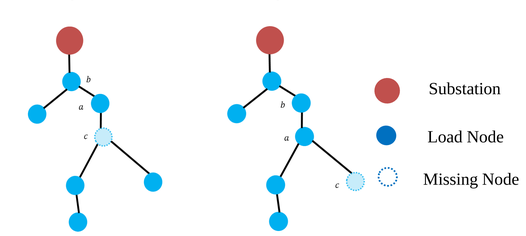
- Estimate means, covariances of power injection using LC power flow equations.

SIMULATION RESULTS



MISSING DATA

- ❑ If separated by greater than two hops
- ❑ Polynomial time algorithm (N^3)
- ❑ Algorithm similar to general case
- ❑ Missing node ***c*** can exist in two cases:
  a) intermediate b) leaf



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