### Neural Interaction Detector - Detecting High-order Interactions via Deep Neural Networks

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Physics Informed Machine Learning

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# Machine learning and AI research can be Thought of as Building the Brain of the 4th Industrial and Revolutions





#### Where we are - Teaching Machines to See





### Where we are - Teaching Machines to Talk





### Next Step - Teaching Machines to Discover



random rewiring (q)



### Next Step - Teaching Machines to Discover





### Research Thrusts in Time Series Analysis





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Sponsors:



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### Research Thrusts in Network Analysis

Novel machine learning models for network analysis and inference:

- Network anomaly detection [SDM 2010; ICDM 2012; KDD 2014]
- Robust network inference [ICML 2015; NIPS 2017; WSDM 2017]
- Network analysis for Recommendation [ICML 2012]
- Network embedding via deep learning models [IJCAI workshop, 2107]

Sponsors:





### NSF CyberSEES Project

**Objectives:** a marriage between deep learning approaches and physics based simulation models

Application: casual attribution of urban heat island from heterogeneous data collections Research problems

- Multi-rate multiresolution
- Heterogeneous data quality
- Interpretation of deep learning models



#### Collaborators







#### Deep Learning as Blackbox





### How Deep Learning is Perceived by Students



 $\sim$ 

I am teaching a Deep Learning graduate course this Fall at CMU with over 300 MSc and PhD students enrolled.

Today, after our midterm, I received the following anonymous feedback: "Did I take the wrong exam? Does this exam cover too little machine learning stuff and focus too much on mathematics?"

I guess there is a common belief that Deep Learning is all about installing TensorFlow or PyTorch and training a gigantic convnet on multiple GPUs (2)





### Importance of Explainable Artificial Intelligence - I





#### Importance of Explainable Artificial Intelligence - I





### Importance of Explainable Artificial Intelligence - II

How can I trust any machine learning algorithm? [Ribeiro et al, 2016]



(a) Husky classified as wolf

(b) Explanation



### Interpretable Model is Necessary

Interpretable predictive models are shown to result in faster adoptability of machine learning models.



- Simple and commonly use models
- Easy to interpret, mediocre performance



- Deep learning solutions
- Superior performance, hard to explain

Can we learn interpretable models with robust prediction performance?



### Ongoing Work on Explainable Machine Learning Models

#### **Direct Interpretation**

- [Garson, 1991]: estimating feature importance directly from network weight connections
- [Hechtlinger, 2016]: computing output gradients with respect to input features
- [Itti et al., 1998; Mnih et al., 2014; Xu et al., 2015]: attention models

#### **Indirect Interpretation**

- [Provost et al., 1997]: sensitivity analysis of feature contributions to a neural network's output
- [Ribeiro et al., 2016]: local interpretability for black-box models
- [Che et al., 2016]: mimicking the blackbox through the prediction scores
- [Maaten and Hinton, 2008; Simonyan et al., 2013; Yosinski et al., 2014; LeCun et al., 2015; Mnih et al., 2015; Mahendran and Vedaldi, 2015]: visualizing the hidden units

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### Interpretable Mimic Learning Framework [Che et al., 2016]

- Main ideas:
  - Borrow the ideas from knowledge distillation [Hinton, et al., 2015] and mimic learning [Ba, Caruana, 2014].
  - Use Gradient Boosting Trees (GBTs) to mimic deep learning models.
- Training Pipeline:



• Benefits: Good performance, less overfitting, interpretations.



### Quantitative Evaluation

AUROC score of prediction on patients with acute hypoxemic respiratory failure.



AUROC score of 20 ICD-9 diagnosis category prediction tasks on MIMIC-III dataset.



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### Model/Feature Interpretation

**Partial dependency plot** for mortality prediction on patients with acute hypoxemic respiratory failure.



- pH value in blood should stay in a normal range around 7.35-7.45.
- Our model predicts a higher mortality change when the patient pH value below 7.325.

#### Most Useful Decision Trees for ventilator free days prediction.



Useful features:

- Lung injury score
- Oxygenation index
- PF ratio change



### Black-Box Problem of Neural Networks

- Can we directly interpret neural networks?
- Existing methods to interpret neural networks do not cover the interpretation of statistical interactions.





#### Interaction Detection

- Statistical interactions: non-additive groupings of variables in function  $F(\mathbf{x})$ .
- Example 1:

$$F_1(\mathbf{x}) = \frac{\pi^{x_1 x_2} \sqrt{2x_3} - \sin^{-1}(x_4) + \log(x_3 + x_5) - \frac{x_9}{x_{10}} \sqrt{\frac{x_7}{x_8}} - \frac{x_2 x_7}{x_8}$$
  
Interactions:  $\{x_1, x_2, x_3\}$ ,  $\{x_3, x_5\}$ ,  $\{x_7, x_8, x_9, x_{10}\}$ ,  $\{x_2, x_7\}$   
• Example 2:

$$F_2(\mathbf{x}) = \log(x_1) + \log(x_2)$$

Interactions: none.



### Main Contributions

- Our contributions:
  - A novel interpretation of the weights of a deep neural network
  - A state-of-the-art framework for detecting arbitrary-order interactions accurately and efficiently
  - A model reduction of deep neural networks via generalized additive models





#### Preliminaries

**Feedforward Neural Network**: consider a feedforward neural network with L hidden layers. Let  $p_{\ell}$  be the number of hidden units in the  $\ell$ -th layer.

- Input features as the 0-th layer and  $p_0 = p$  is the number of input features.
- L weight matrices  $\mathcal{W}^{(\ell)} \in \mathcal{P}_{\ell} \times \mathcal{P}_{\ell-1}$  and L bias vectors  $\mathbf{b}^{(\ell)} \in \mathcal{P}_{\ell}$ .
- $\phi(\cdot)$  is the activation function (non-linearity),  $\mathbf{w}^y \in p_L$  and  $b^y \in are$  the coefficients and bias for the final output.
- Formulation:

$$\mathbf{h}^{(0)} = \mathbf{x}, \quad y = (\mathbf{w}^y)^\top \mathbf{h}^{(L)} + b^y, \quad \text{ and } \mathbf{h}^{(\ell)} = \phi\left(\mathcal{W}^{(\ell)}\mathbf{h}^{(\ell-1)} + \mathbf{b}^{(\ell)}\right), \quad \forall \ell = 1, 2, \dots, L.$$



#### Motivations

**Key observation:** any input features interacting with each other must follow strongly weighted connections to a common hidden unit before the final output.

An example:  $F(\mathbf{x})$  has interaction  $\{x_1, x_3\}$ 





#### Lemma (Interactions at Common Hidden Units)

Consider a feedforward neural network with input feature  $x_i, i \in [p]$ , where  $y = \varphi(x_1, \ldots, x_p)$ . For any interaction  $\mathcal{I} \subset [p]$  in  $\varphi(\cdot)$ , there exists a vertex  $v_{\mathcal{I}}$  in the associated directed graph such that  $\mathcal{I}$  is a subset of the ancestors of  $v_{\mathcal{I}}$  at the input layer (i.e.,  $\ell = 0$ ).



### Neural Interaction Detector (NID)

Proposed Algorithm:

- 1 Train feedforward neural networks with sparsity regularization
- 2 Rank interactions by interpreting weights
- S Find cutoff on the ranking (if desired)



Interaction Strength Per Hidden Unit

$$\omega_i(\mathcal{I}) = z_i^{(1)} \mu\left(\left|\mathbf{W}_{i,\mathcal{I}}^{(1)}
ight|
ight)~~ ext{for hidden unit}~i$$

$$\mathbf{z}^{(1)} = \left|\mathbf{w}^{y}\right|^{\top} \left|\mathbf{W}^{(L)}\right| \cdot \left|\mathbf{W}^{(L-1)}\right| \cdots \left|\mathbf{W}^{(2)}\right|$$



Interaction Strength Per Hidden Unit

$$\omega_i(\mathcal{I}) = z_i^{(1)} \mu\left(\left|\mathbf{W}_{i,\mathcal{I}}^{(1)}
ight|
ight)~$$
 for hidden unit  $i$ 

$$\mathbf{z}^{(1)} = \left|\mathbf{w}^{y}\right|^{\top} \left|\mathbf{W}^{(L)}\right| \cdot \left|\mathbf{W}^{(L-1)}\right| \cdots \left|\mathbf{W}^{(2)}\right|$$





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### **Theoretical Analysis**

#### Lemma (Neural Network Lipschitz Estimation)

Let the activation function  $\phi(\cdot)$  be a 1-Lipschitz function. Then the output y is  $z_i^{(\ell)}$ -Lipschitz with respect to  $h_i^{(\ell)}$ .

- Lipschitz constants provides upper bounds the gradient magnitudes of hidden units.
- The upper bound on the gradient magnitude approximates how important the variable can be.



Interaction Strength Per Hidden Unit

$$\begin{split} \omega_i(\mathcal{I}) &= z_i^{(1)} \mu\left(\left|\mathbf{W}_{i,\mathcal{I}}^{(1)}\right|\right) \; \text{ for hidden unit } i \\ \mu\left(\cdot\right) &= \min\left(\cdot\right) \end{split}$$

$$\mathbf{z}^{(1)} = \left|\mathbf{w}^{y}\right|^{\top} \left|\mathbf{W}^{(L)}\right| \cdot \left|\mathbf{W}^{(L-1)}\right| \cdots \left|\mathbf{W}^{(2)}\right|$$



#### A Simple Example

**Definition of**  $\mu$ : Let  $\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2$  be the best quadratic approximation, measured by square loss, to the ReLu function  $\max\{\alpha_1 x_1 + \alpha_2 x_2, 0\}$  on  $(x_1, x_2) \in (-1, 1) \times (-1, 1)$ , for the coefficient of interaction  $\{x_1, x_2\}$ ,  $\beta_5$ , we have:

$$|\beta_5| = \frac{3}{4} \left( 1 - \frac{\min\{\alpha_1^2, \alpha_2^2\}}{5 \max\{\alpha_1^2, \alpha_2^2\}} \right) \min\{|\alpha_1|, |\alpha_2|\}$$





























































 $|w_1| > |w_2| > |w_3| > |w_4|$ 







































### Sample Interaction Ranking

Interactions	Strengths		
{1 2 3}	1 2/21		
(1,2,3)	1.5421		
{1,2,3,4}	0.8241		
{1,2}	0.3415		
{1,3}	0.2310		



#### Theorem (Improving the ranking of higher-order interactions)

Let  $\mathcal{R}$  be the set of interactions proposed by NID, let  $\mathcal{I} \in \mathcal{R}$  be a *d*-way interaction where  $d \geq 3$ , and let  $\mathcal{S}$  be the set of subset (d-1)-way interactions of  $\mathcal{I}$  where  $|\mathcal{S}| = d$ . Assume that for any hidden unit *j* which proposed  $s \in \mathcal{S} \cap \mathcal{R}$ ,  $\mathcal{I}$  will also be proposed at the same hidden unit, and  $\omega_j(\mathcal{I}) > \frac{1}{d}\omega_j(s)$ . Then, one of the following must be true: a)  $\exists s \in \mathcal{S} \cap \mathcal{R}$  ranked lower than  $\mathcal{I}$ , i.e.,  $\omega(\mathcal{I}) > \omega(s)$ , or b)  $\exists s \in \mathcal{S}$  where  $s \notin \mathcal{R}$ .



### Find a Cutoff on the Ranking

• Use a generalized additive model with interactions (MLP-Cutoff)

$$c_{K}(\mathbf{x}) = \sum_{i=1}^{p} g_{i}(x_{i}) + \sum_{i=1}^{K} g'_{i}(\mathbf{x}_{\mathcal{I}})$$

$$g_{i}(\cdot): \text{ main effects}$$

$$g'_{i}(\cdot): \text{ interactions}$$

$$Cutoff$$

$$0 \quad 2 \quad 4 \quad 6 \quad 8$$

$$K$$



### Test Suite of Data-Generating Functions

$F_1(\mathbf{x})$	$\pi^{x_1x_2}\sqrt{2x_3} - \sin^{-1}(x_4) + \log(x_3 + x_5) - \frac{x_9}{x_{10}}\sqrt{\frac{x_7}{x_8}} - x_2x_7$
$F_2(\mathbf{x})$	$\pi^{x_1x_2}\sqrt{2 x_3 } - \sin^{-1}(0.5x_4) + \log( x_3 + x_5  + 1) + \frac{x_9}{1 +  x_{10} }\sqrt{\frac{x_7}{1 +  x_8 }} - x_2x_7$
$F_3(\mathbf{x})$	$\exp x_1 - x_2  +  x_2x_3  - x_3^{2 x_4 } + \log(x_4^2 + x_5^2 + x_7^2 + x_8^2) + x_9 + \frac{1}{1 + x_{10}^2}$
$F_4(\mathbf{x})$	$\exp x_1 - x_2  +  x_2x_3  - x_3^{2 x_4 } + (x_1x_4)^2 + \log(x_4^2 + x_5^2 + x_7^2 + x_8^2) + x_9 + \frac{1}{1 + x_{10}^2}$
$F_5(\mathbf{x})$	$rac{1}{1+x_1^2+x_2^2+x_3^2}+\sqrt{\exp(x_4+x_5)}+ x_6+x_7 +x_8x_9x_{10}$
$F_6(\mathbf{x})$	$\exp\left( x_1x_2 +1\right) - \exp( x_3+x_4 +1) + \cos(x_5+x_6-x_8) + \sqrt{x_8^2 + x_9^2 + x_{10}^2}$
$F_7(\mathbf{x})$	$\left(\arctan(x_1) + \arctan(x_2)\right)^2 + \max(x_3x_4 + x_6, 0) - \frac{1}{1 + (x_4x_5x_6x_7x_8)^2} + \left(\frac{ x_7 }{1 +  x_9 }\right)^5 + \sum_{i=1}^{10} x_i$
$F_8(\mathbf{x})$	$x_1x_2 + 2^{x_3 + x_5 + x_6} + 2^{x_3 + x_4 + x_5 + x_7} + \sin(x_7\sin(x_8 + x_9)) + \arccos(0.9x_{10})$
$F_9(\mathbf{x})$	$\tanh(x_1x_2+x_3x_4)\sqrt{ x_5 }+\exp(x_5+x_6)+\log\left((x_6x_7x_8)^2+1\right)+x_9x_{10}+\frac{1}{1+ x_{10} }$
$F_{10}({f x})$	$\sinh(x_1 + x_2) + \arccos(\tanh(x_3 + x_5 + x_7)) + \cos(x_4 + x_5) + \sec(x_7 x_9)$

#### Complex functions are used in our evaluation



### AUC of Pairwise Interaction Strengths

	ANOVA <sup>1</sup>	HierLasso <sup>2</sup>	AG <sup>3</sup>	NID. MLP	NID. MLP-M
$F_1(\mathbf{x})$	0.992	1.00	$1 \pm 0.0$	$0.970 \pm 9.2e{-3}$	$0.995 \pm 4.4e - 3$
$F_2(\mathbf{x})$	0.468	0.636	$0.88 \pm 1.4\mathrm{e}{-2}$	$0.79\pm3.1\mathrm{e}{-2}$	$0.85\pm3.9\mathrm{e}{-2}$
$F_3(\mathbf{x})$	0.657	0.556	$1\pm0.0$	$0.999\pm2.0\mathrm{e}{-3}$	$1\pm0.0$
$F_4(\mathbf{x})$	0.563	0.634	$0.999 \pm 1.4\mathrm{e}{-3}$	$0.85\pm6.7\mathrm{e}{-2}$	$0.996\pm4.7\mathrm{e}{-3}$
$F_5(\mathbf{x})$	0.544	0.625	$0.67\pm5.7\mathrm{e}{-2}$	$1\pm0.0$	$1\pm0.0$
$F_6(\mathbf{x})$	0.780	0.730	$0.64\pm1.4\mathrm{e}{-2}$	$0.98\pm6.7\mathrm{e}{-2}$	$0.70\pm4.8\mathrm{e}{-2}$
$F_7(\mathbf{x})$	0.726	0.571	$0.81\pm4.9\mathrm{e}{-2}$	$0.84\pm1.7\mathrm{e}{-2}$	$0.82\pm2.2\mathrm{e}{-2}$
$F_8(\mathbf{x})$	0.929	0.958	$0.937\pm1.4\mathrm{e}{-3}$	$0.989\pm4.4\mathrm{e}{-3}$	$0.989 \pm 4.5\mathrm{e}{-3}$
$F_9(\mathbf{x})$	0.783	0.681	$0.808\pm5.7\mathrm{e}{-3}$	$0.83\pm5.3\mathrm{e}{-2}$	$0.83\pm3.7\mathrm{e}{-2}$
$F_{10}(\mathbf{x})$	0.765	0.583	$1\pm0.0$	$0.995 \pm 9.5 e{-3}$	$0.99\pm2.1\mathrm{e}{-2}$
average	0.721	0.698	$0.87 \pm 1.4 \mathrm{e}{-2}$	$0.92* \pm 2.3e - 2$	$0.92 \pm 1.8e - 2$

<sup>1</sup>Fisher 1925, <sup>2</sup>Bien et al. 2013, <sup>3</sup>Sorokina et al. 2008

 $*F_6$  plays an important role for this result



#### Higher-Order Interaction Detection for Synthetic Data



$$\begin{array}{|c|c|c|c|c|c|c|}\hline F_1(\mathbf{x}) & \pi^{x_1x_2}\sqrt{2x_3} - \sin^{-1}(x_4) + \log(x_3 + x_5) - \frac{x_9}{x_{10}}\sqrt{\frac{x_7}{x_8}} - x_2x_7 \\\hline F_3(\mathbf{x}) & \exp|x_1 - x_2| + |x_2x_3| - x_3^{2|x_4|} + \log(x_4^2 + x_5^2 + x_7^2 + x_8^2) + x_9 + \frac{1}{1 + x_{10}^2} \\\hline F_5(\mathbf{x}) & \frac{1}{1 + x_1^2 + x_2^2 + x_3^2} + \sqrt{\exp(x_4 + x_5)} + |x_6 + x_7| + x_8x_9x_{10} \\\hline F_7(\mathbf{x}) & (\arctan(x_1) + \arctan(x_2))^2 + \max(x_3x_4 + x_6, 0) - \frac{1}{1 + (x_4x_5x_6x_7x_8)^2} + \left(\frac{|x_7|}{1 + |x_9|}\right)^5 + \sum_{i=1}^{10} x_i \\\hline \end{array}$$



### Pairwise Heap-Maps for Real-World Data



#### {1,2}: longitude and latitude

<sup>1</sup>Pace et al. 1997, <sup>2</sup>Fanaee-T et al. 2014, <sup>3</sup>Adam-Bourdarios et al. 2014, <sup>4</sup>Frey et al. 1991



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### Pairwise Heap-Maps for Real-World Data



### {4,7}: hour and working day

<sup>1</sup>Pace et al. 1997, <sup>2</sup>Fanaee-T et al. 2014, <sup>3</sup>Adam-Bourdarios et al. 2014, <sup>4</sup>Frey et al. 1991



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#### Deciphering the Black Box





### USC-Melady Research Group



Michael Tsang, Dehua Cheng, and Yan Liu, Detecting Statistical Interactions from Neural Mascle Network Weights, arXiv:1705.04977.

Yan Liu (USC)

### Thank You! Questions and Comments?



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