Stochastic Models of Load (and Renewables)

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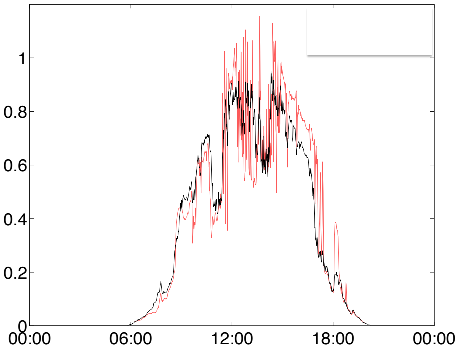
Name that process (1)



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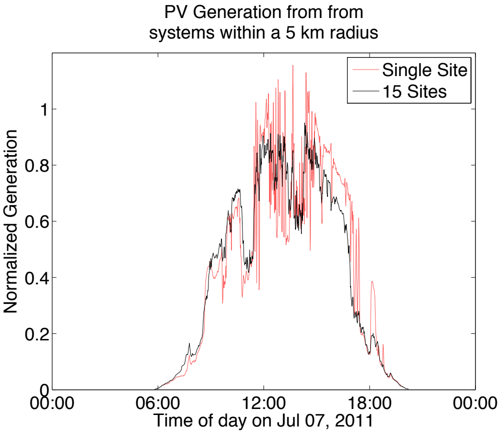
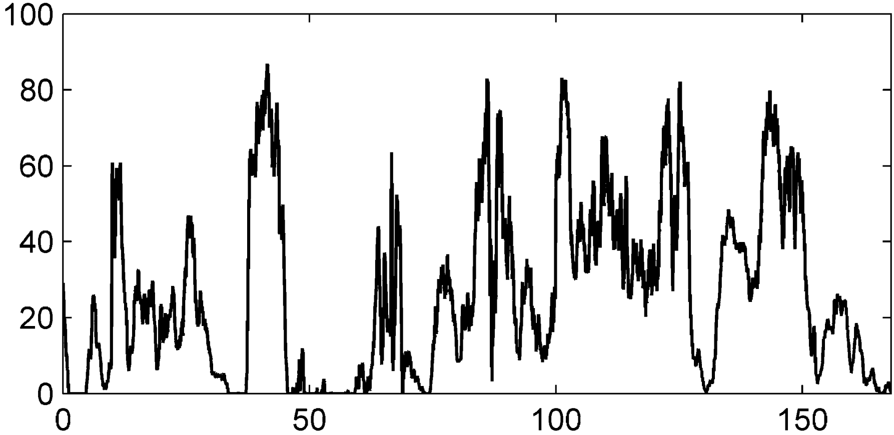


1 second solar production from 1 to 15 residential PV systems in California (data from SolarCity)

Name that process (2)



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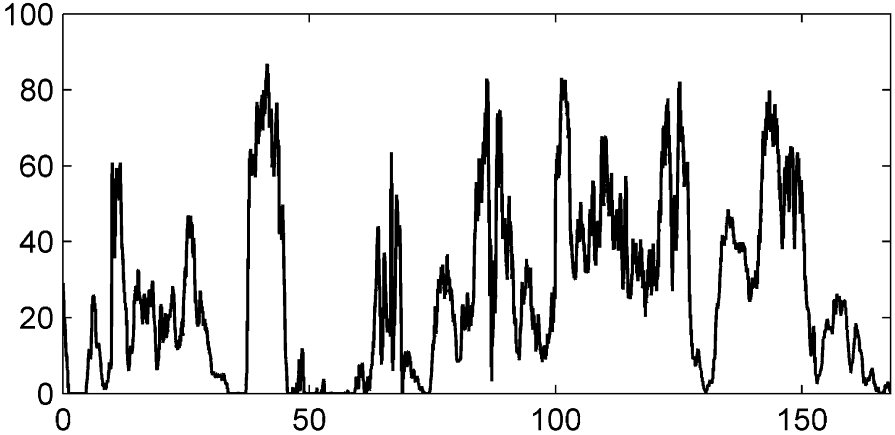
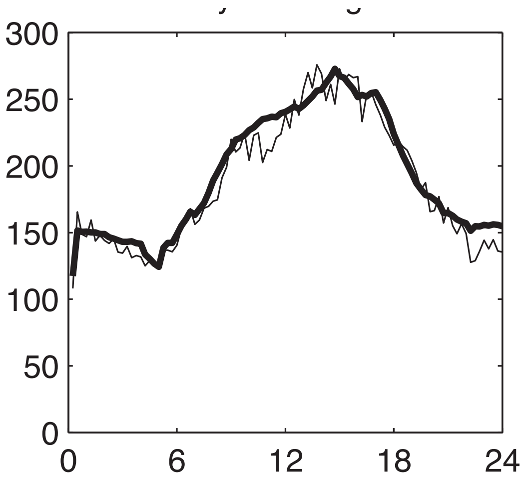


1 second wind production at 100 MW wind farm in MN (courtesy Yi Wei Wan, NREL)

Name that process (3)



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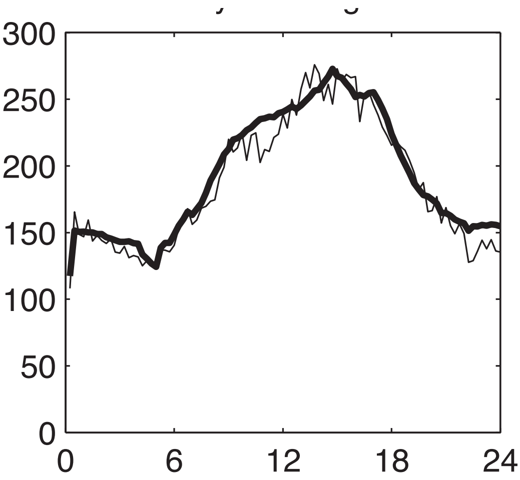
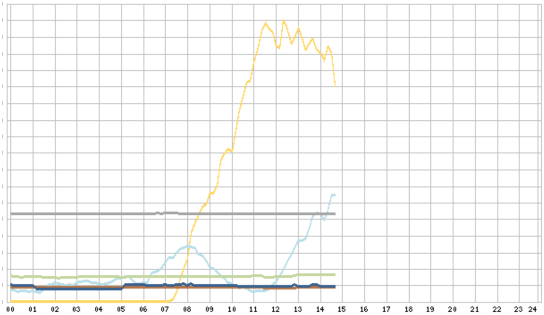


15 minute average electricity consumption from individual building (x-axis = hours, y-axis = kW) (Mathieu et al Energy and Buildings, 2011)

Name that process (4)



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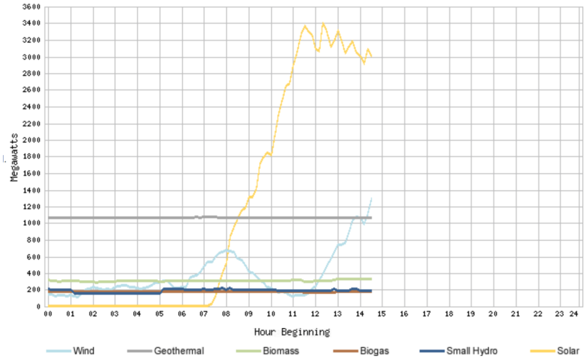
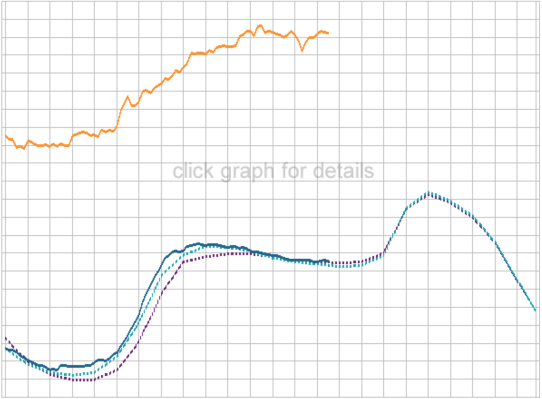
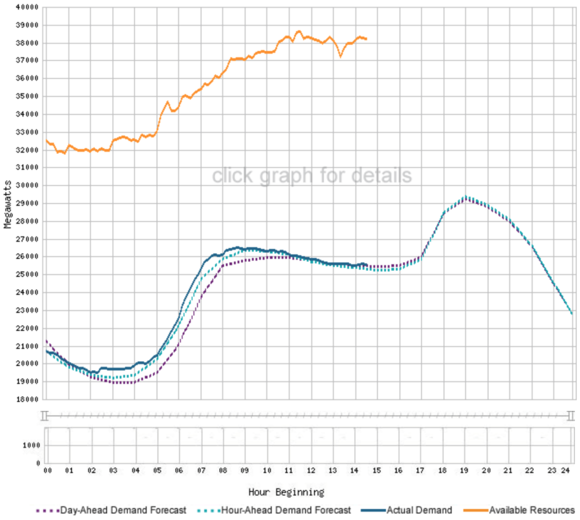


Renewables production aggregated across the state of California (CAISO)

Name that process (5)



Name that process (5)



Electricity demand aggregated across the state of California (CAISO)

Early question: what drives variability in wind, solar, electricity demand?

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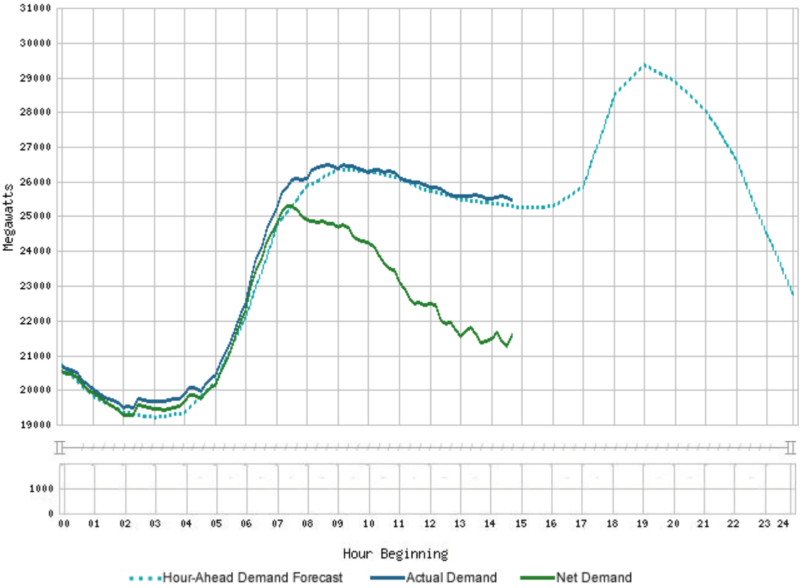
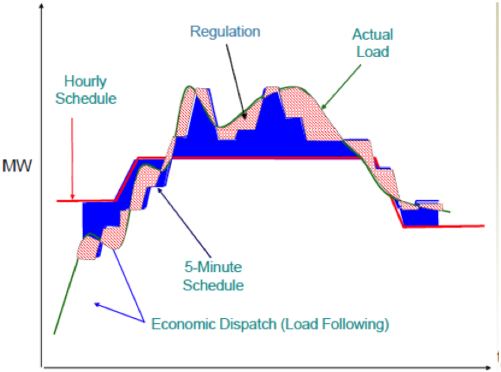
- Weather
  - diurnal, annual periodicity
  - free convection, turbulence $\rightarrow$ chaos
- People
  - diurnal, annual periodicity
  - Random fluctuations in when people do things: turning on light switches, plugging in car, etc.

How do all these things come together?



How system operators think about variability & uncertainty



(source: CAISO)

- System operators dispatch generation to follow 5 min schedules and provide frequency control
- More renewables ⇒ more variability and uncertainty. What to do?

Tools for managing variability and uncertainty

Tools for managing variability and uncertainty

- Better (or different) forecasts
- New technologies for ramping quickly
  - Faster generators
  - "demand response"
  - storage
- New market structures that provide financial compensation for faster generators.

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Focus of the lecture:

- Survey some methods for describing load and renewables variability
- Put them in the context of applications

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  - "demand response"
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- Survey some methods for describing load and renewables variability
- Put them in the context of applications

Outline

Physical load models

- Introduction
- Individual TCLs – Stochastic differential & difference equations
- Aggregations of TCLs – ARMAX, PDE and state space formulations
- Whole building models – Stochastic differential equations
- Whole building models – disaggregation with smart meters

Renewables production models

- Capturing solar PV *variability*

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A framework for categorizing loads (cf Lijun Chen *et al* 2012

1. Energy-constrained tasks
   - Dishwasher, dryer
   - EV charging
2. Thermostically controlled loads (TCLs)
   - Refrigeration, water heating, space conditioning

These can be modeled with an internal **energy** state

- Energy over time defines the performance of the load
- Can turn on or off without impacting overall performance of the load.
- ...so they are good for load shaping

A framework for categorizing loads, ctd

3. Loads that support other basic activities
    - Lighting, elevators
4. Loads that *are* the activity
    - Computers, video games, televisions.

Depend on instantaneous **power** consumption

- Turning them ON or OFF means electricity consumers go without
- They are also much harder (if not impossible) to model from a first-principles perspective
- We'll leave them out of further discussion.

What I will and won't talk about

Load modeling has two major veins:

1. Those that focus on describing how real power varies in time in order to serve end-use functions
2. Those designed to describe dynamics of the loads as a function of power system state variables (frequency, voltage...).
    - Ian Hiskens spoke about this on Monday.

I'll focus on #1 and not #2.

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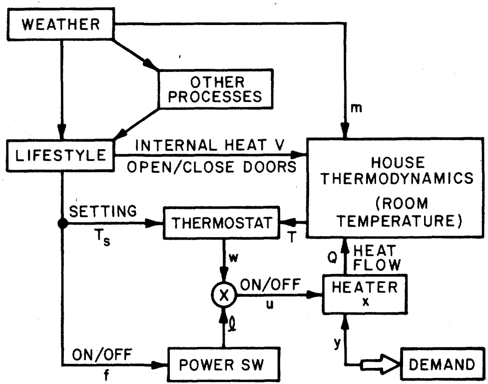
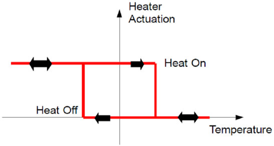
Physically based TCL models

Ihara and Schweppe (1981) posed an early conceptual and mathematical model of TCLs



Their interest was in describing the phenomenon of "cold load pickup"

TCL models as hybrid state deterministic ODEs (Ihara and Schweppe 1981)



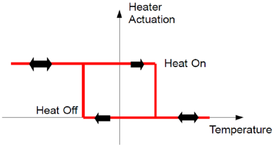
(as shown here, this is a heating model)

In this model, power changes due to ON/OFF switches and average power over time changes due to varying ambient temp.

TCL models as hybrid state deterministic ODEs (Ihara and Schweppe 1981)

$$C_i \frac{d\theta_i(t)}{dt} = -R_i\left(\theta_i(t) - \theta_a\right) + m_i(t)P_i$$

$$m_i(t) = \begin{cases} 0, & \theta_i(t) > \theta_{i,+} \\ 1, & \theta_i(t) < \theta_{i,-} \\ m_i(t), & \text{otherwise} \end{cases}$$



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$\theta_i(t)$	= temperature of i$^{\text{th}}$ load	C_i	= thermal capacitance
$m_i(t)$	= OFF/ON state of the load $\in \{0,1\}$	R_i	= thermal resistance
θ_a	= ambient temp	$\theta_{i,\pm}$	= themostat limits

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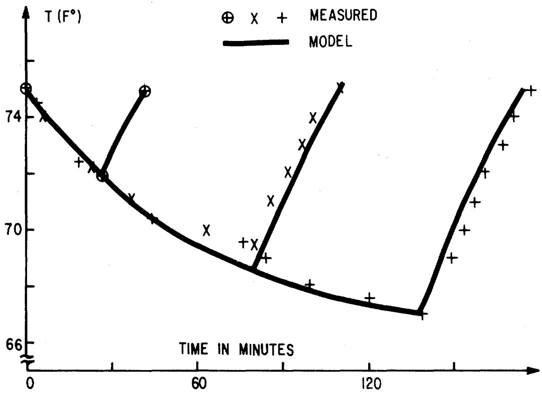
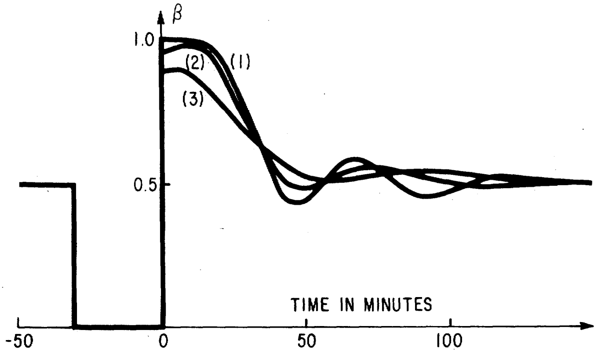
Model performance (Ihara and Schweppe 1981)



Application (1): Cold load pickup – thousands of loads' aggregated power (Ihara and Schweppe 1981)



Parameters drawn from Gaussian distribution. Which has the largest standard deviation on the distribution?

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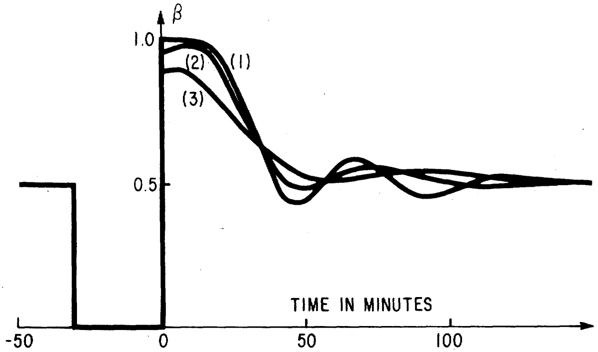
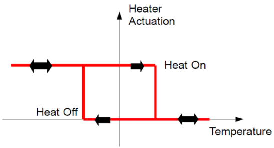


Parameters drawn from Gaussian distribution. Which has the largest standard deviation on the distribution? **Ans: (3)**. Diversity damps oscillations.

TCL models as hybrid state stochastic ODEs (Malhame and Chong TAC 1985)



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$$C_i \frac{d\theta_i(t)}{dt} = -R_i\left(\theta_i(t) - \theta_a\right) + m_i(t)P_i + w_i(t)$$

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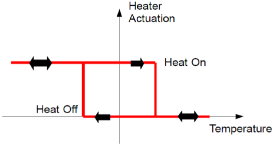
$$y(t) = \sum_{i=1}^{N} \frac{1}{\eta_i} P_i m_i(t)$$



(as shown here, this is a heating model)

$\theta_i(t)$	= temperature of i$^{\text{th}}$ load
$m_i(t)$	= OFF/ON state of the load $\in \{0, 1\}$
$w_i(t)$	= noise
$y(t)$	= total power consumed
η_i	= conversion efficiency
θ_a	= ambient temp
C_i	= thermal capacitance
R_i	= thermal resistance
P_i	= power to ON load (after conversion losses)
$\theta_{i,\pm}$	= themostat limits

What's the noise for?

Motivating examples

- Weather
- Opening / closing windows and doors
- Internal heat gain driven by occupants

Practical reasons

- Facilitates a very nice aggregation approximation (more to come)
- Facilitates estimation via Kalman filter (more later)

Application (2): Receding horizon control Mathieu, Kamgarpour *et al* (forthcoming)

Converting to discrete time:

$$\min_{\xi^i \in \Xi} h \sum_{k=k_0}^{k_0+N_b} l_k y_k^i$$

s.t. TCL dynamics

h=time step,

l_k = energy price,

ξ = overrides local ON/OFF control (only within deadband)

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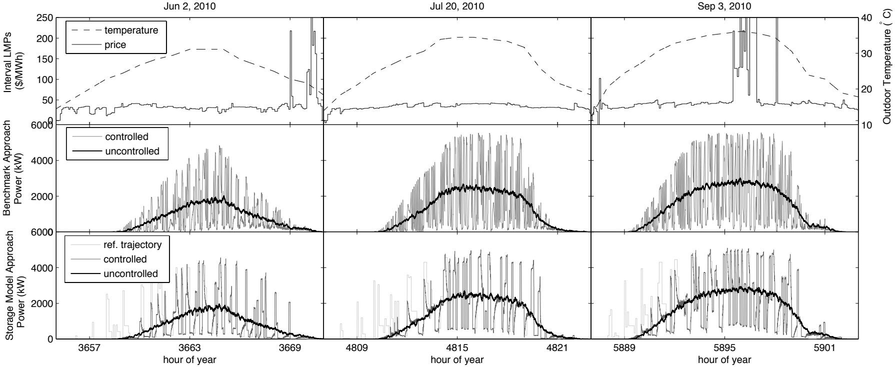
$$\min_{\xi^i \in \Xi} \; h \sum_{k=k_0}^{k_0+N_b} l_k y_k^i$$

s.t. TCL dynamics

Saves 5-35% of energy costs without disrupting comfort.



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Exploring perturbations to thermostatically controlled loads

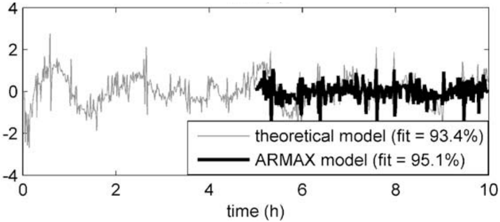
(TCLmovie)

How to describe the dynamics of the aggregation?

Option 1: Simple time series model

$$A(q)y_k = B(q)u_k + C(q)\varepsilon_k$$

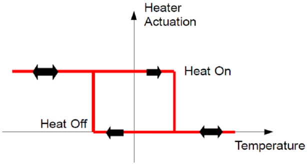
In this case, y is aggregate power, u is temperature setpoint change.



Problem: Model doesn't track system states (internal temp) and performs poorly if perturbations are large.

But aren't there better ways to describe the aggregation?

Option 2: PDE models



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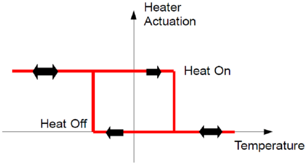
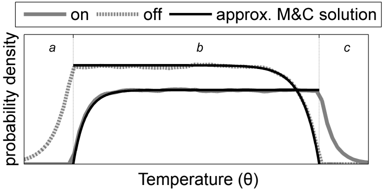




- *Population* can be modeled as a pdf in temperature
- Fokker-planck coupled PDEs describe dynamics
  - Derivation via Chapman-Komogorov
  - Requires load homogeneity, stochastic differential equation for derivation.

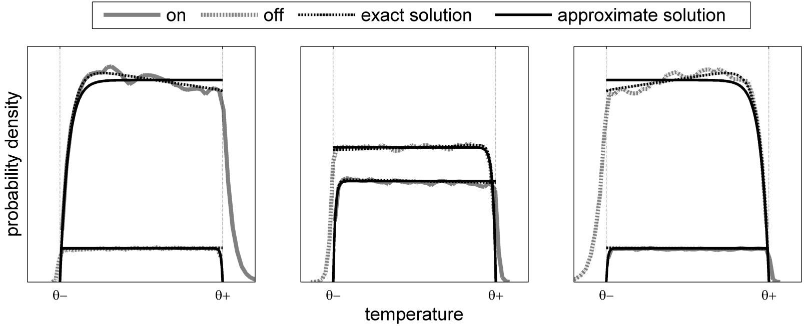
$$\frac{\partial f_0}{\partial t} = \frac{\partial}{\partial \theta}\left[\left(\frac{1}{CR}(\theta(t) - \theta_a)\right) f_0\right] + \frac{\sigma^2}{2}\frac{\partial^2}{\partial \theta^2} f_0$$

$$\frac{\partial f_1}{\partial t} = \frac{\partial}{\partial \theta}\left[\left(\frac{1}{CR}(\theta(t) - \theta_a) + \frac{P}{C}\right) f_1\right] + \frac{\sigma^2}{2}\frac{\partial^2}{\partial \theta^2} f_1$$

(Malhame & Chong, TAC '85)

Fokker-Planck approximation solution

- Non-stationary solution eigenvalues will determine how quickly disturbances decay to steady state.
  - unable to find nonstationary solutions under M&C assumption of constant indoor temp
- However, the original system can in fact be solved by separation of variables ($f(\theta, t) = \varphi(\theta)e^{-\lambda t}$) and the series method. The result:



Application (3): PDEs yield insight into the dynamics (Callaway '09)

Nonstationary solution to the homog. PDE with noise has modes with eigenvalues

$$\lambda_k = \frac{k}{CR}, k = 0, 1, 2...$$

$\Rightarrow$ if CR (thermal time constant) is small enough, steady state distribution informs control

- Figure: control law based on steady state approximation
- Result: small setpoint change $\rightarrow$ large, accurate response

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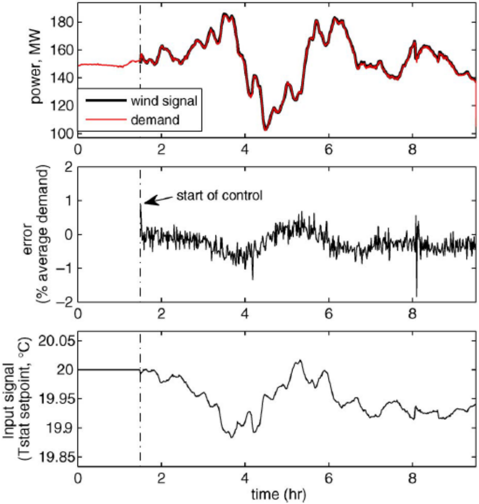
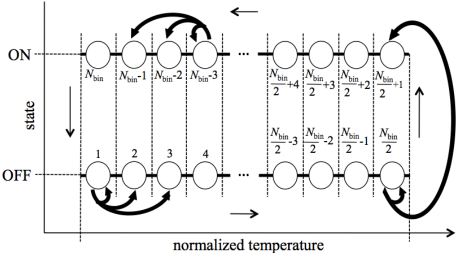


Challenges to the Fokker-Planck approach

- Incorporating parameter heterogeneity
- Available control tools relatively small
- System identification tends to be computationally intensive.

A more flexible framework

Option 3: State space models



$\mathbf{x}$ = fraction of TCLs in each bin

$\mathbf{A}$ = state transition matrix

$\mathbf{y}$ = system output: aggregate power only, or power and *all* $\mathbf{x}$

$$x(k+1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k)$$

(Liu & Chassin, TPWRS '05; Bashash & Fathy TCST '12; various Callaway pubs.)

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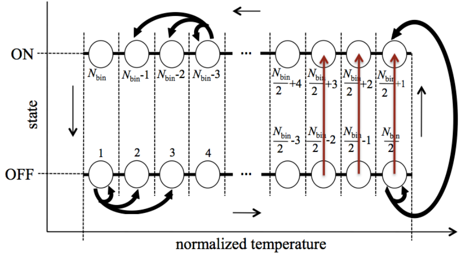

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$\mathbf{u}$ = $N_{\text{bin}}/2 \times 1$, switch OFF/ON

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$$B = \left[\begin{array}{cc} -1 & 0 \\ & \ddots \\ 0 & -1 \\ \hline 0 & 1 \\ & \unicode{x22F0} \\ 1 & 0 \end{array}\right].$$

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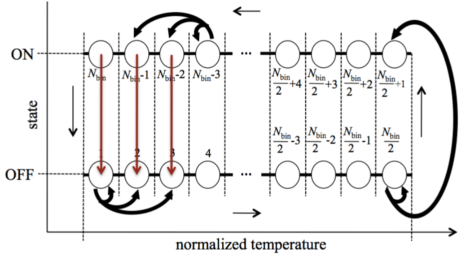

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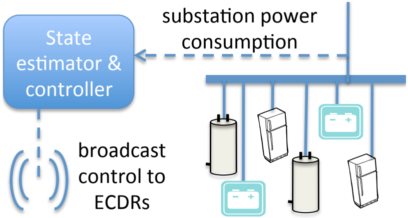
State space models: Opportunities

- Can derive model analytically from heterogeneous SDEs (Mathieu *et al* TPWRS '13, Kamgarpour *et al* IREP '13)
- Simple to model a variety of control signals
- Kalman filtering yields states; nonlinear filter can deliver parameters (Mathieu *et al* TPWRS '13)
- Wide array of control tools apply for this and similar models:
  - MPC (e.g. Mathieu TPWRS '14)
  - LQR (e.g. Kundu *et al* PSCC '11)
  - Sliding mode (e.g. Bashash and Fathy TCST '12)
- Alternatives to get to LTI plant model: aggregate step function laplace transfers and invert (Kundu *et al* PSCC '11)

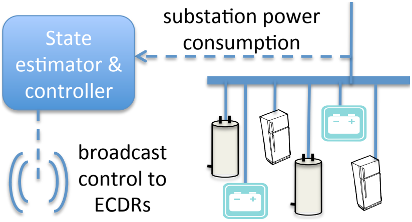
Application (4) What communication and sensing equipment do we need? (Mathieu *et al* TPWRS 2013)

Reference case: Meter power and temperature at all controlled loads, error following dispatch signal = 0.6% RMS (smaller is better)
Case 1: Meter the ON/OFF state at all loads, measure aggregate power at the distribution substation. Result: error = 0.76% RMS
Case 2: Meter only aggregate power at distribution substation. Result: error = 5% RMS (this compares favorably to conventional generators)



(c.f. Mathieu *et al* TPWRS '13)

Centralized control via state space models: Open problems



- State space model is time-varying; depends on outdoor temperature and occupancy (Mathieu et al ECC13), but more work needed
- What is the process noise covariance? Abstraction methods can address (Soudjani and Abate, TCST'14); more work needed
- Parameter heterogeneity (Mathieu *et al* TPWRS '13, Kamgarpour *et al* IREP '13). Very few satisfying answers here!
- Filter observer error can depend on forecast of uncontrolled loads ⇒ short term, small spatial scale forecasts needed.
- Validation!

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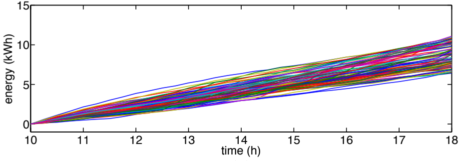
Modeling total customer load as an SDE

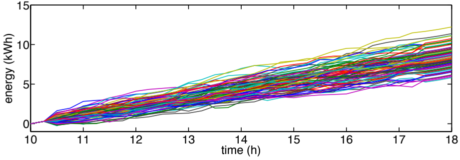
Customer energy consumption: e_t^i is the total energy consumption up to time t by customer i

$$de_t^i = (l_i(t) + u_t^i)dt + \tilde{\sigma}_i(t) dW_t^i$$

(more validation to come)



(a)



(b)

Load state dynamics: $dx_t^i = f_i(x_t^i, u_t^i)dt$

Example: First order temperature dynamics for air conditioning

Electricity price dynamics also modeled as SDE

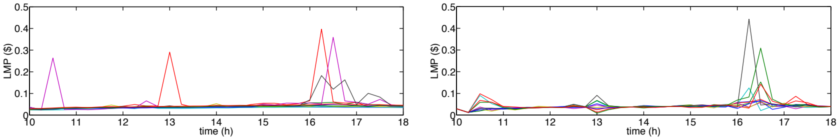
- λ_t: Electricity price in the real-time market (LMP)
- mean-reverting model [Deng, Johnson, Sogomonian, DSS, 2001], [Kamat, Oren, OR, 2002]

$$d\lambda_t = r_0(\nu(t) - \ln \lambda_t)\lambda_t dt + \sigma_0(t)\lambda_t dW_t^0$$

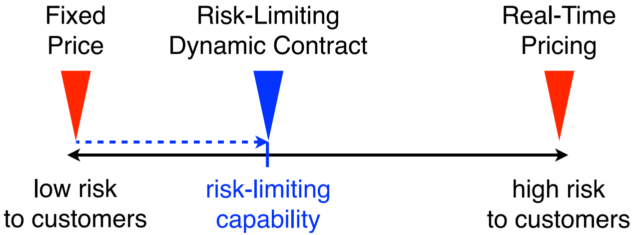
- change of variable $\Longrightarrow$ linear model

$$dw_t = r_0(\nu(t) - w_t)dt + \sigma_0(t)dW_t^0$$

- data (ERCOT LMP) vs. identified model



Application (5): Financial Risk-Sharing with Risk-Limiting Dynamic Contracts (Yang, Callaway, Tomlin Allerton '14)



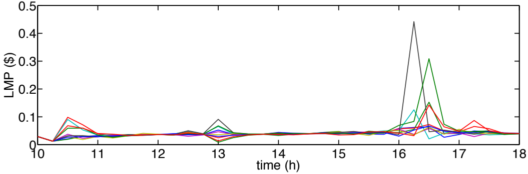
Key Idea: Direct load control + Contract

Goal:

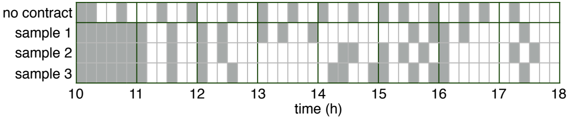
- Capture the benefits of real time pricing
- But also manage concerns over risk

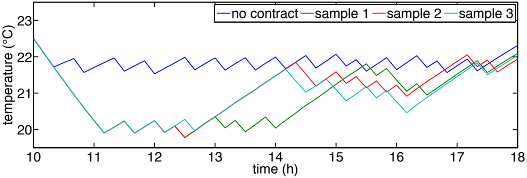
Application (5): Risk management: price volatility

Energy price is high and volatile from 4pm to 5pm:



Result: Precooling under contract



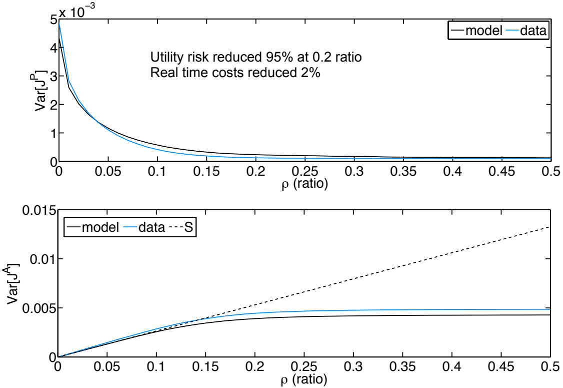


Application (5): Comparison to customer optimal control

- optimal control with no contract

$$\max_{u^i \in \mathbb{U}^i} \quad \mathbb{E}[\hat{J}_i^A[u^i]]$$
$$\text{subject to} \quad dx_t^i = f_i(x_t^i, u_t^i)dt,$$

- nominal mean and variance:
$\bar{b} = \mathbb{E}[\hat{J}_i^A[u^{*i}]]$,
$\bar{S} = \mathrm{Var}[\hat{J}_i^A[u^{*i}]]$
- contract with $(b, S) = (\bar{b}, \rho\bar{S})$
- Blue line: control law (from model) applied against actual price and load data



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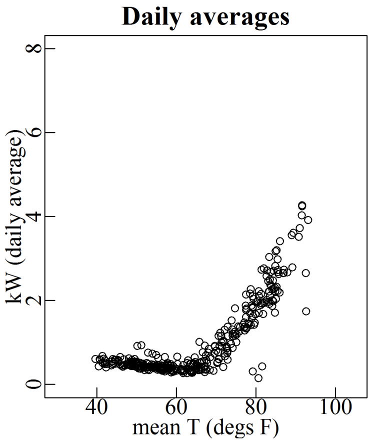
Renewables production models

- Capturing solar PV *variability*

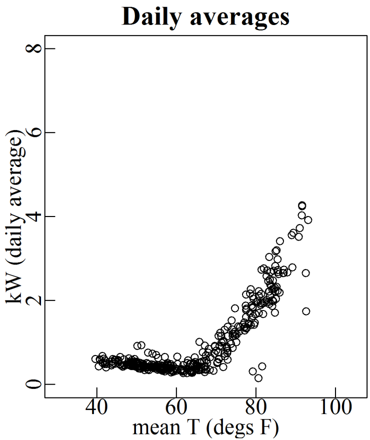
Smart meters

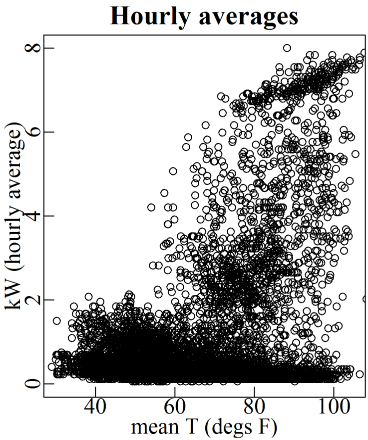
- Provide whole building electricity consumption on 15- to 60-minute intervals.
- Can they be used to identify customers for demand response, or to issue commands for demand response?
- Major target for demand response: air conditioning.

Key challenge: *WHEN* is air conditioning operating?



Key challenge: *WHEN* is air conditioning operating?





Model

$$
\begin{aligned}
W_t &= \text{hourly average power} \\
&\approx \sum_{h=1}^{24} [Hr_{h,t}(weekday_t \gamma_h + weekend_t \delta_h)] \\
&\quad + C_t(\eta + \beta \max(0, T_t - CP))
\end{aligned}
$$

Known:

$Hr_{h,t}$, $weekday_t$ and $weekend_t$: indicator variables

T_t, outside air temperature

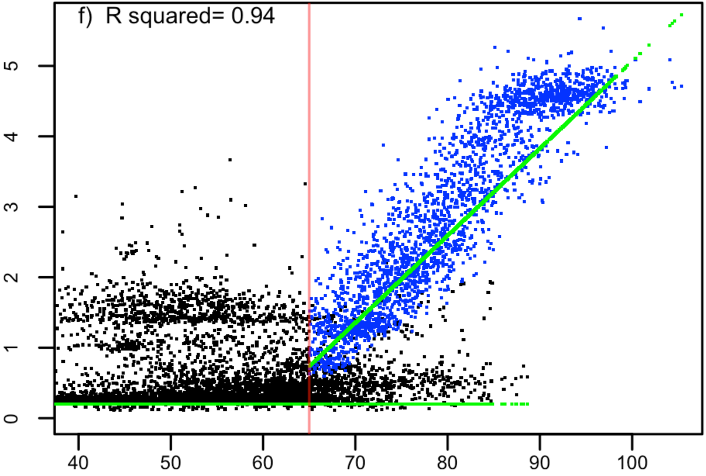
To estimate:

γ_h and δ_h: hourly fixed effects

η and β determine cooling intensity

$C_t \in \{0, 1\}$ and CP: cooling ON/OFF status and "change point"

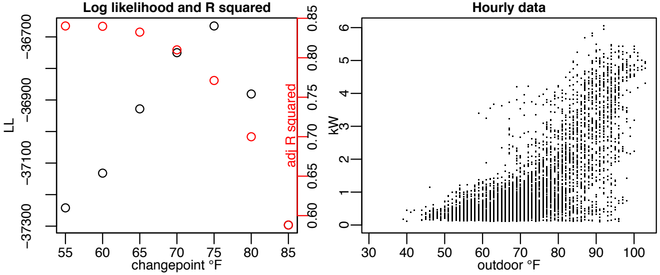
Model performance



Estimating the model

It's easy to estimate this model *IF* C_t and CP are known.

Estimating CP**:** Standard method (ASHRAE inverse modeling toolkit) is to choose best model in a plausible range of CPs.



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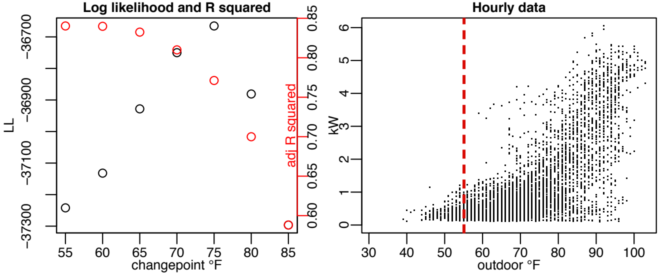
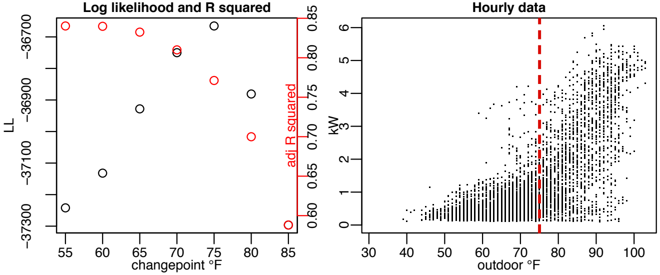


Estimating the model

It's easy to estimate this model *IF* C_t and CP are known.

Estimating CP**:** Standard method (ASHRAE inverse modeling toolkit) is to choose best model in a plausible range of CPs. We adapt this by maximizing likelihood:

$$L_t = \begin{cases} \mathcal{N}(e_t; 0, \sigma^2)(f_{clg}/f_{>CP}) & C_t = 1 \\ \mathcal{N}(e_t; 0, \sigma^2)(1 - f_{clg}/f_{>CP}) & C_t = 0, T_t > CP \\ \mathcal{N}(e_t; 0, \sigma^2) & T_t \leq CP \end{cases}$$



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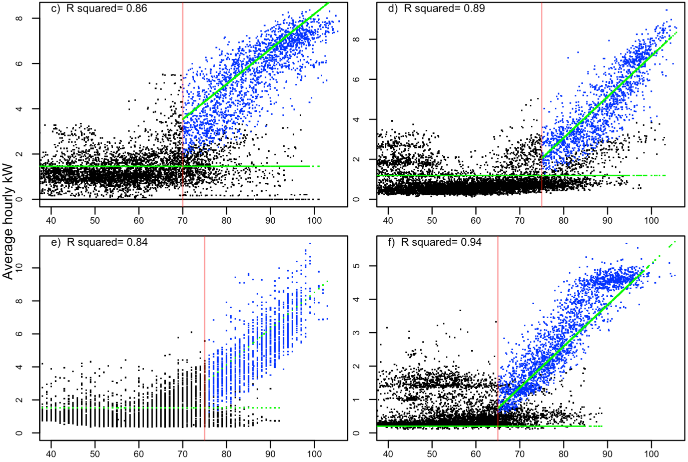
To determine C_t we follow an approach analogous to k-means clustering

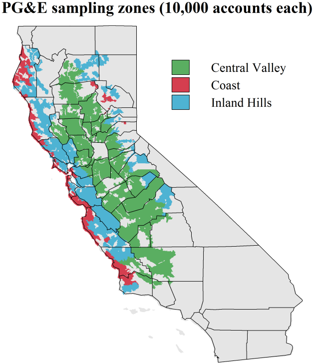
1. Make an initial guess for which hours are $C_t = 1$, which $C_t = 0$.
2. Estimate the regression equation with the likelihood function
3. Calculate residuals for points with $T_t > CP$ using cooling and non-cooling models. Re-assign C_t if the other model fits better.
4. Repeat Steps 2 and 3; exit when assignments change by <0.5%

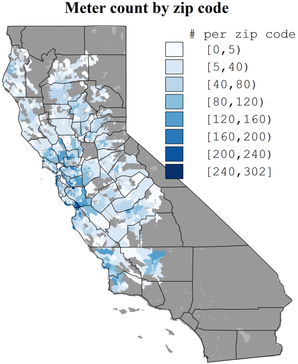
model performance

Typical results:



Application (6) Exploring the demand side resource (Dyson *et al* 2014)





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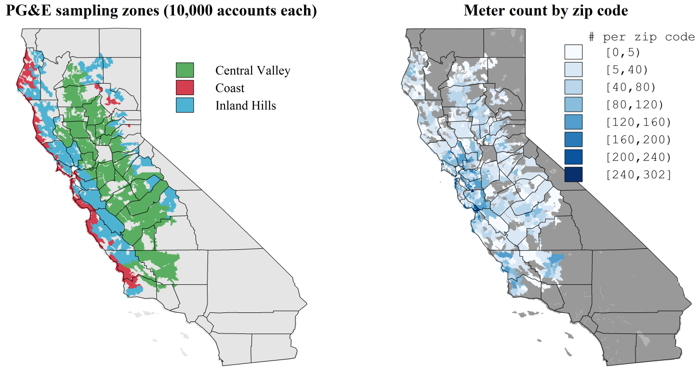
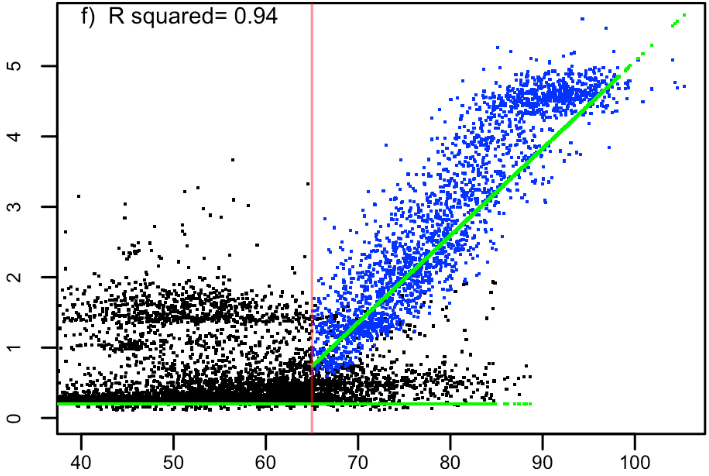


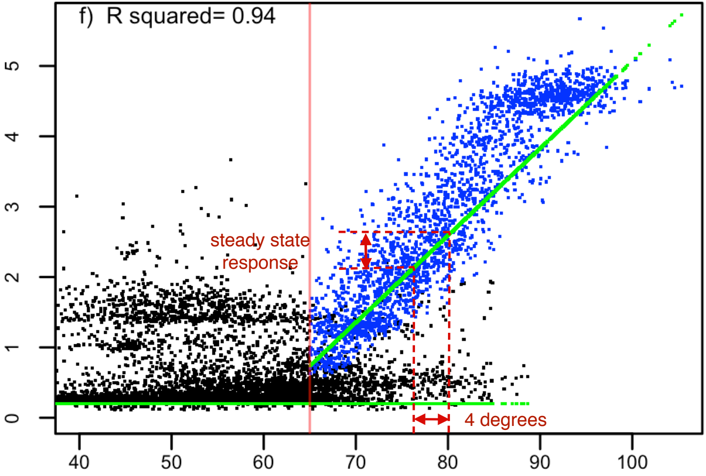
Key questions:

- How do AC setpoint shifts change demand (instantaneously, steady state)?
- How do these changes in demand correlate with renewables?

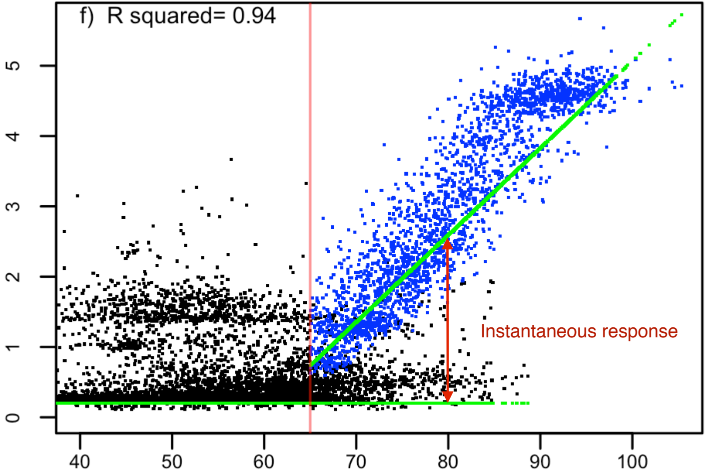
Computing potential



Computing potential

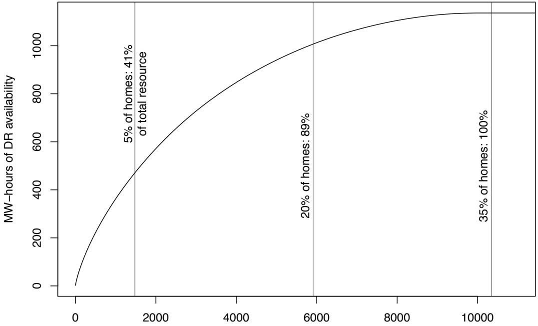


Computing potential



Application (6): Making the case for targeting customers for DR

...a large fraction of the resource is in a small fraction of customers.

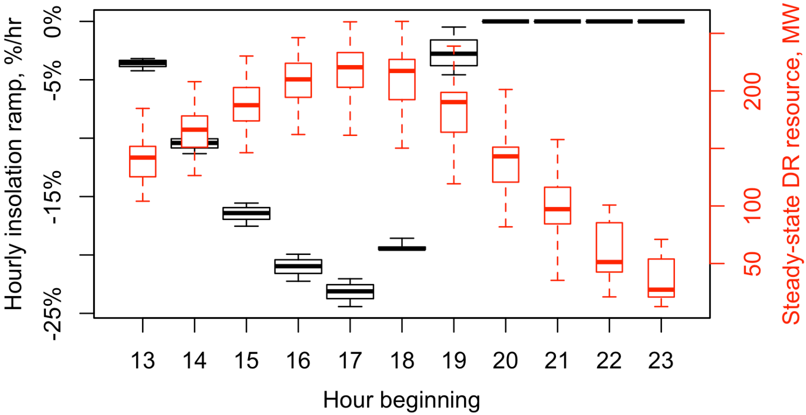


The result is for the PG&E sample (30,000 customers; total PG&E is roughly $150\times$ larger)

MW-hour is the sum of MW available in each hour of the year if buildings increased their temperature setpoints by 4 degrees. (Note this is about 13% of annual electricity consumed for cooling.)

Application (6) Does the resource correlate with solar ramps?

Comparing summer solar ramps to DR availability (scaled to all PG&E)



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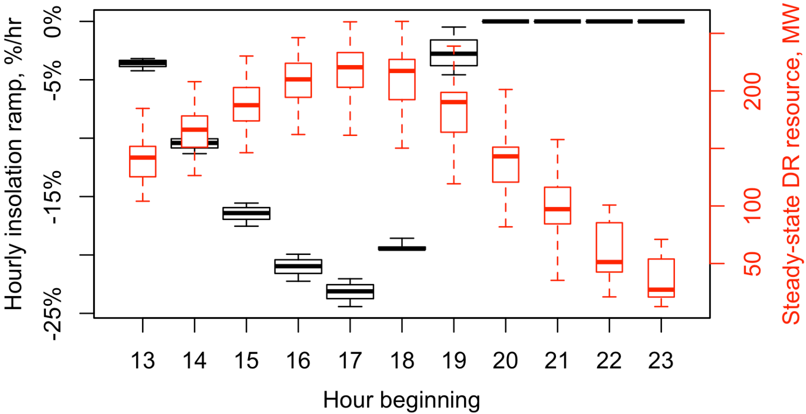
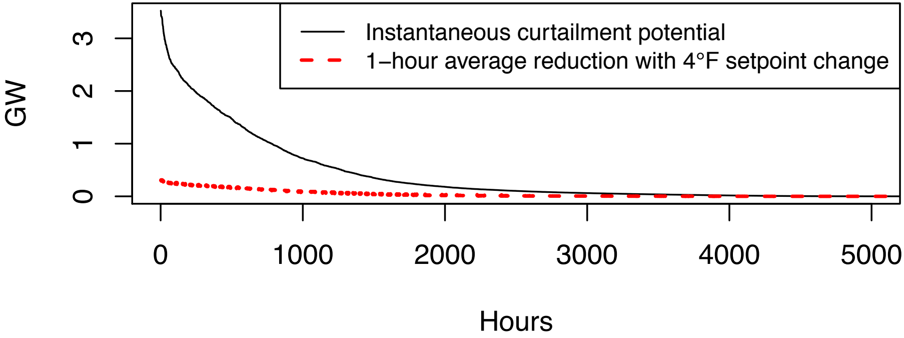


Resource not as big in spring, when *net load* ramp may be worst.

Application (6): Instantaneous vs. steady state

The instantaneous resource is much larger than the steady state resource



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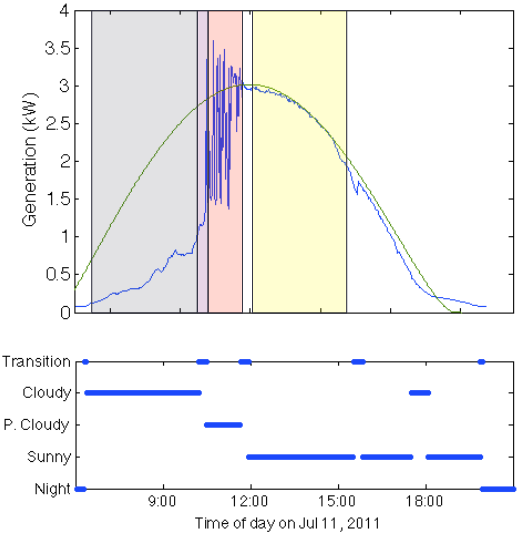
Renewables production models

- Capturing solar PV *variability*

Solar Volatility



Hypothesis: Generation can be described as a *mixture* of models

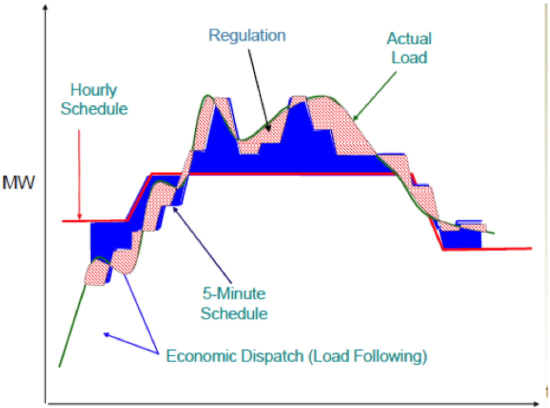
- It is easy to imagine that cloud cover "regimes" determine distributions
- But we don't directly observe cloudiness

The figure shows eyeballed categorization

- But we'd like to estimate regimes endogenously

Decomposition

- Automatic generation control (AGC, a.k.a. regulation):
  - 5 min average minus raw data
- Load following (LF)
  - hour average minus 5 min average
- Using averages ignores forecast error
  - Will add forecast error in future
- Linear operations $\rightarrow$ do statistics on indiv. parts, add them later



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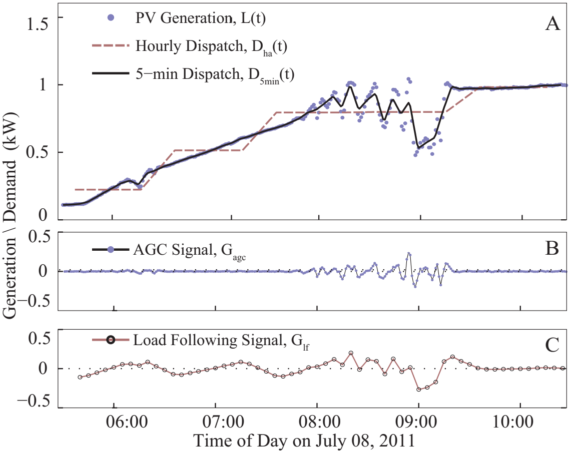
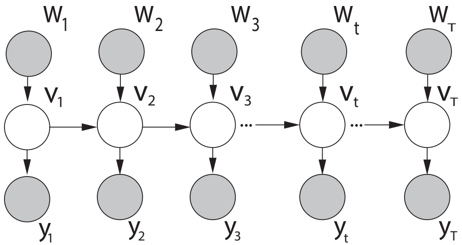


Observations, hidden states and parameters



$W_{t,k} \in \{1, 2, \dots K\} \triangleq$ exogenous data inputs

$\mathbf{Y}_t =$ vector of measured AGC or LF requirements at each site

$\mathbf{v}_t =$ "hidden" volatility state of the AGC or LF metric

The model: equations to estimate

Define $\mathbf{Y}_{CL,t}$ as the AGC or LF requirement due to clear sky variation only.

Assume that the variability at a network of sites is multivariate Gaussian:

$$\mathbf{Y}_t \sim MVG\left(\mathbf{Y}_{CL,t}, \mathbf{\Sigma}_Y(\mathbf{v}_t;\phi)\right) \cdot \max_{t \in HE} CL_t$$

$\mathbf{\Sigma}$ is conditionally dependent on

- the volatility state of each system, $\mathbf{v}_t$,
- a geographic autocorrelation function $\rho_{m,n}$

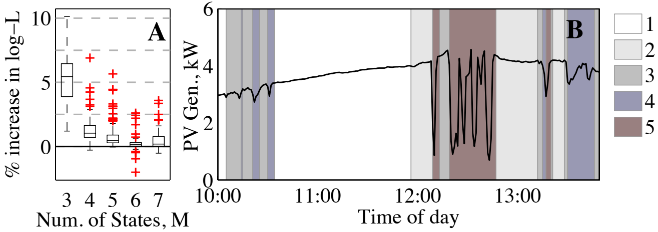
$$\rho_{m,n}(d_{i,j};\phi) = a_{m,n} \cdot \exp\left\{-d_{i,j}/\tau_{m,n}\right\}$$

where m and n are any two hidden volatility states and $\phi = \{\mathbf{a}, \tau\}$

Estimating the model

1. Estimate $\mathbf{v}_t$, σ and $P(v_{t,n} = j | v_{t-1,n} = i, W_{t,m} = k)$ using hidden Markov models with a fixed number of states v_t.
    - All data in a region used to find the parameters – so an estimated model applies to an entire region
    - Estimate via expectation-maximization
    - Fit the HMM using a transition matrix that is conditional on the input state from $\mathbf{W}_t$.
2. Fit $a_{m,n}$ and $\tau_{m,n}$ for each pair of volatility states using a weighted least squares regression

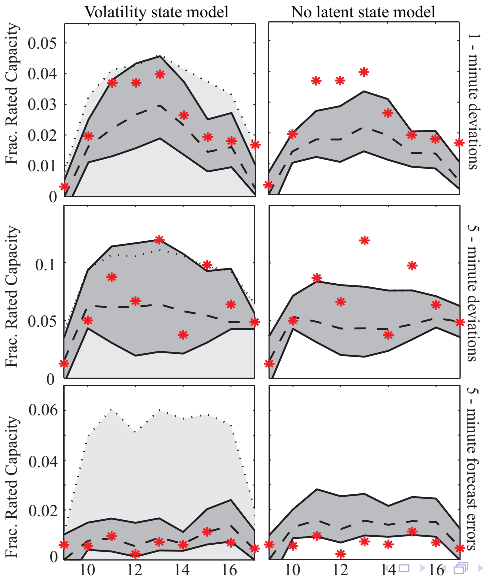
Identifying volatility states



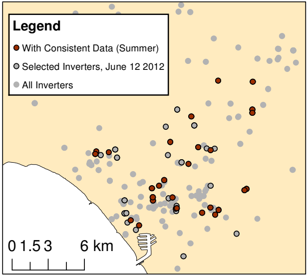
Panel A: Percent increase in log-likelihood of test data given HMMs with M states versus M-1 sates

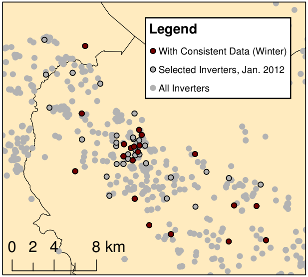
Panel B: Example of volatility states and measured PV generation for a system in San Jose (M=5).

Model validation (all models 5 state)



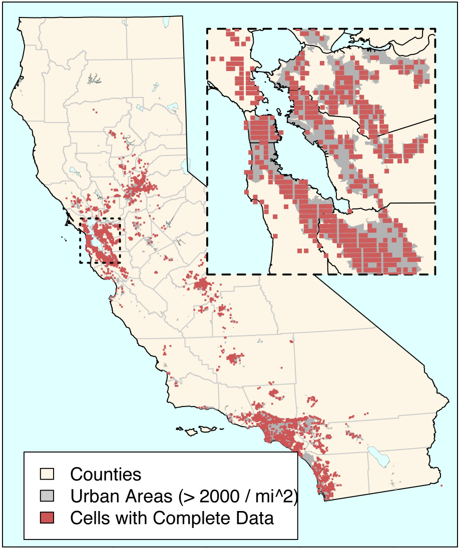
Application (7) Data on small spatial and temporal scales





- Instantaneous voltage and current from small ($<$ 15kW) installations, once per minute.
- $\sim$50 systems in three 256 km^2 areas
  - 256 km^2 $\sim$ smallest spatial area for energy markets.
  - Systems selected to give stratified sample of distances between pairs of locations & geographical random sampling.
- Will use to create small spatial scale model

Application (7) What should be the exogenous inputs?



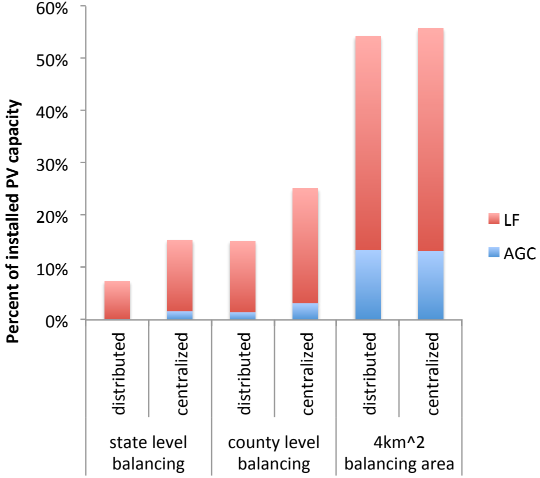
- Instantaneous voltage & current from ∼7,000 sites throughout CA, every 15 min.
- Condition short time scale volatility model with discrete indicators of 15 minute volality
- Use these 15 minute data to predict statewide impacts for geographies and times for which we have no short time scale data

Application (7) What balancing capacity do we need in the future?

Basic approach

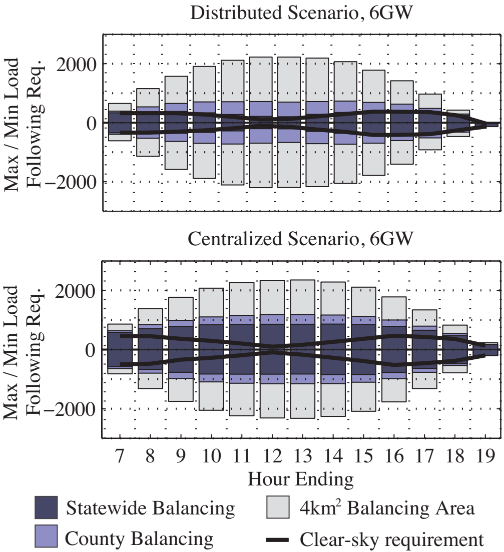
- Divide regions of state with 15 minute data into 2km by 2km cells
  - This covers 40% of California's population
- Use the 15 minute data point closest to the centroid of each cell as inputs to simulate the HMM model
- Investigate the following PV arrangements:
  - *Distributed*: small-scale PV (several kW) located behind the meter
  - *Centralized*: concentrate all PV in region of the state with the best resource
- Make AGC and LF predictions for PV penetration of 6 and 12 GW (CA's 2020 goal)

Application (7) Results: Capacity required to balance PV



- Centralized PV systems more than double requirements
- Local balancing requires much more!
- AGC a tiny part of the impact on larger scales
- Best case: 7% reserve requirement
  - Contrast to NREL 3+5 rule
  - But most is predictable (clear sky variability) → challenge lies in scheduling ramps

Application (7) Hourly Results



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Supplemental slides

Application (2): Energy arbitrage Mathieu, Kamgarpour *et al* TPWRS (forthcoming)

$$\theta^i_{k+1} = a^i\theta^i_k + (1-a^i)(\theta^i_{\mathrm{a},k} - m^i_k\theta^i_{\mathrm{g}}) + \epsilon^i_k,$$

$$m^i_{k+1} = \begin{cases} 0, & \theta^i_{k+1} < \theta^i_-, \\ 1, & \theta^i_{k+1} > \theta^i_+, \\ m^i_k, & \text{otherwise} \end{cases}$$

$$y^i_k = m^i_k \frac{P^i_{\mathrm{trans}}}{\eta^i} = m^i_k P^i,$$

k indexes time

ξ is a new control variable (see next slide)

ϵ is Gaussian noise

$$m^i_{k+1} = \begin{cases} 0, & \theta^i_{k+1} < \theta^i_- \text{ or} \\ & \theta^i_{k+1} \in [\theta^i_-, \theta^i_+] \text{ and } \xi^i_k = 0 \\ 1, & \theta^i_{k+1} > \theta^i_+ \text{ or} \\ & \theta^i_{k+1} \in [\theta^i_-, \theta^i_+] \text{ and } \xi^i_k = 1 \\ m^i_k, & \text{otherwise.} \end{cases}$$

Interpretation, caveats and next steps

Interpretation

- Distributed PV may have *no* impact on LF and AGC requirements beyond (forecastable) clear sky variation.
- Contrast: Centralized PV may require 2 GW of capacity
  - This works out to *double* earlier estimates

Caveats, next steps

- We have only looked at requirements from PV, but system operators balance *net load*.
  - **Next step:** Add model of short time scale load and wind variability
- The estimates are upper bounds assuming a perfect forecast
  - **Next step:** Add forecast error model (will increase the estimates
- How well does the need for solar balancing correlate with DR resource?
  - **Next step:** Smart meter data analysis to characterize resource