

Reinforcement Learning in Phases of Quantum Control



A. G.R. Day



D. Sels



P. Mehta



P. Weinberg



A. Polkovnikov

[arXiv: 1705.00565 \(2017\)](#)

[arXiv: 1711.09109 \(2017\)](#)

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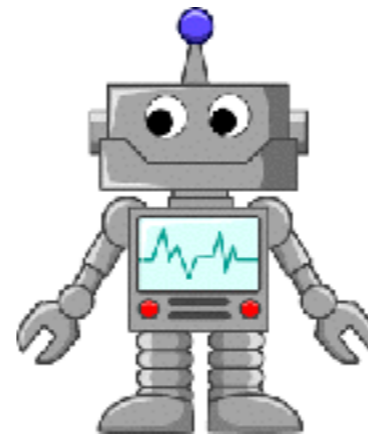
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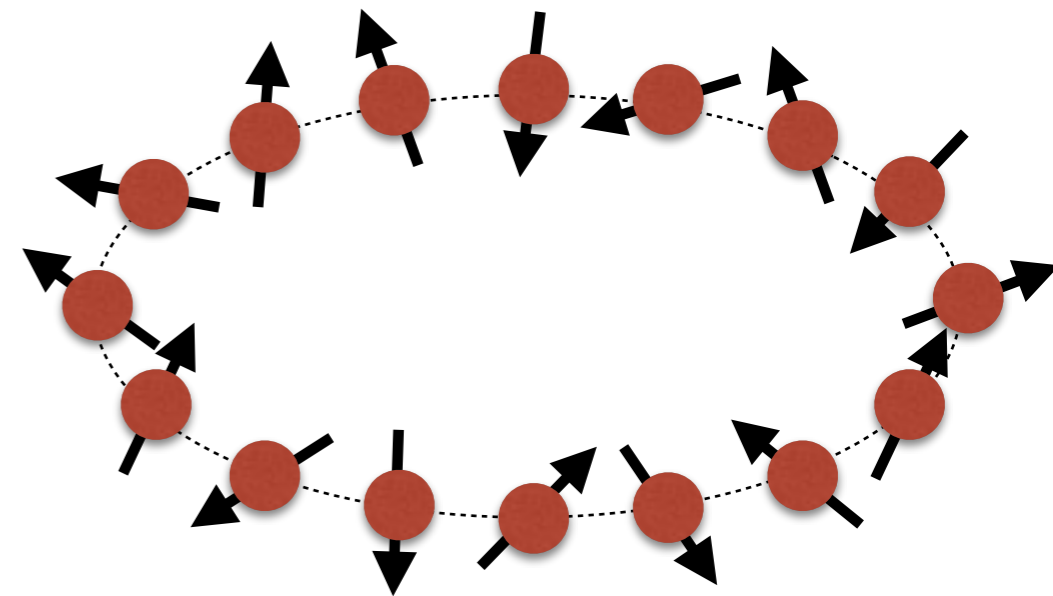
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GOAL:

teach reinforcement learning agent to prepare states
in non-integrable quantum Ising model

$$H(t) = - \sum_j J S_{j+1}^z S_j^z + h_z S_j^z + h_x(t) S_j^x$$



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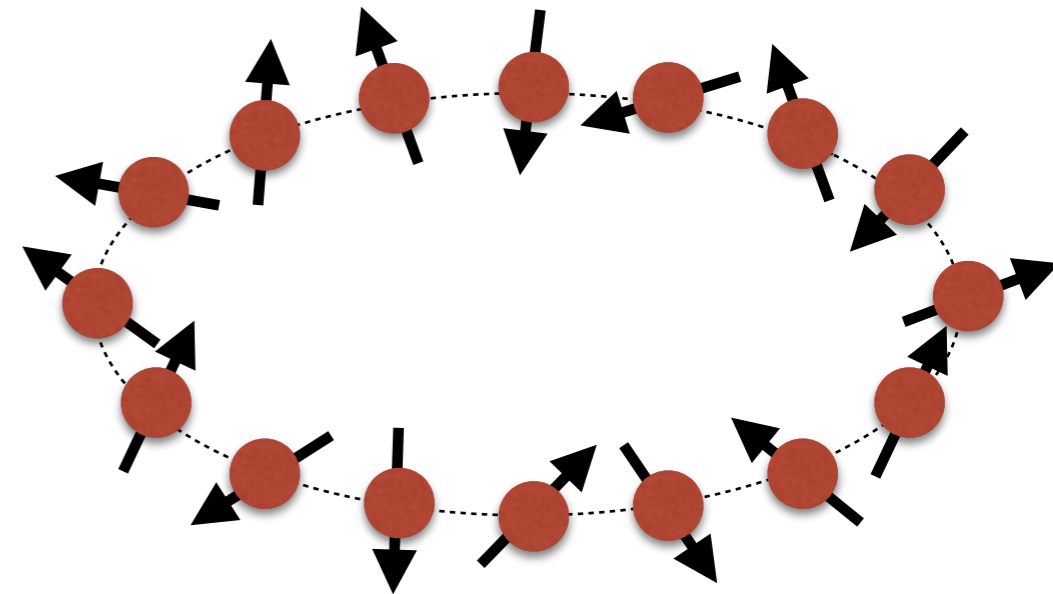
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1. Single-qubit System $J = 0$ (THIS TALK)

→ problem setup, RL agent solution

→ control phase transitions
overconstrained phase,
correlated (glassy) phase,
controllable phase



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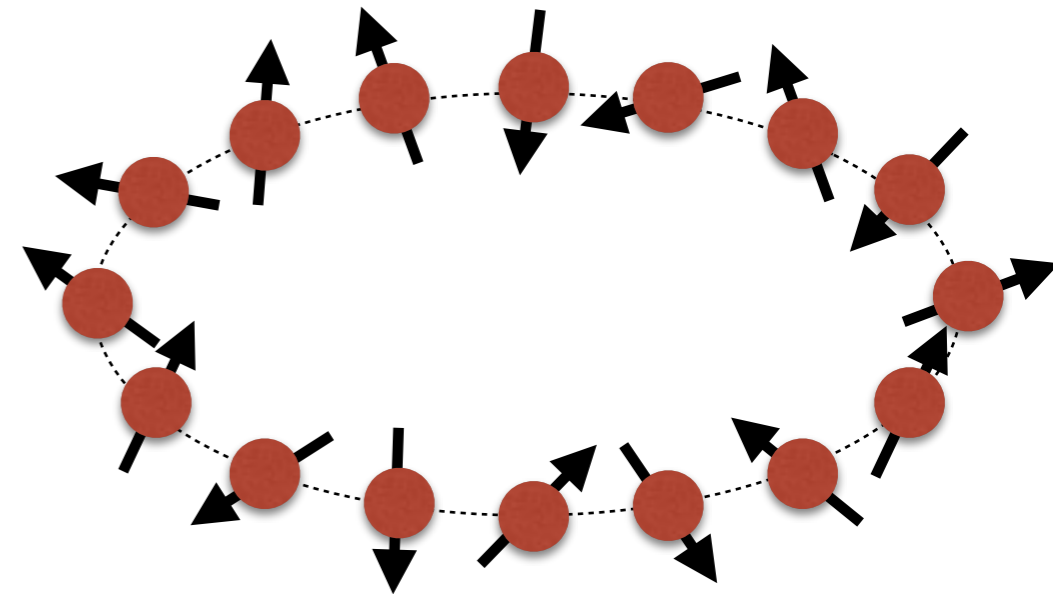
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2. Two-qubit System $L = 2$ (POSTER)

→ spontaneous symmetry breaking
in optimal control landscape



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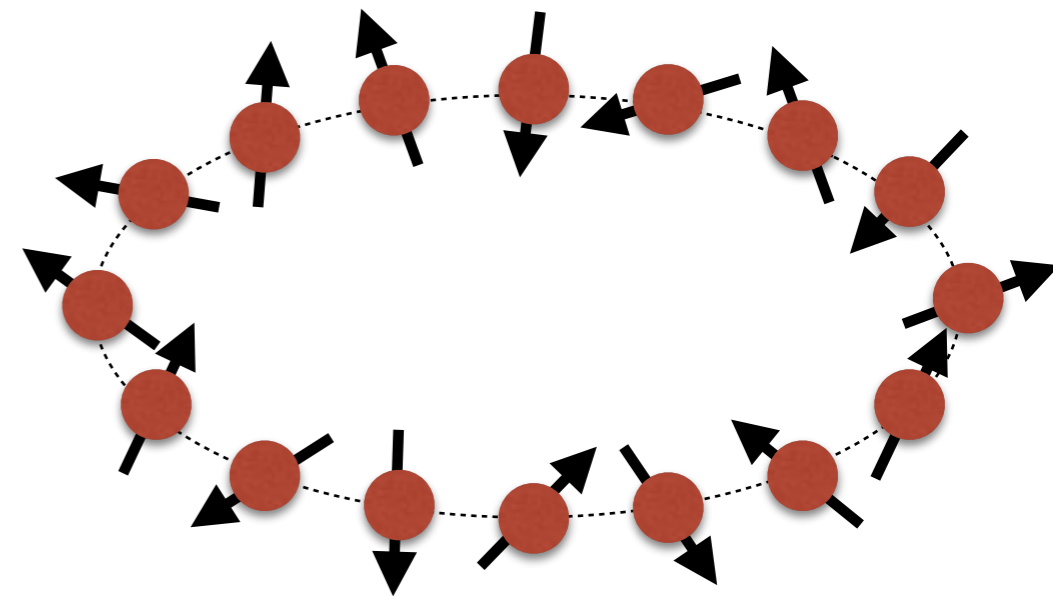
→ spontaneous symmetry breaking in optimal control landscape

3. Many-Body System (POSTER)

→ control phase diagram: *overconstrained phase, glassy/correlated phase*



Alex

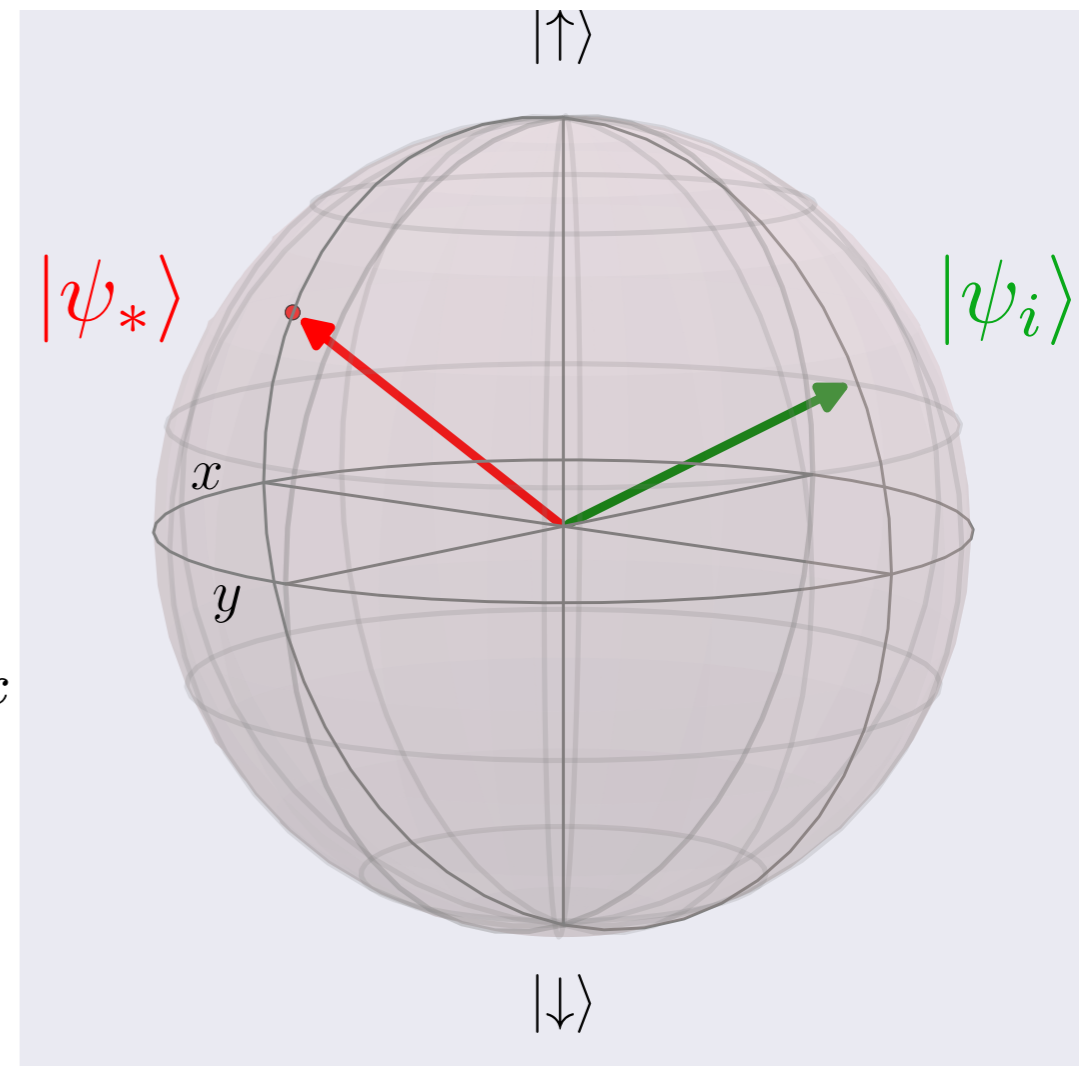


Optimal Qubit State Preparation

→ Hamiltonian: $H(t) = -S^z - h_x(t)S^x$

initial state: $|\psi_i\rangle$: GS of $H_i = -S^z - 2S^x$

target state: $|\psi_*$: GS of $H_* = -S^z + 2S^x$

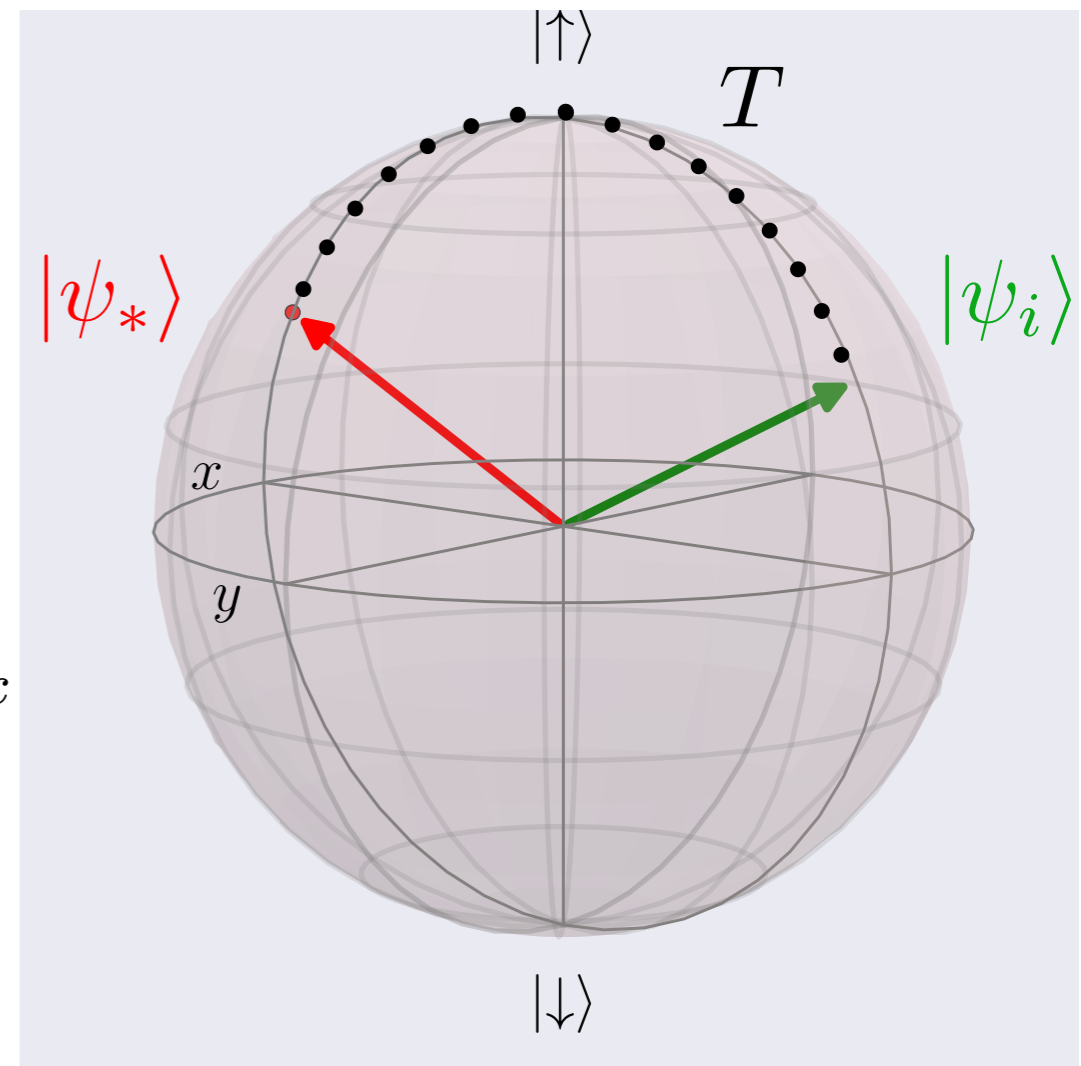


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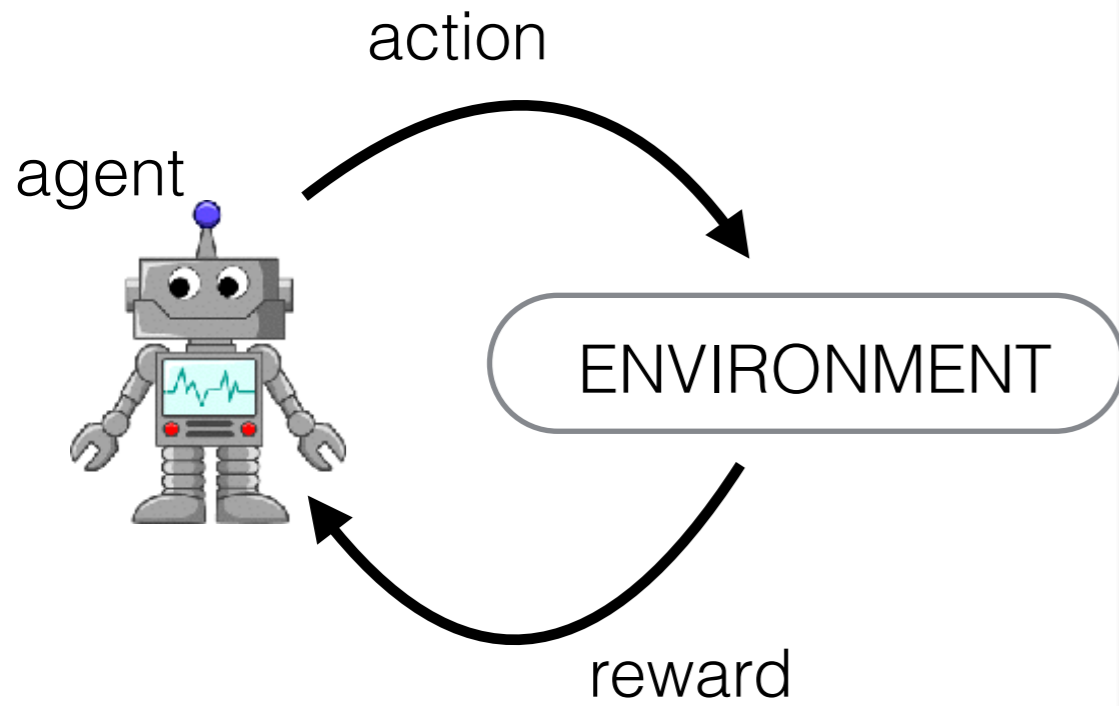


GOAL: find protocol $h(t) \in [-4, 4]$

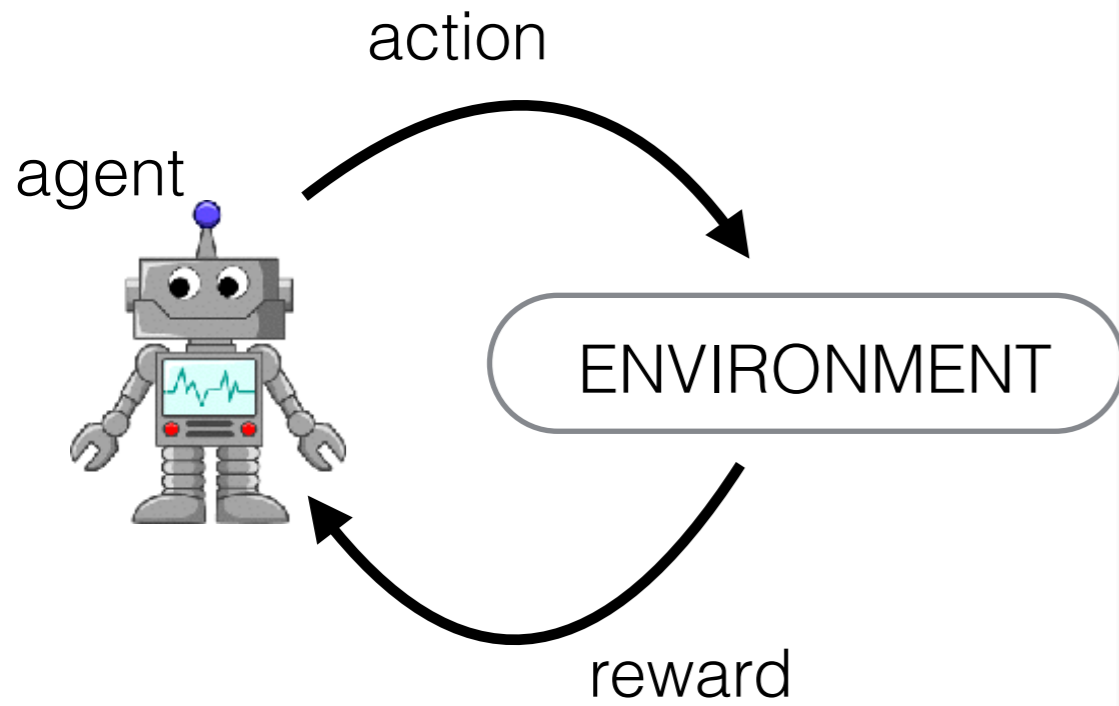
such that $|\psi(t=0)\rangle = |\psi_i\rangle$, $|\psi(t=T)\rangle = |\psi_*\rangle$

measure: fidelity $F_h(T) = |\langle \psi(T) | \psi_* \rangle|^2$

What is Reinforcement Learning?

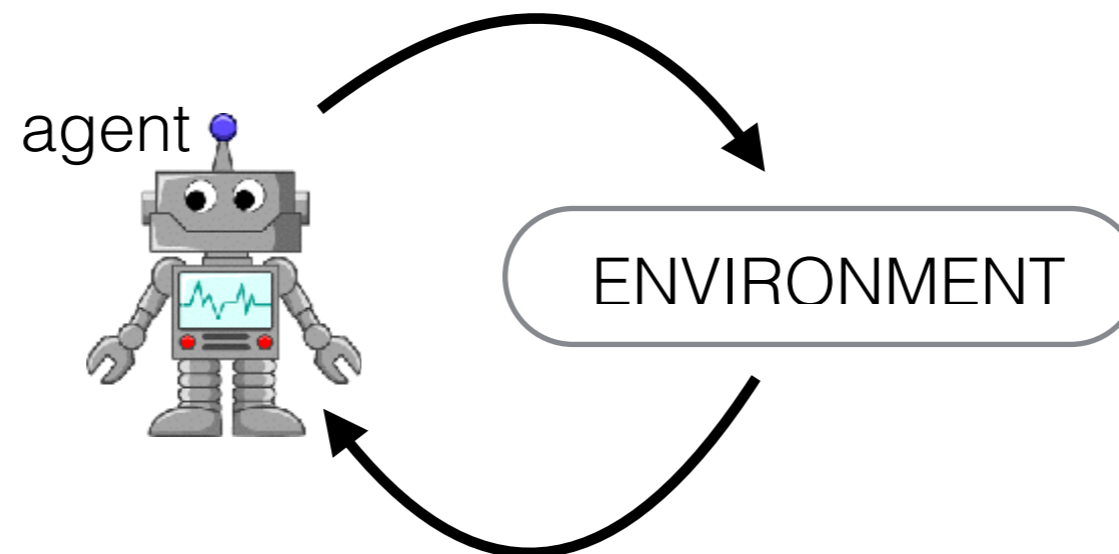


What is Reinforcement Learning?



Reinforcement Learning in a Nutshell

- **Machine Learning**
 - Supervised Learning
 - • **Reinforcement Learning (RL)**
 - Unsupervised Learning



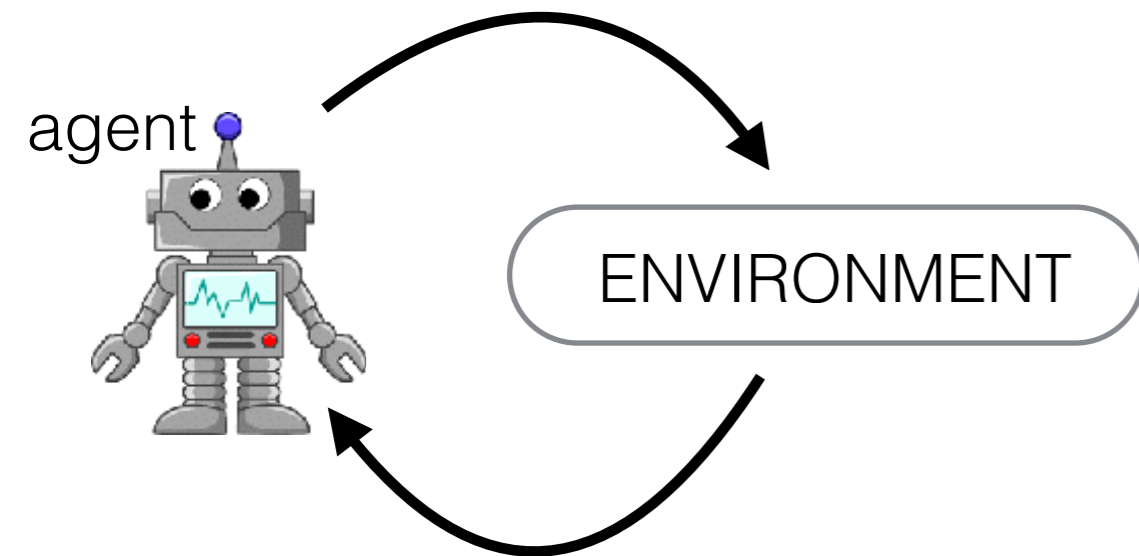
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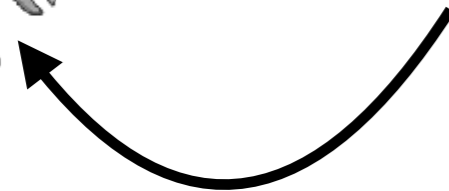
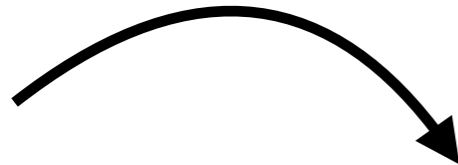
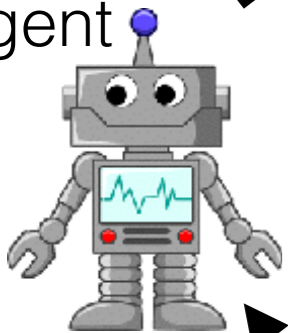
- **Reinforcement Learning (RL)**

- Unsupervised Learning

RL states $\mathcal{S} = \{s\}$



agent



RL states $\mathcal{S} = \{(t, h(t))\}$

Reinforcement Learning in a Nutshell

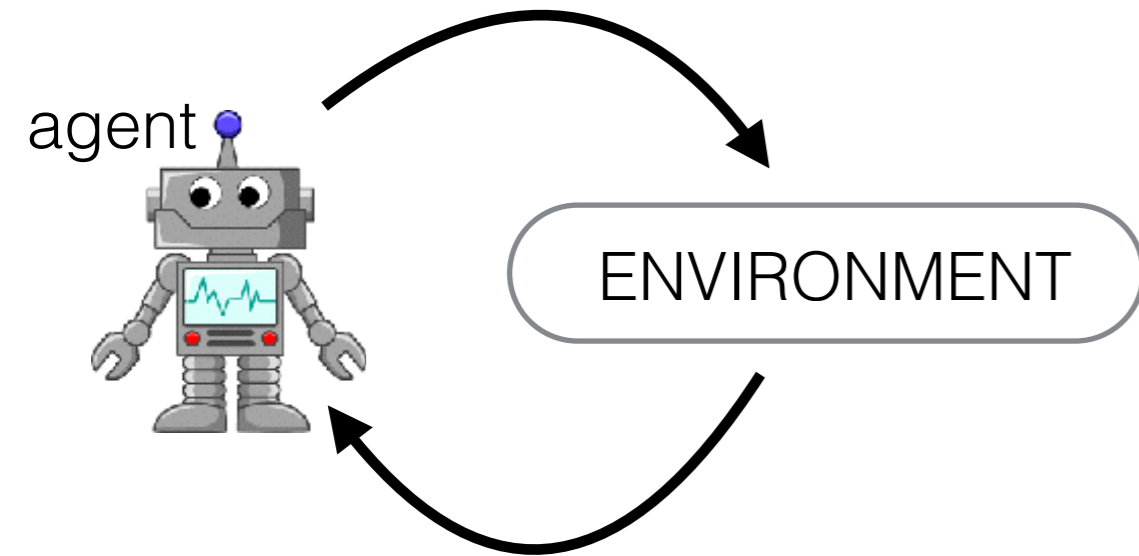
- **Machine Learning**

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→ available actions $\mathcal{A} = \{a\}$



RL states $\mathcal{S} = \{(t, h(t))\}$

available actions $\mathcal{A} = \{a = \delta h\} = \{0, \pm 0.1, \pm 0.2, \pm 0.5, \pm 1.0, \pm 2.0, \pm 4.0, \pm 8.0\}$

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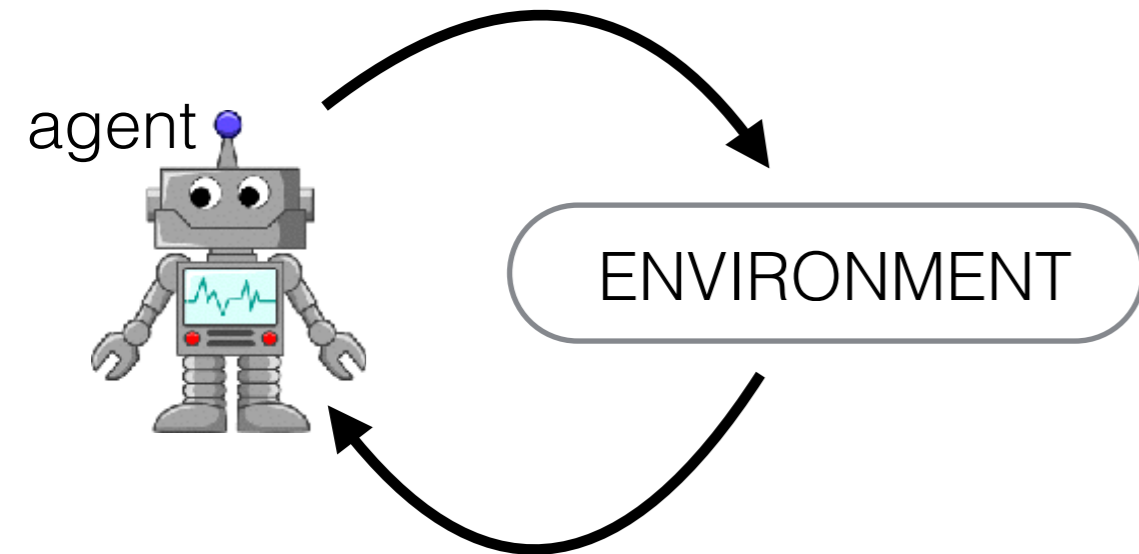
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RL states $\mathcal{S} = \{s\}$

available actions $\mathcal{A} = \{a\}$

rewards $\mathcal{R} = \{r\}$



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rewards $\mathcal{R} = \{r(t) \in [0, 1]\}$

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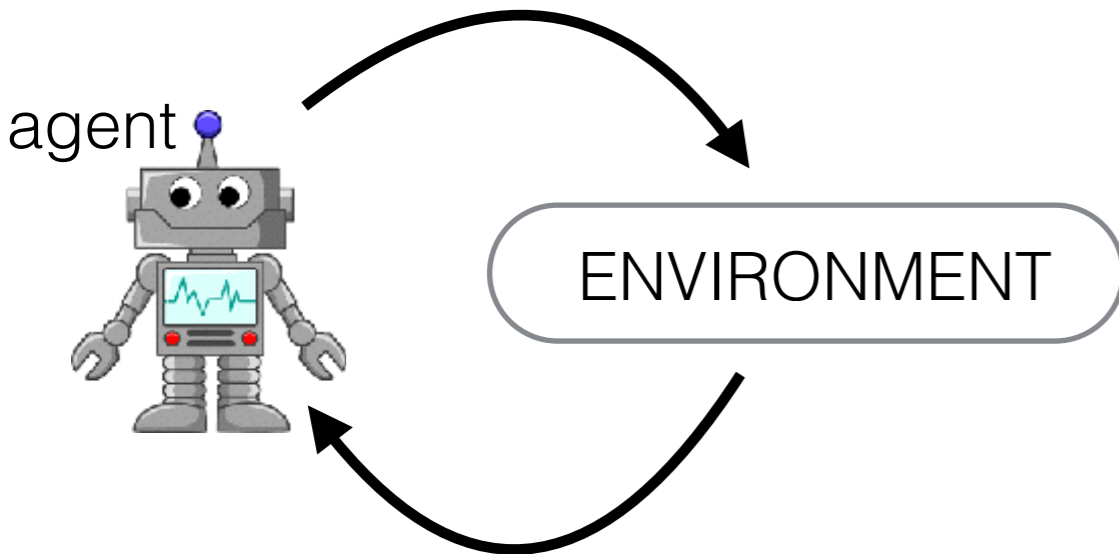
RL states $\mathcal{S} = \{s\}$

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cumulative expected reward
or Q-function $Q(s, a)$

encodes experience/knowledge



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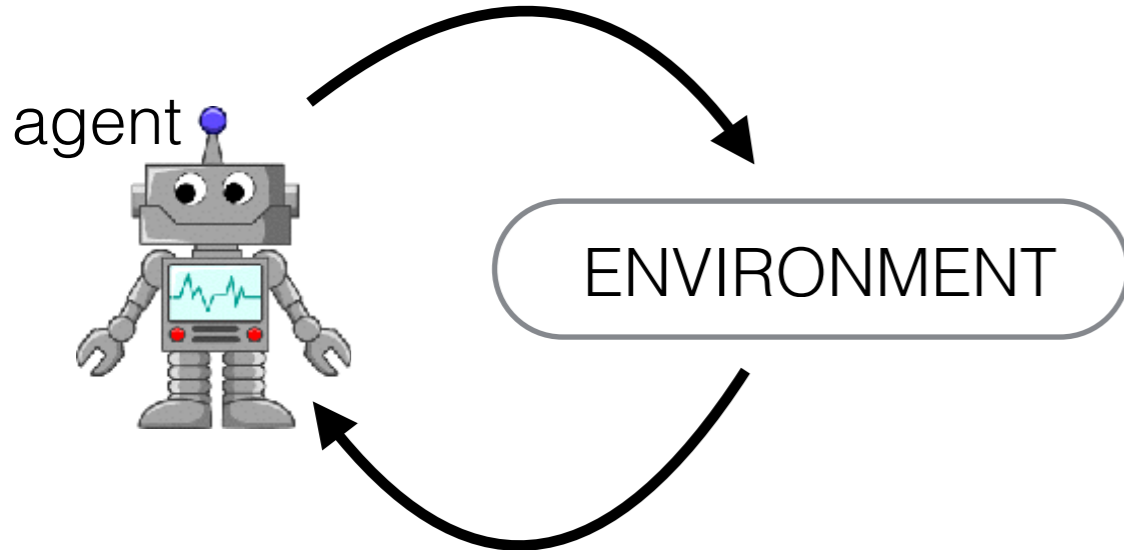
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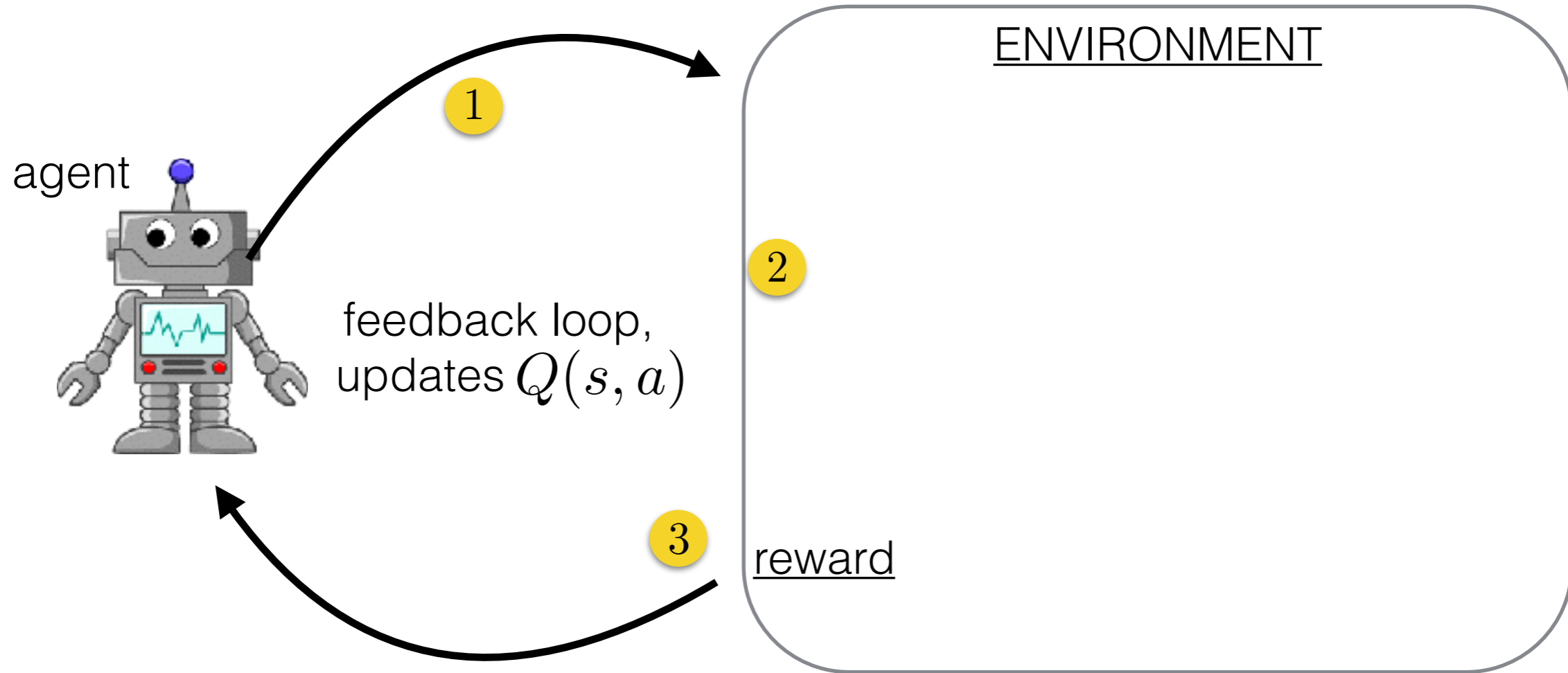
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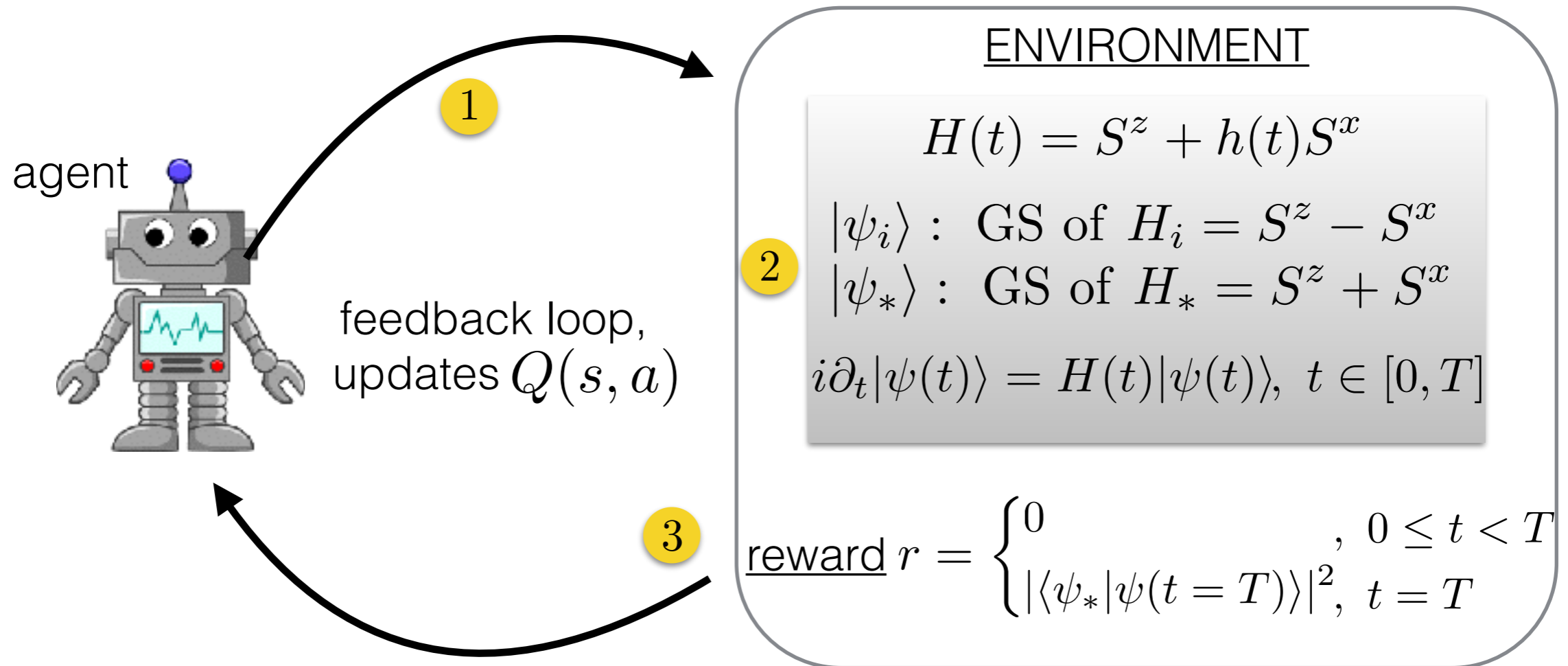
GOAL: maximise cumulative expected reward / Q-function $Q(s, a)$

starting from state s and taking action a

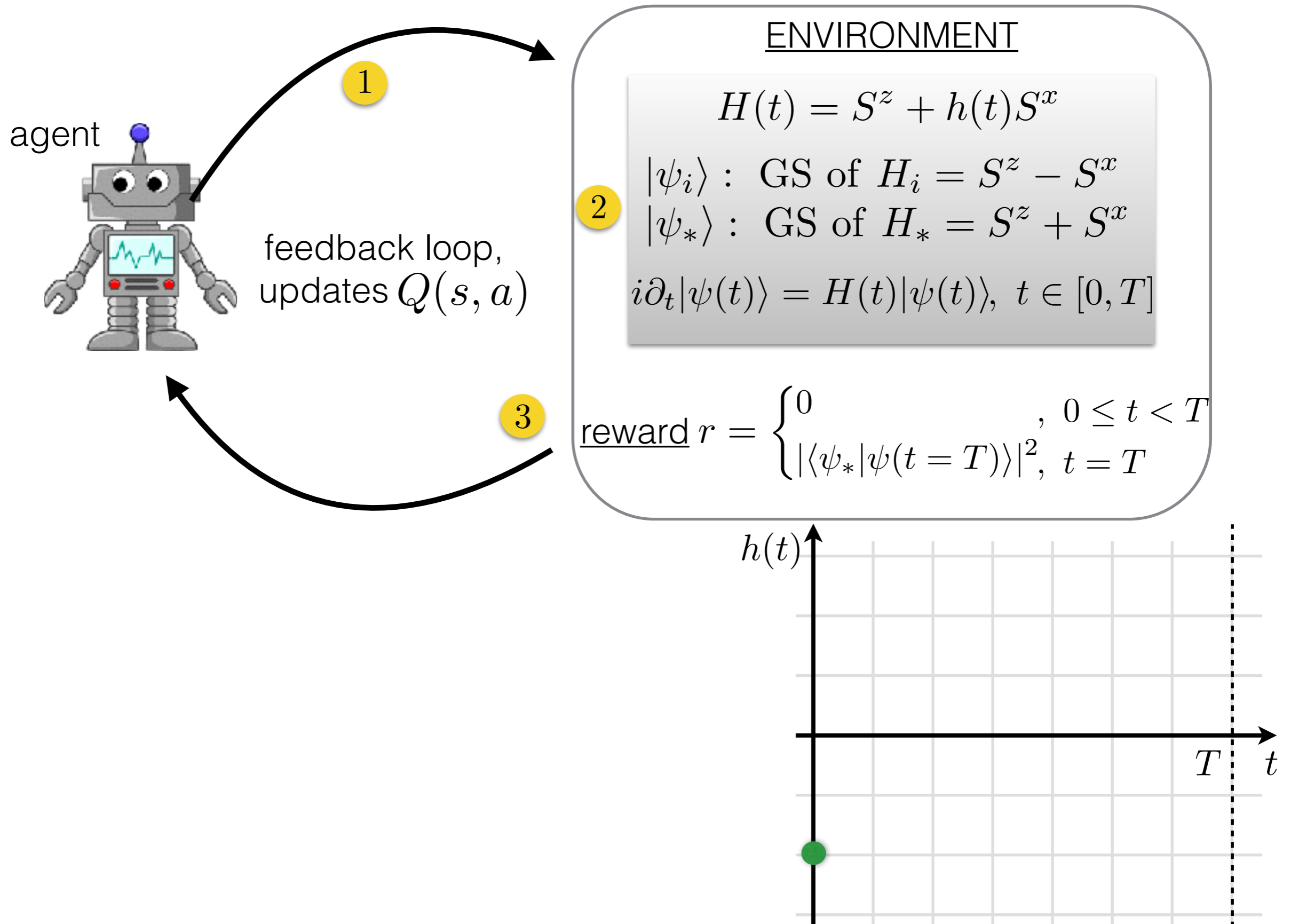
RL Applied to Quantum State Preparation



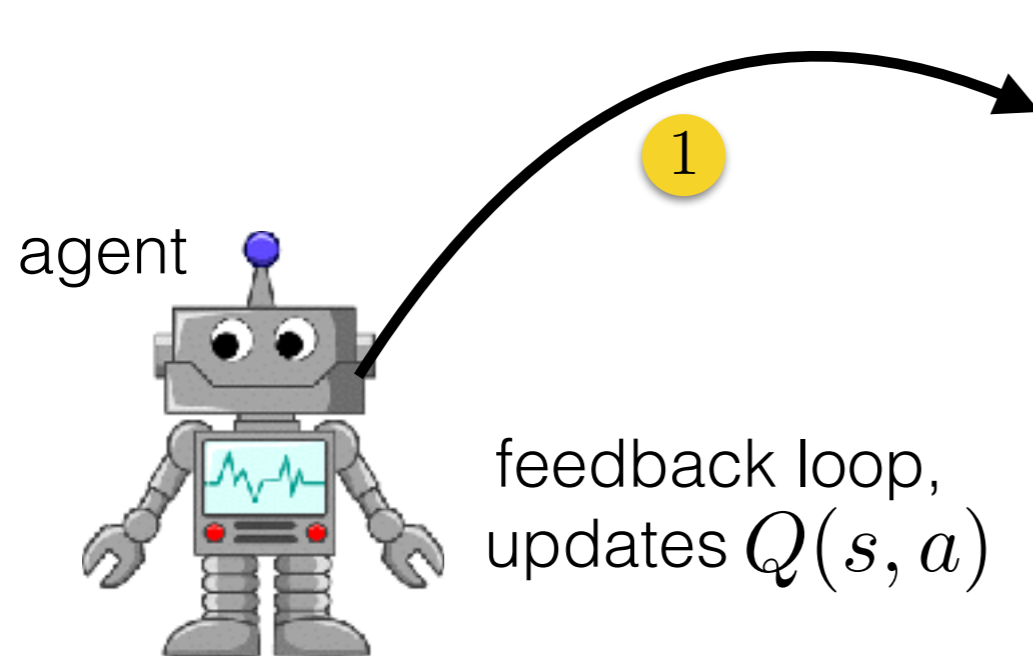
RL Applied to Quantum State Preparation



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RL Applied to Quantum State Preparation



ENVIRONMENT

$$H(t) = S^z + h(t)S^x$$

2

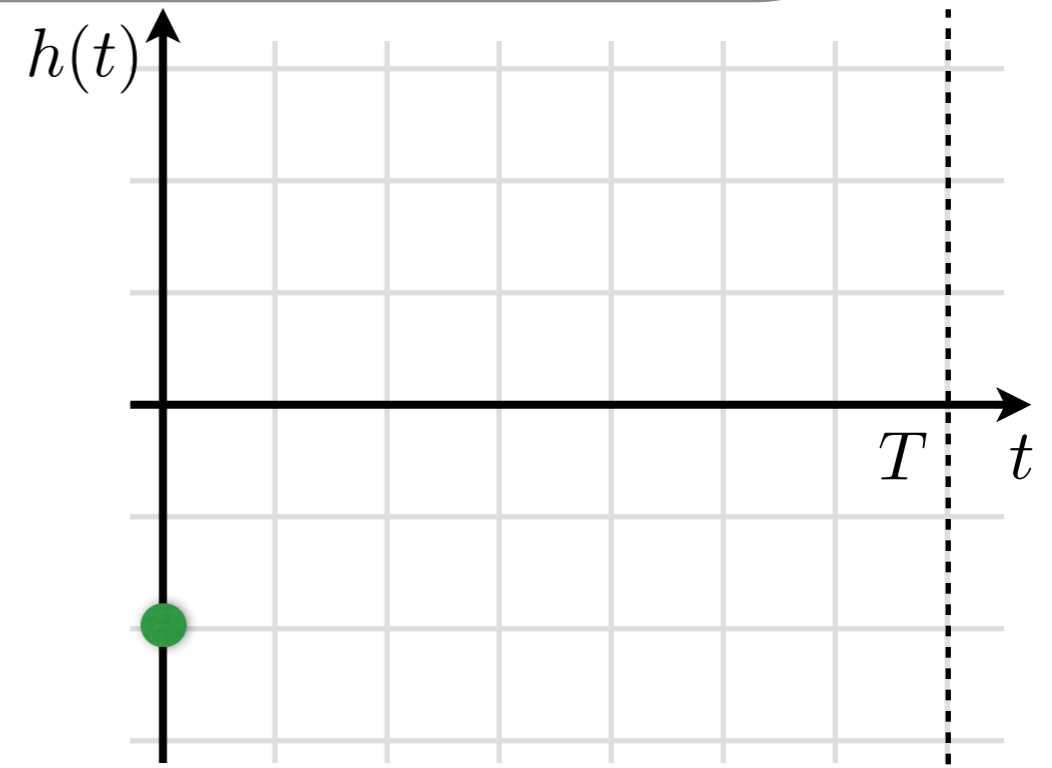
$$|\psi_i\rangle : \text{GS of } H_i = S^z - S^x$$

$$|\psi_*\rangle : \text{GS of } H_* = S^z + S^x$$

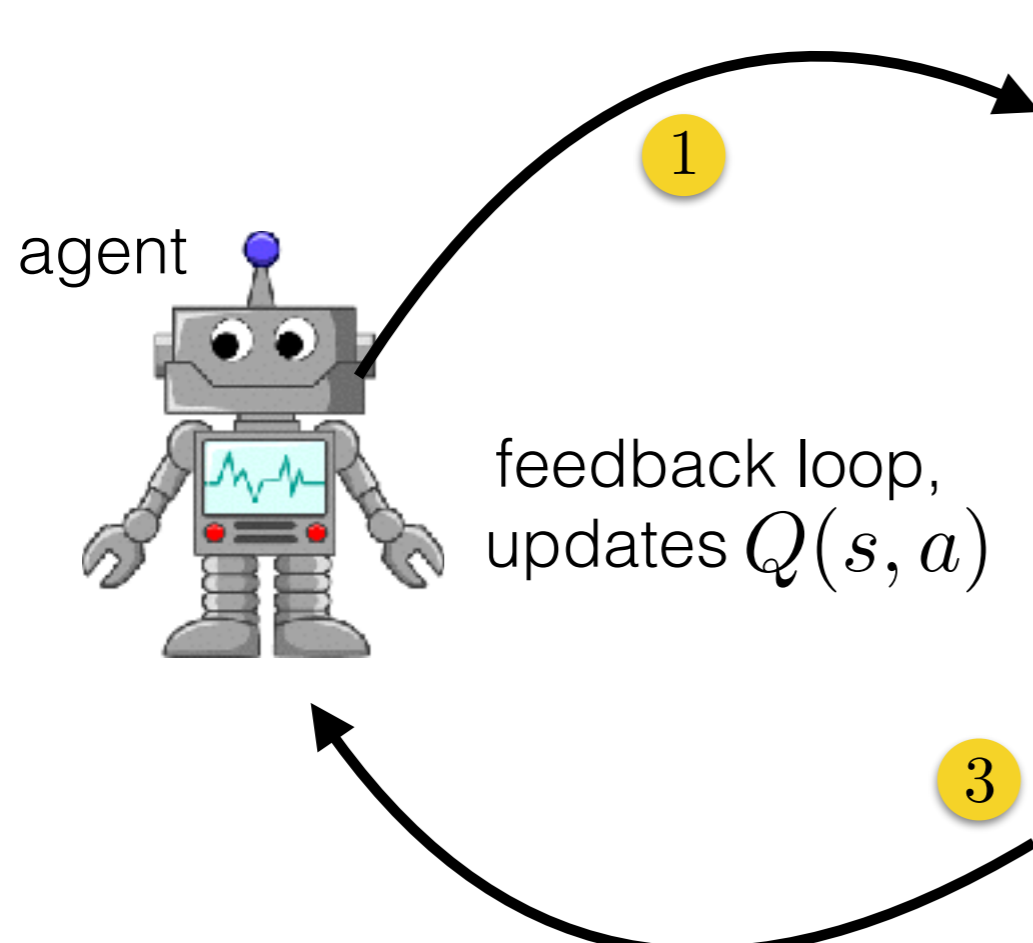
$$i\partial_t|\psi(t)\rangle = H(t)|\psi(t)\rangle, t \in [0, T]$$

$$\text{reward } r = \begin{cases} 0 & , 0 \leq t < T \\ |\langle\psi_*|\psi(t=T)\rangle|^2 & , t = T \end{cases}$$

1 start from state $s_0 = (t = 0, h = -1)$



RL Applied to Quantum State Preparation



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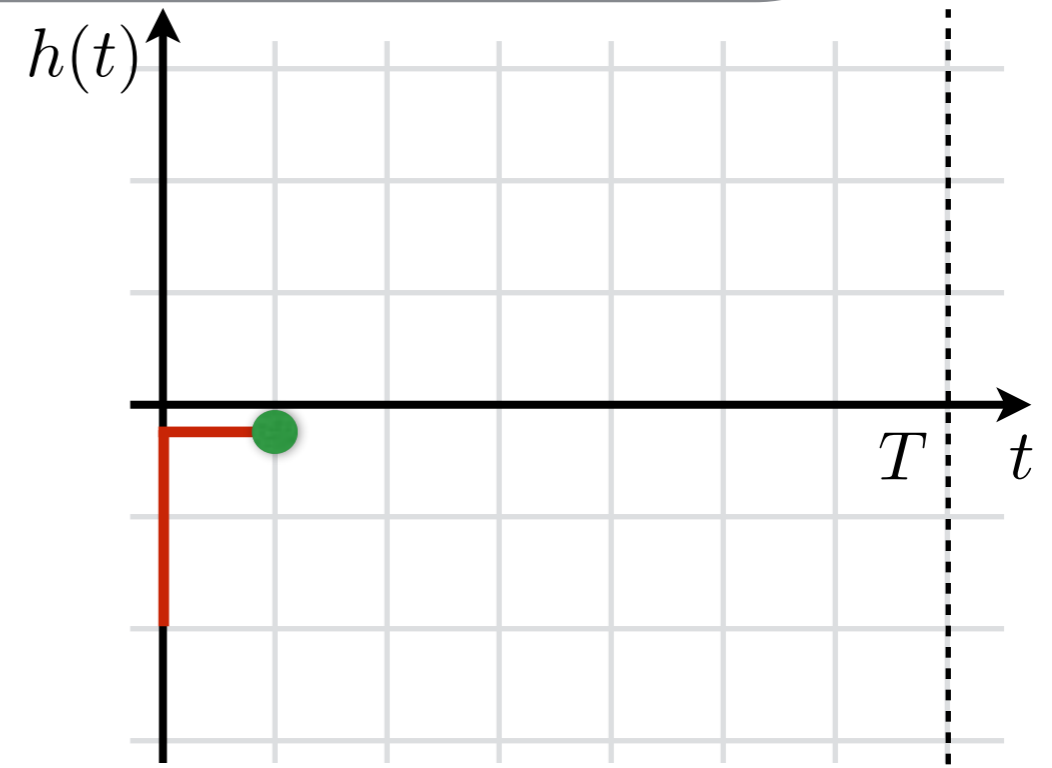
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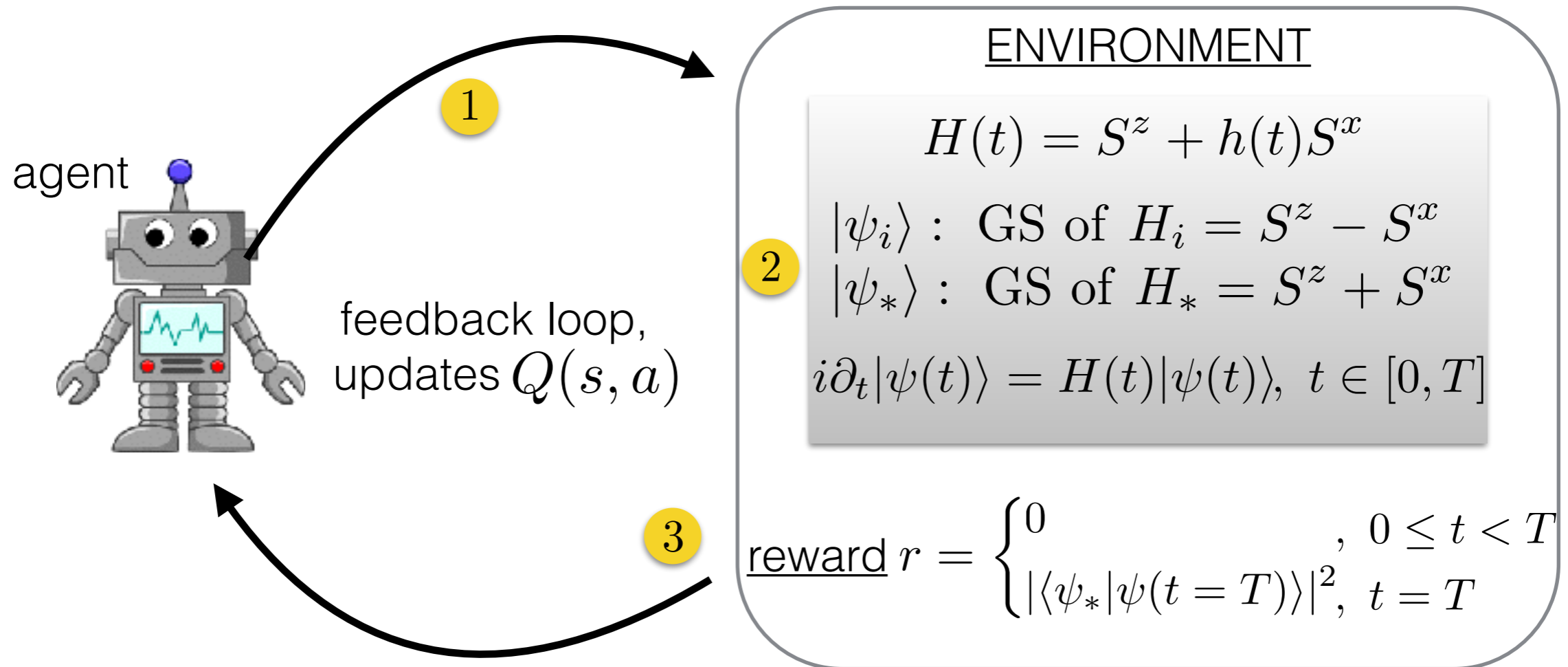
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- 1 start from state $s_0 = (t = 0, h = -1)$
- take action $a_0 : \delta h = 0.8$
- go to state $s_1 = (t = 0.05, h = -0.2)$

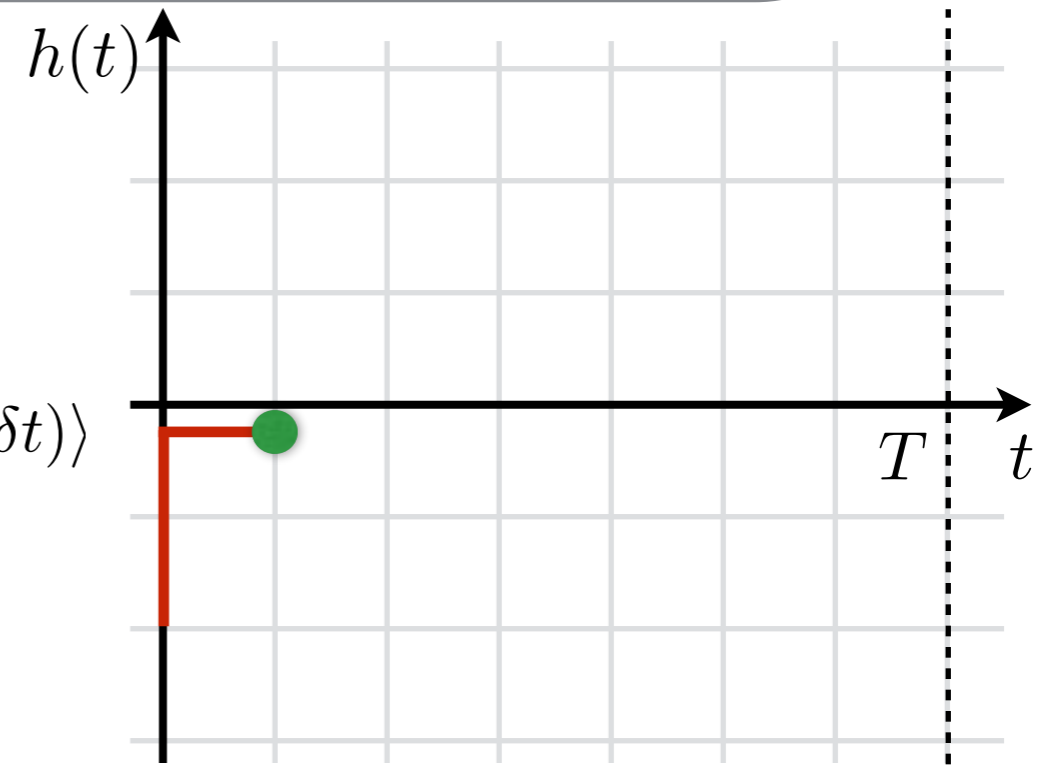


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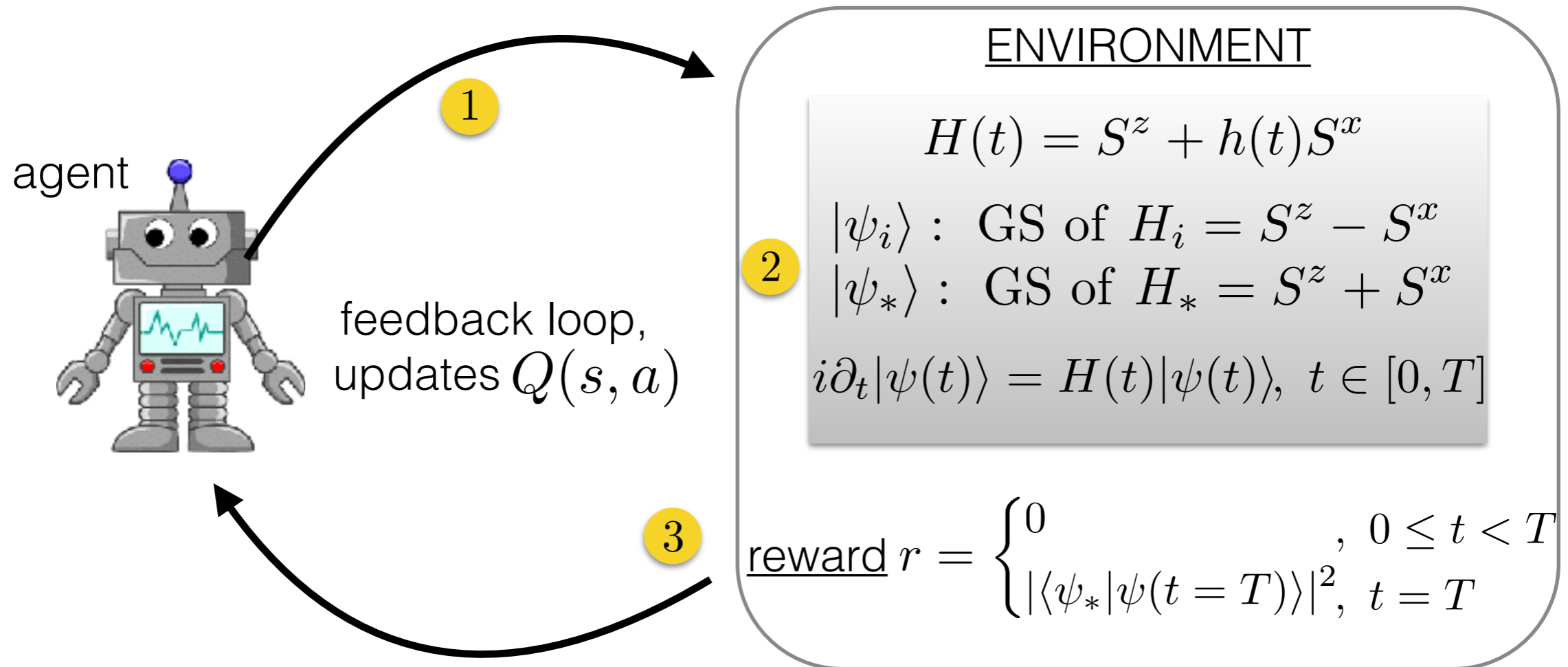


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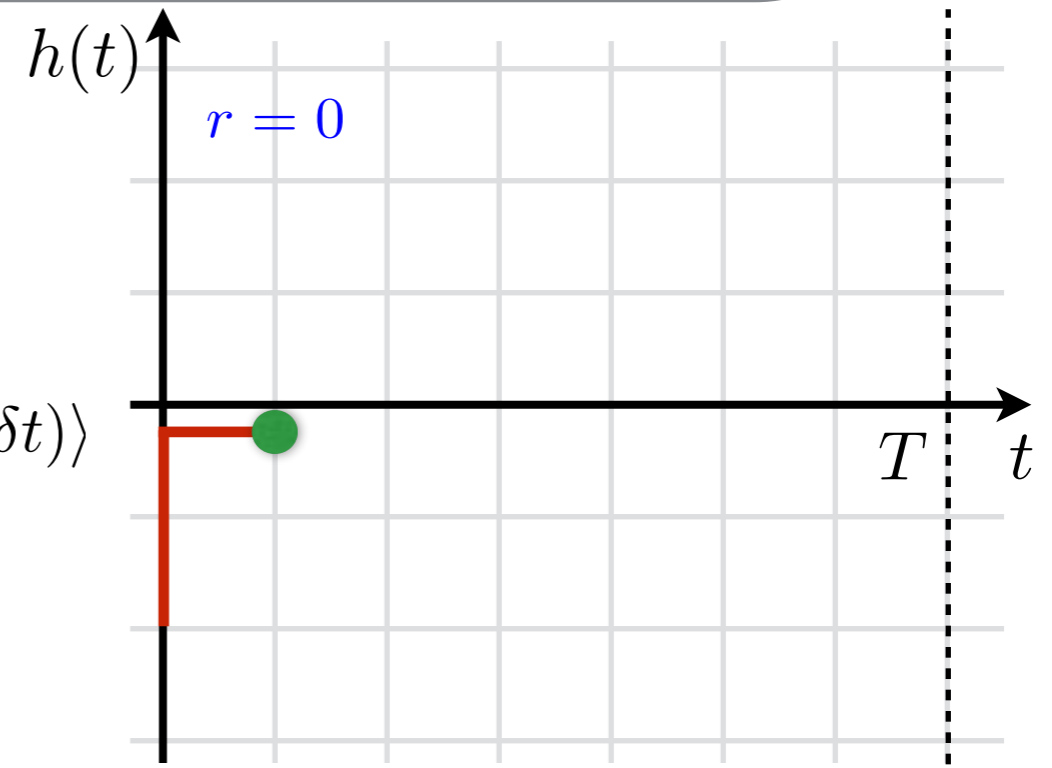
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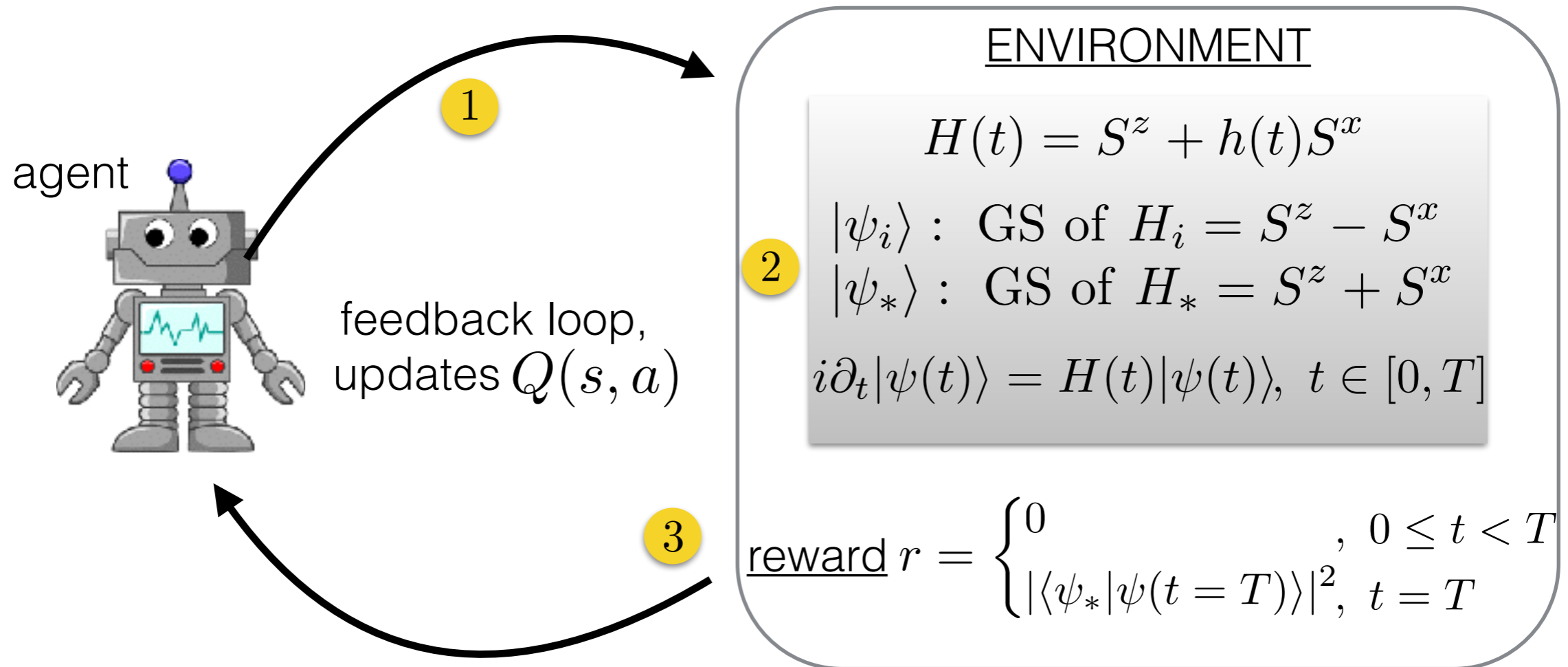
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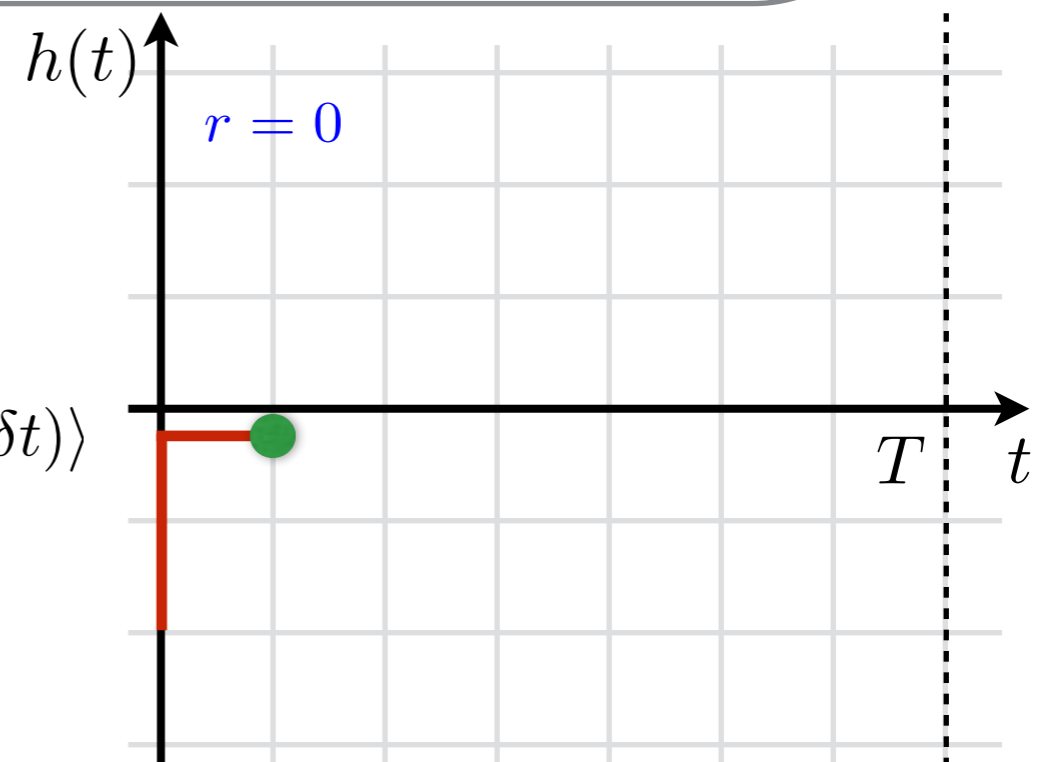
3 calculate reward r
 and use it to update $Q(s, a)$
 which in turn is used to choose subsequent actions



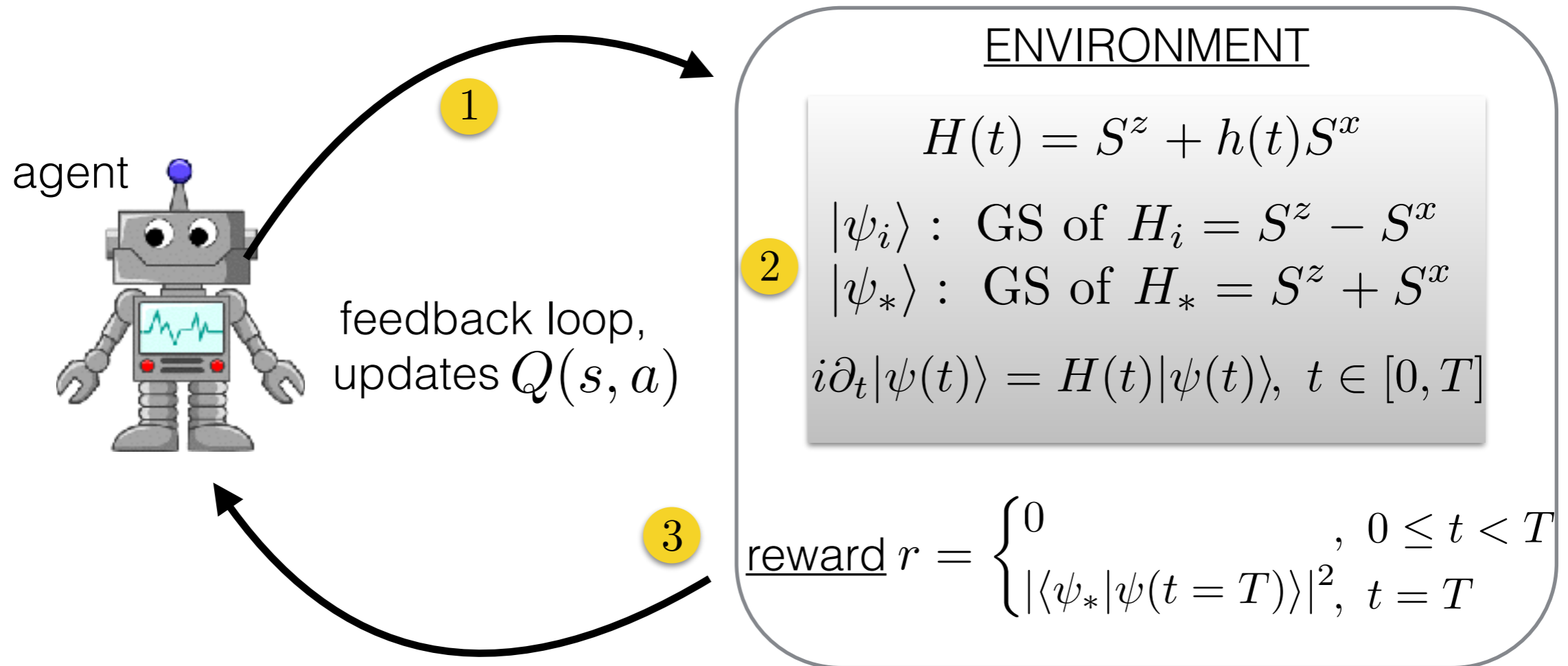
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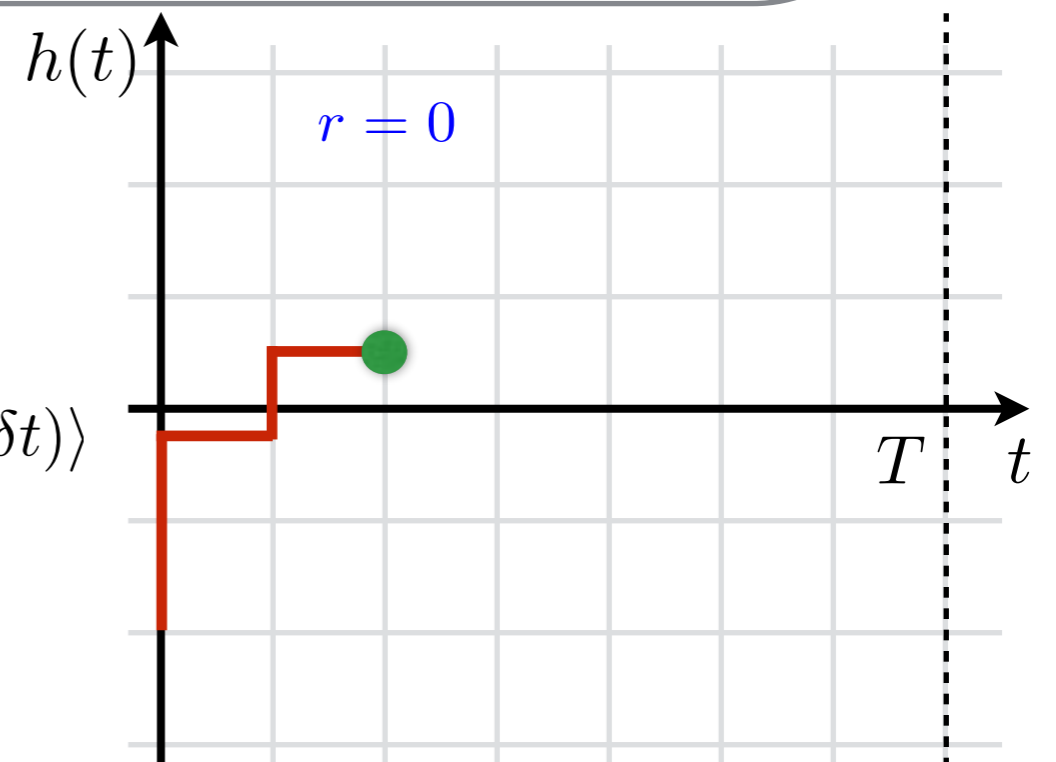
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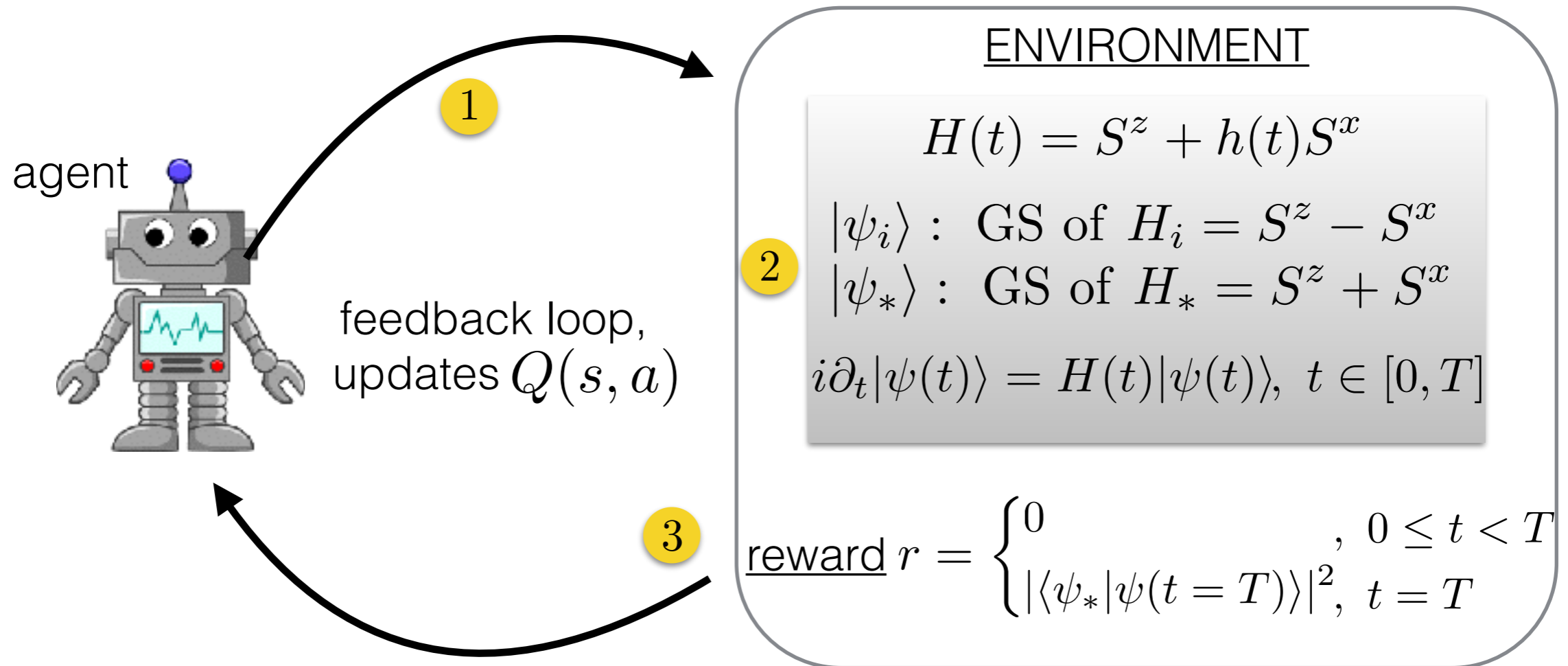
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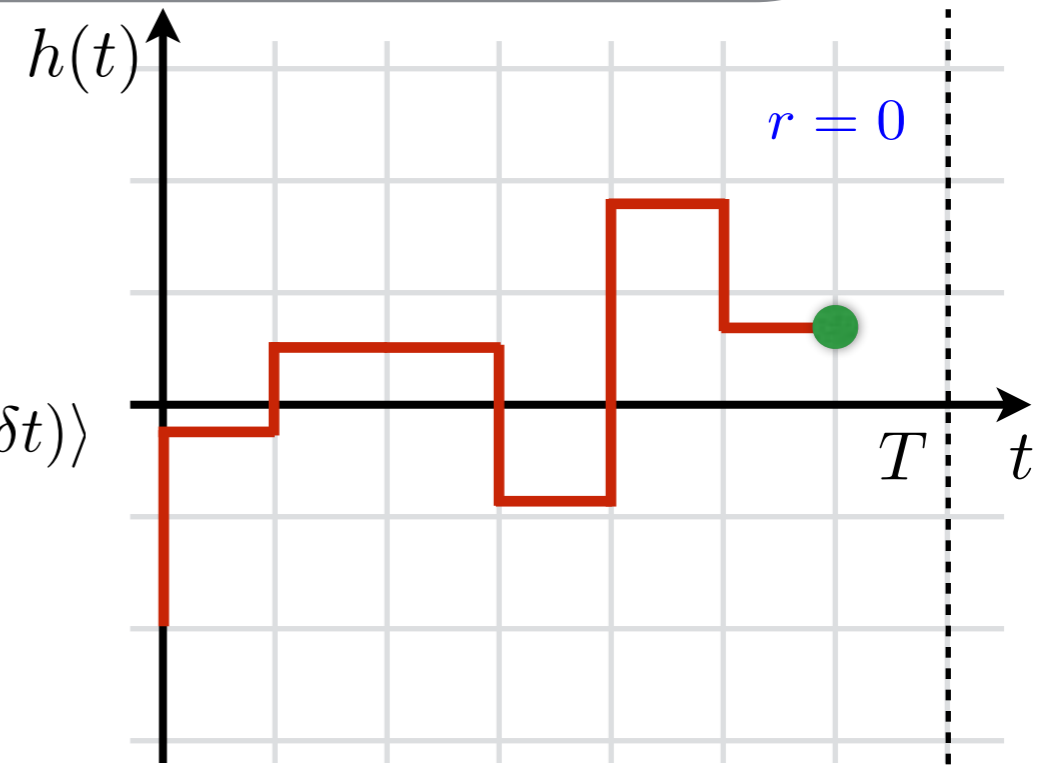
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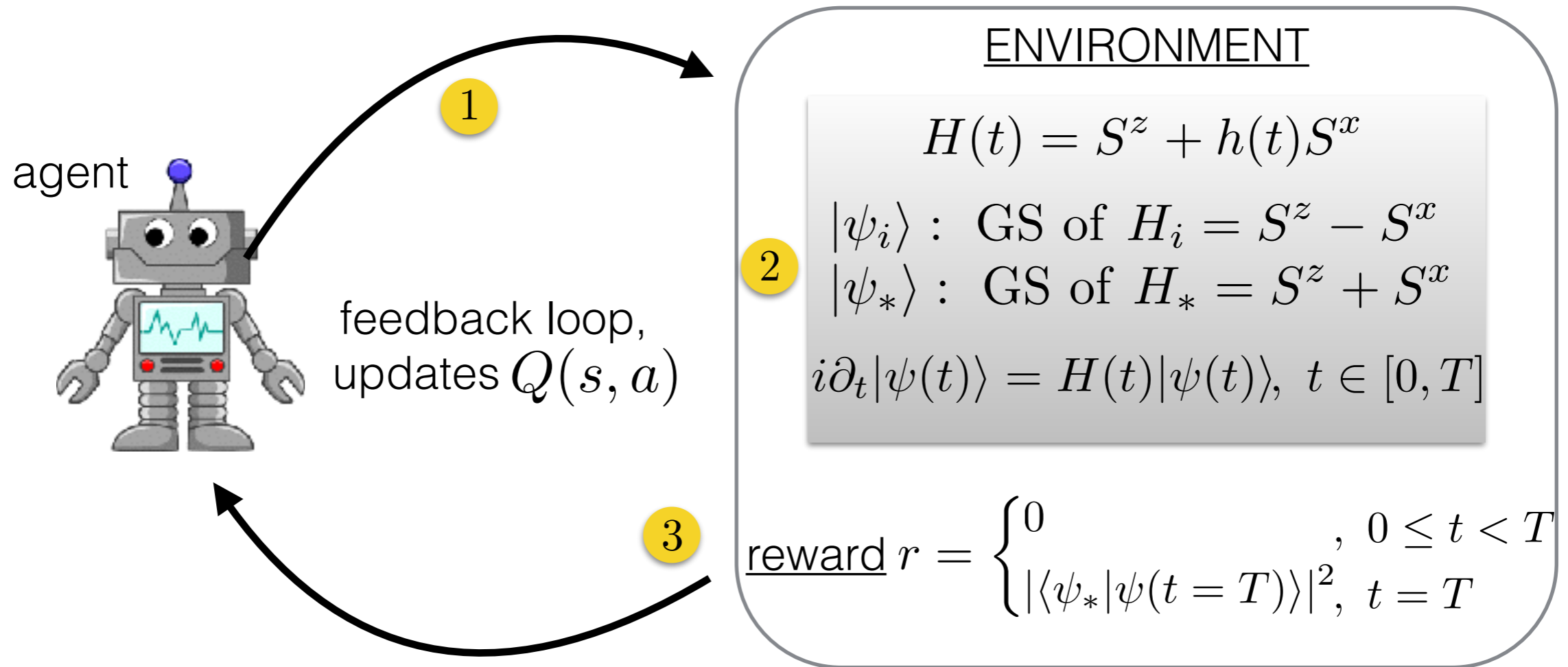
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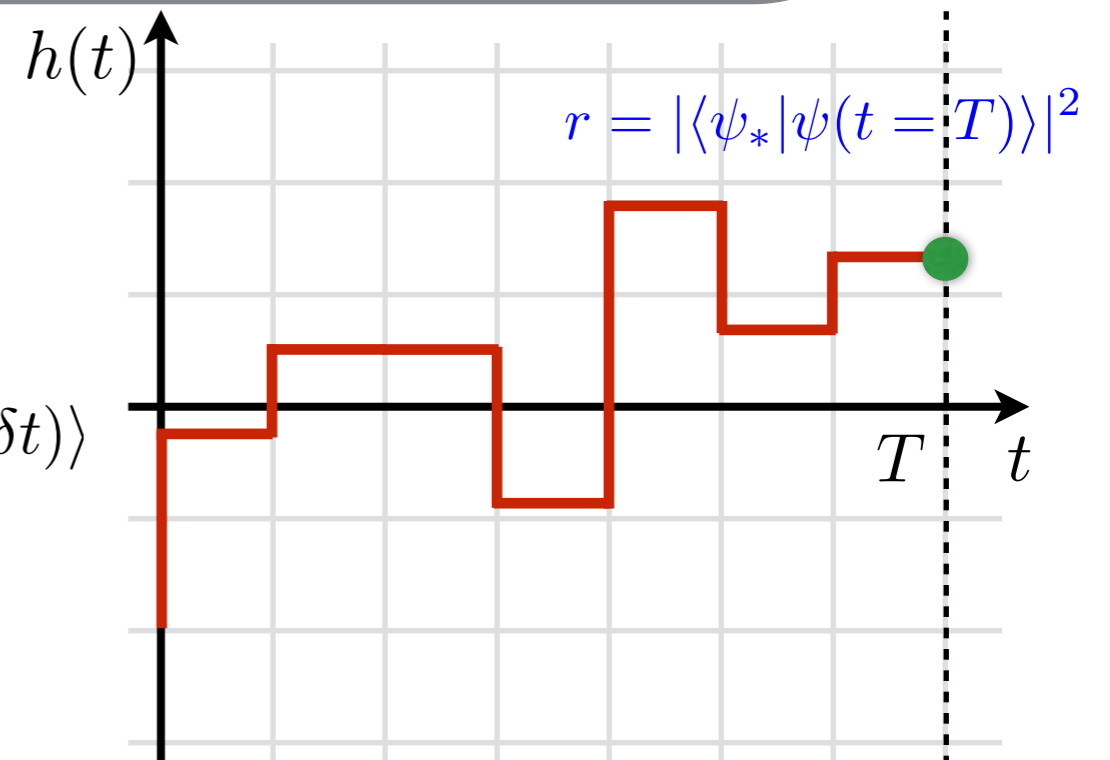
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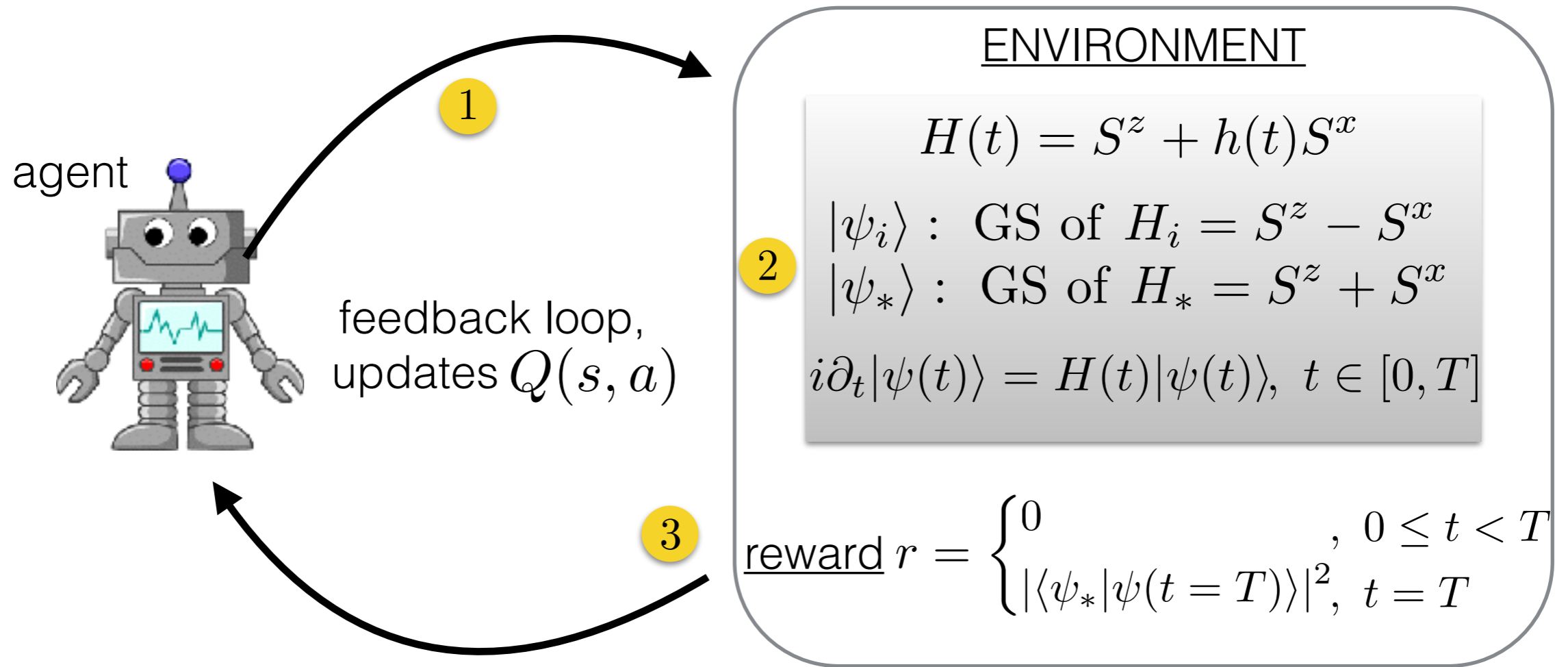
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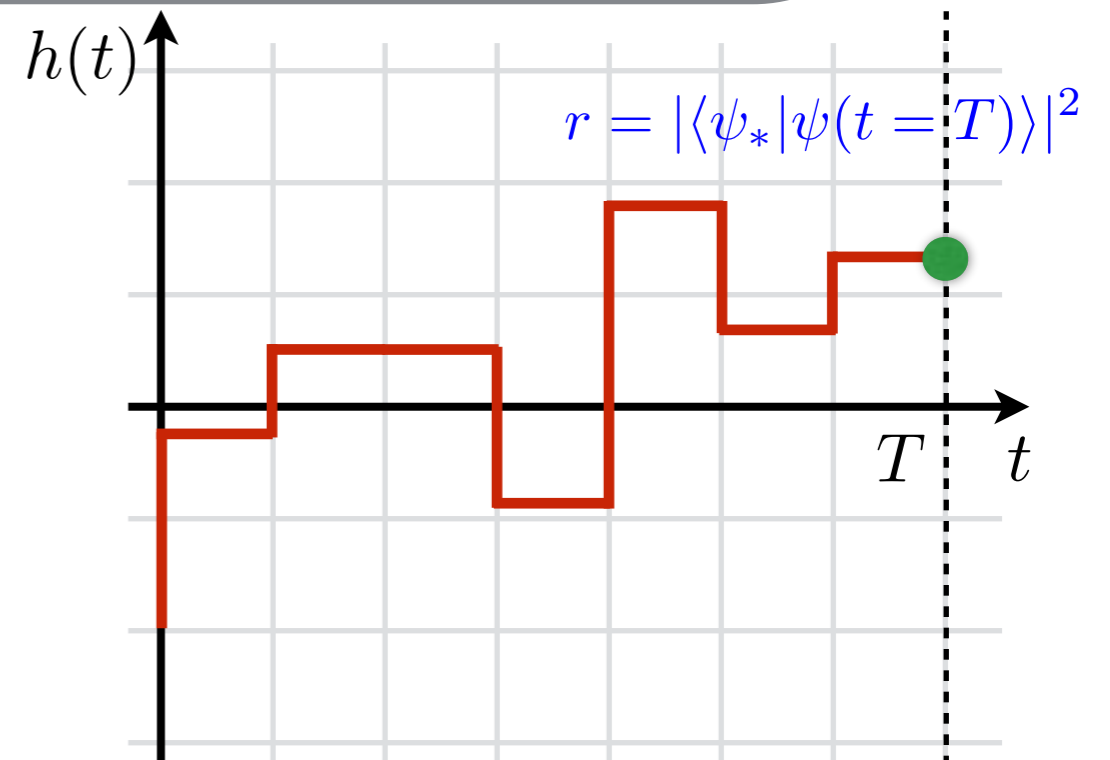
episode completed

RL Applied to Quantum State Preparation

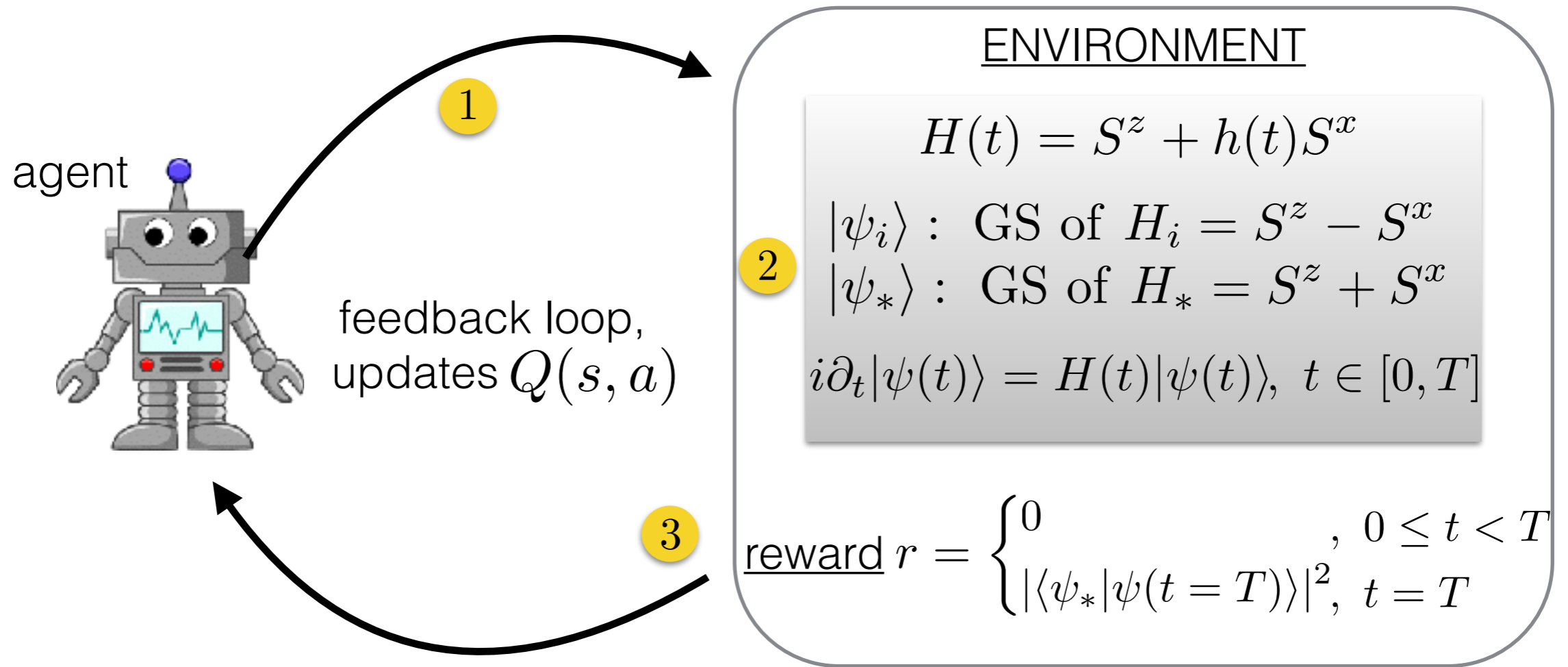


problems:

- state space *exponentially* big
- how do we choose actions?

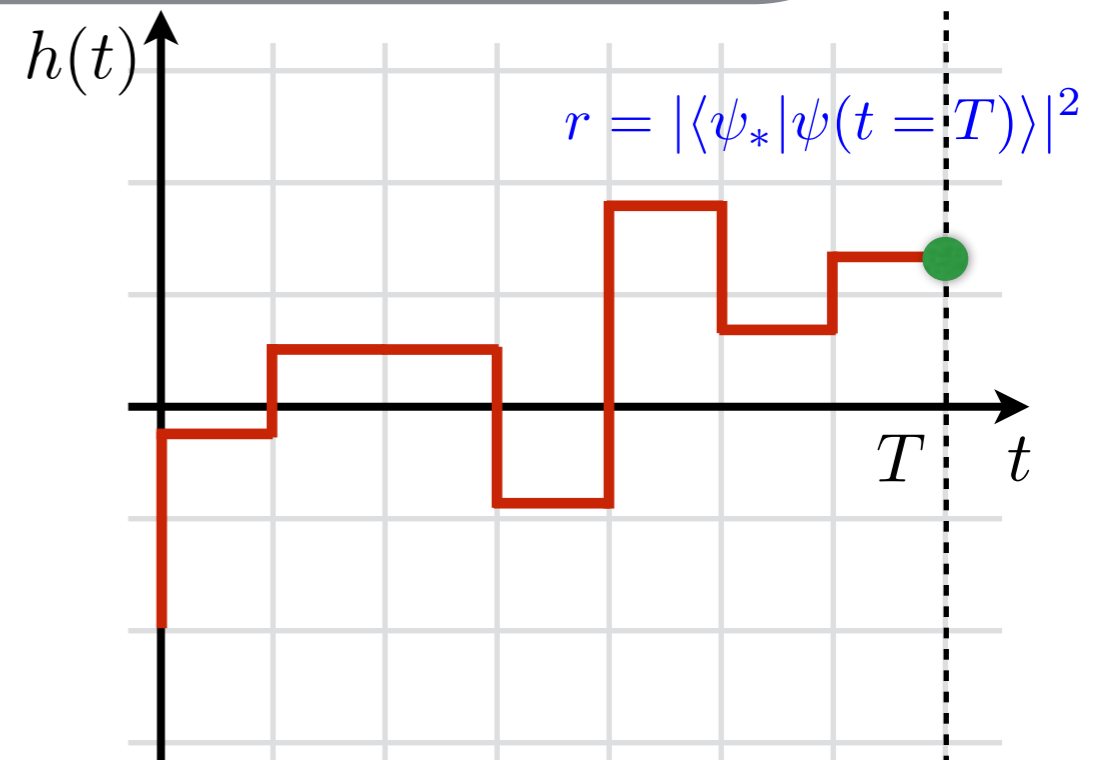


RL Applied to Quantum State Preparation

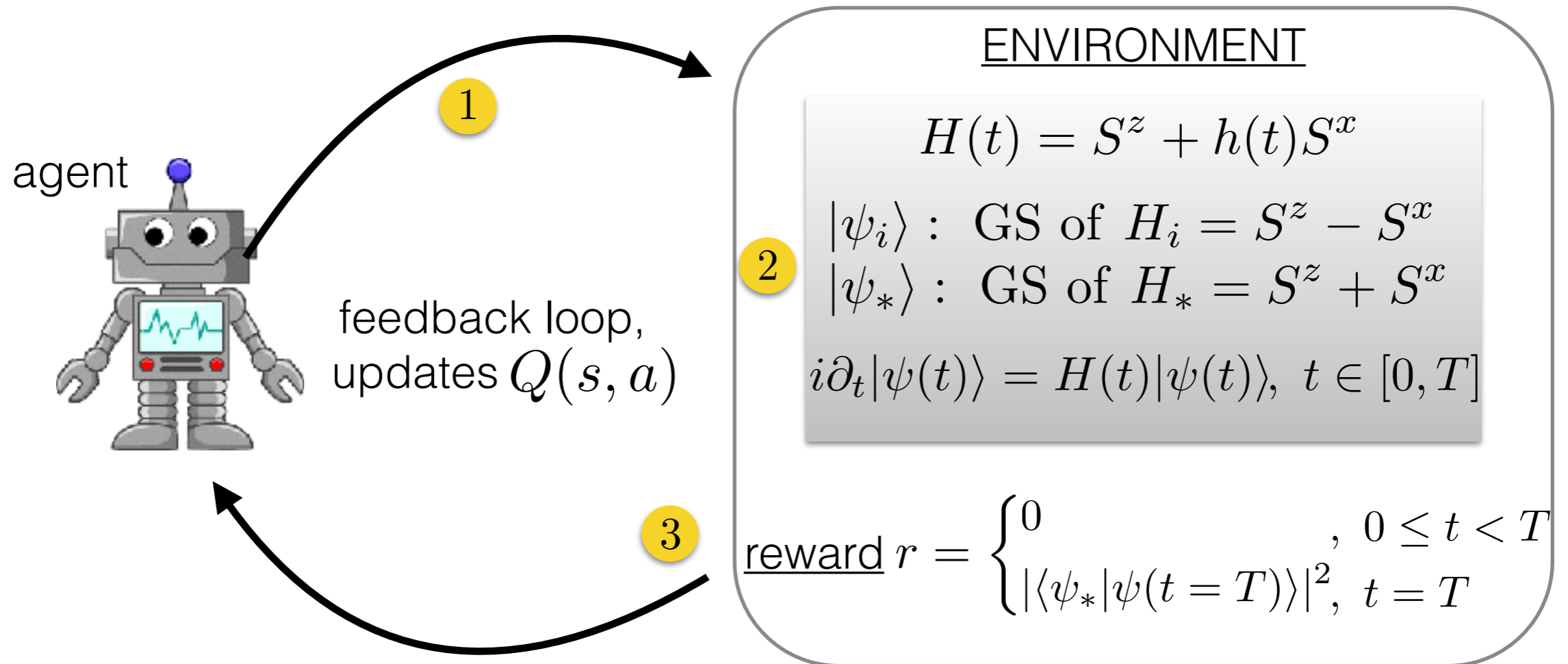


problems:

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RL Applied to Quantum State Preparation



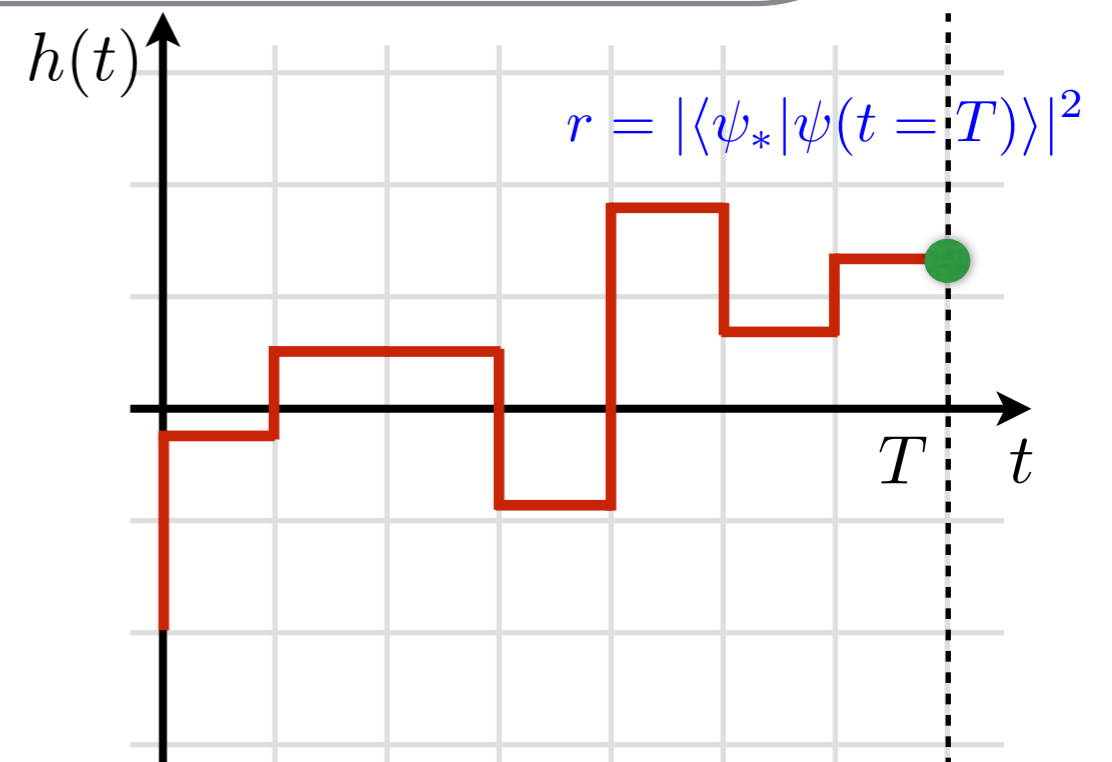
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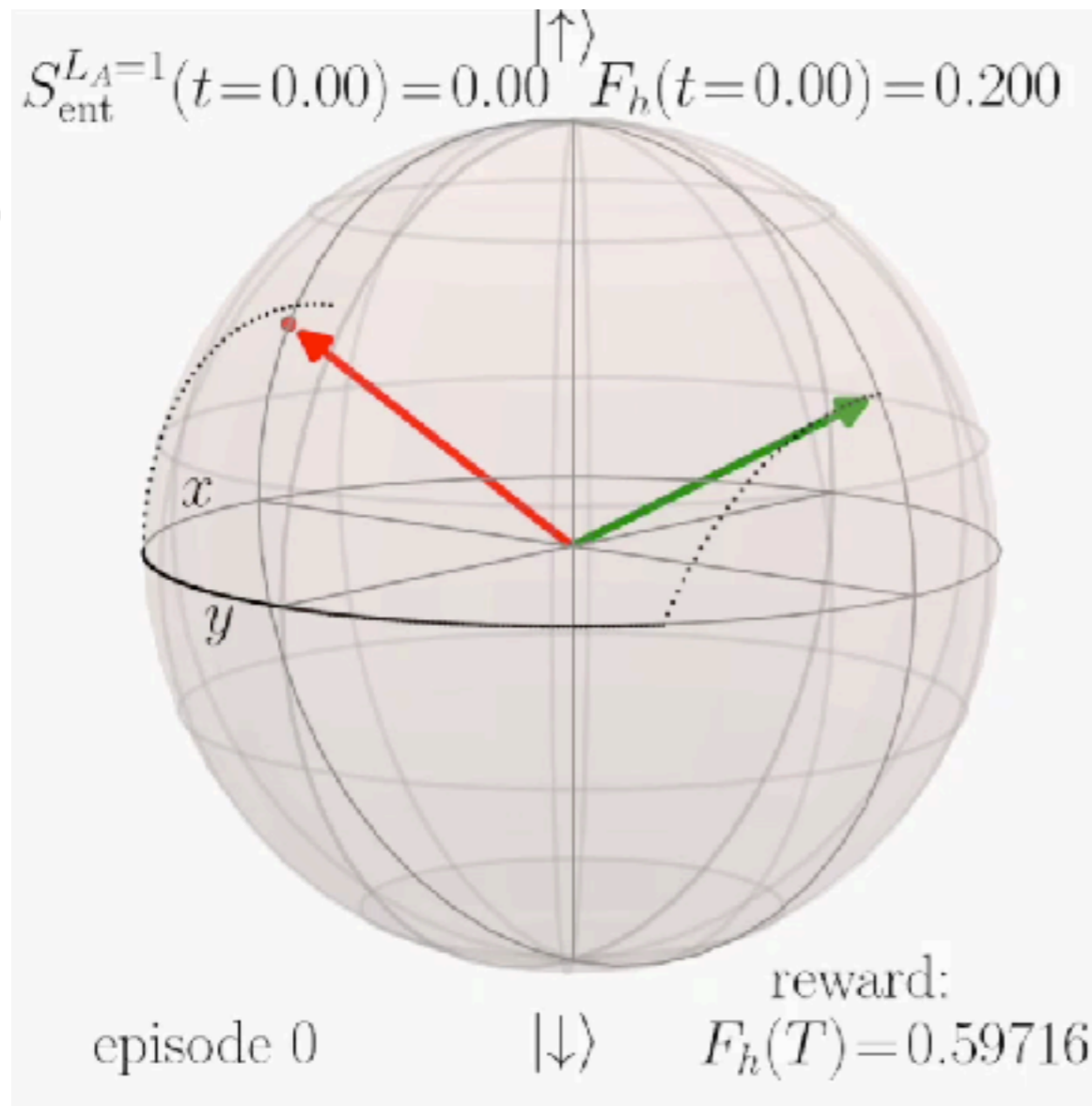
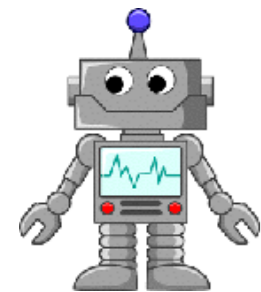
exploration exploitation dilemma



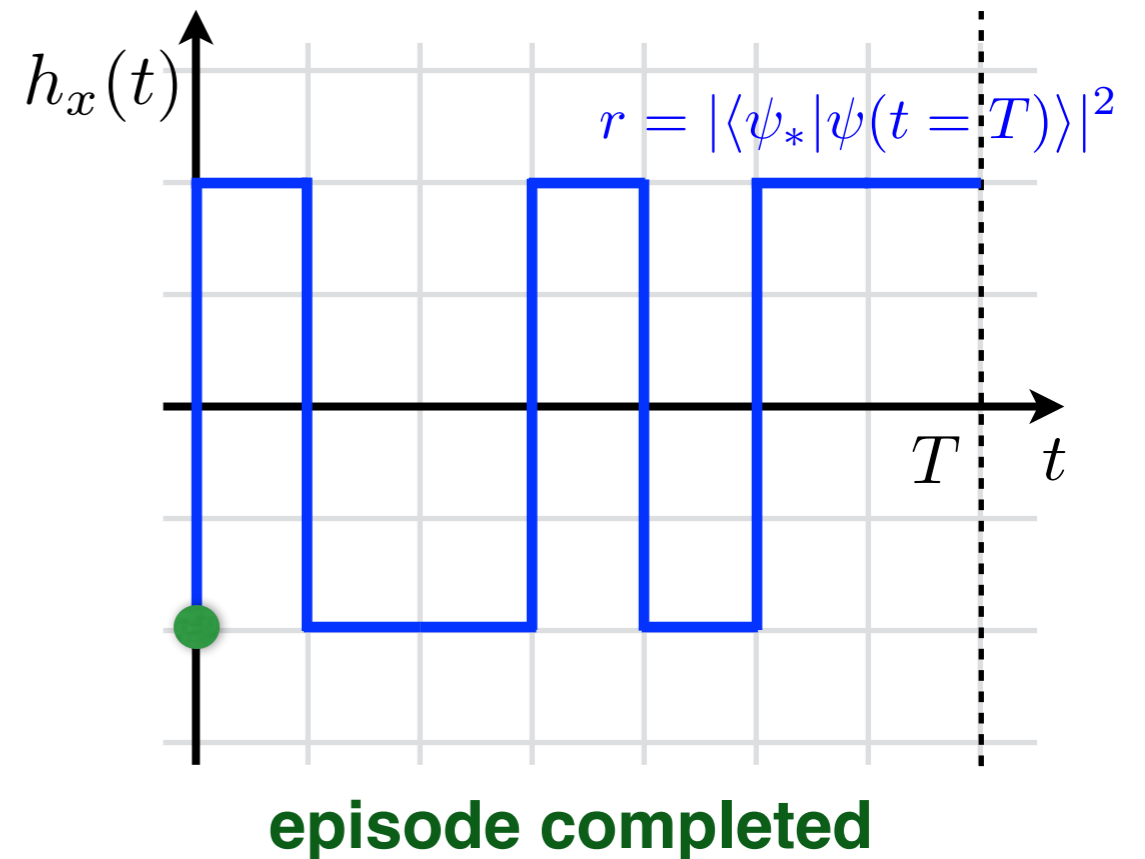
Reinforcement Learning Quantum State Preparation

$$H(t) = -S^z - h_x(t)S^x$$

the learning process



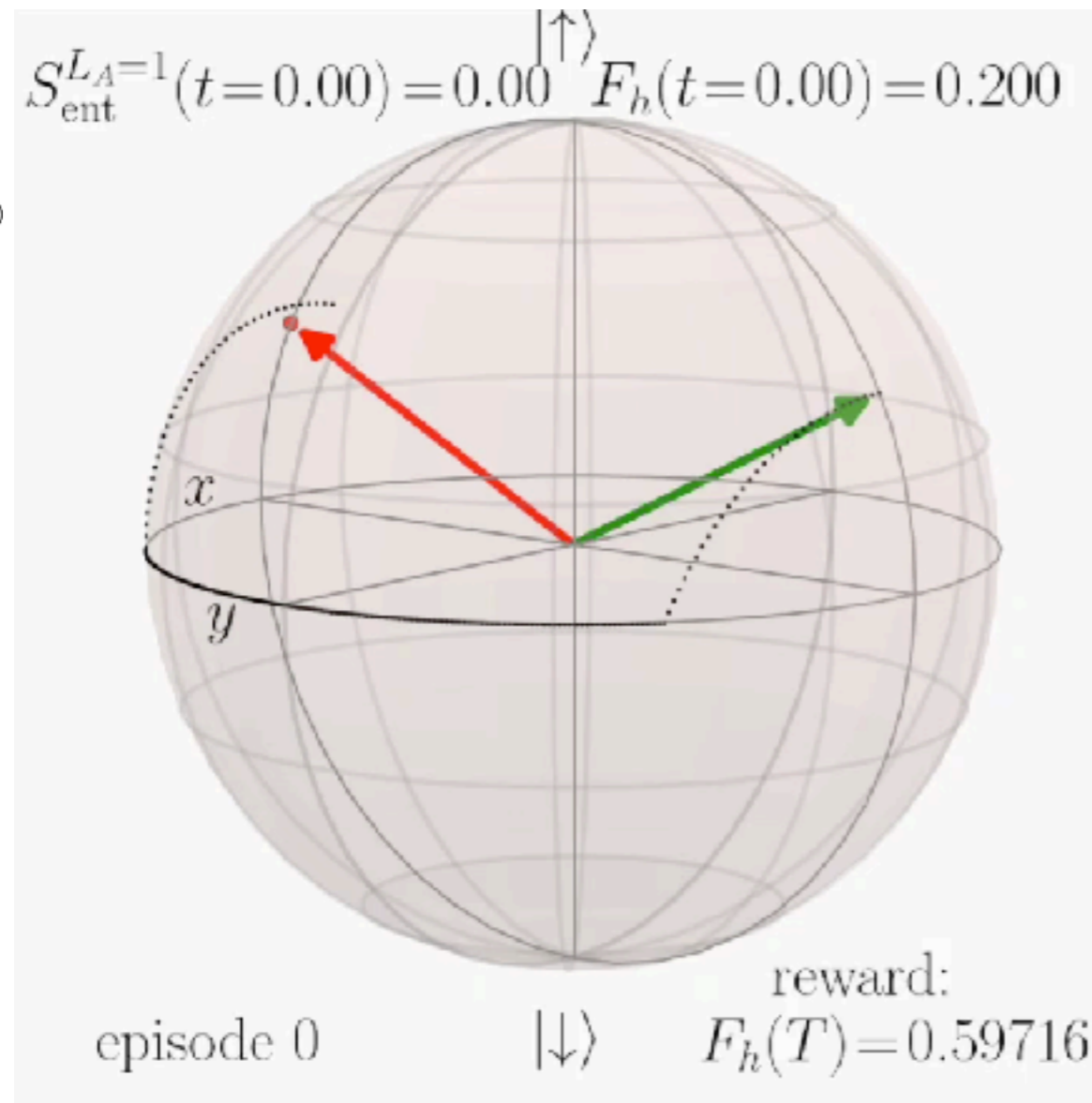
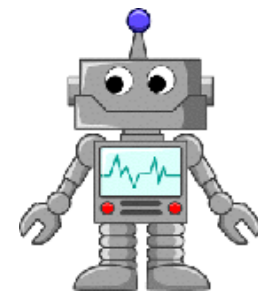
$h \in \{\pm 4\}$ bang-bang protocols



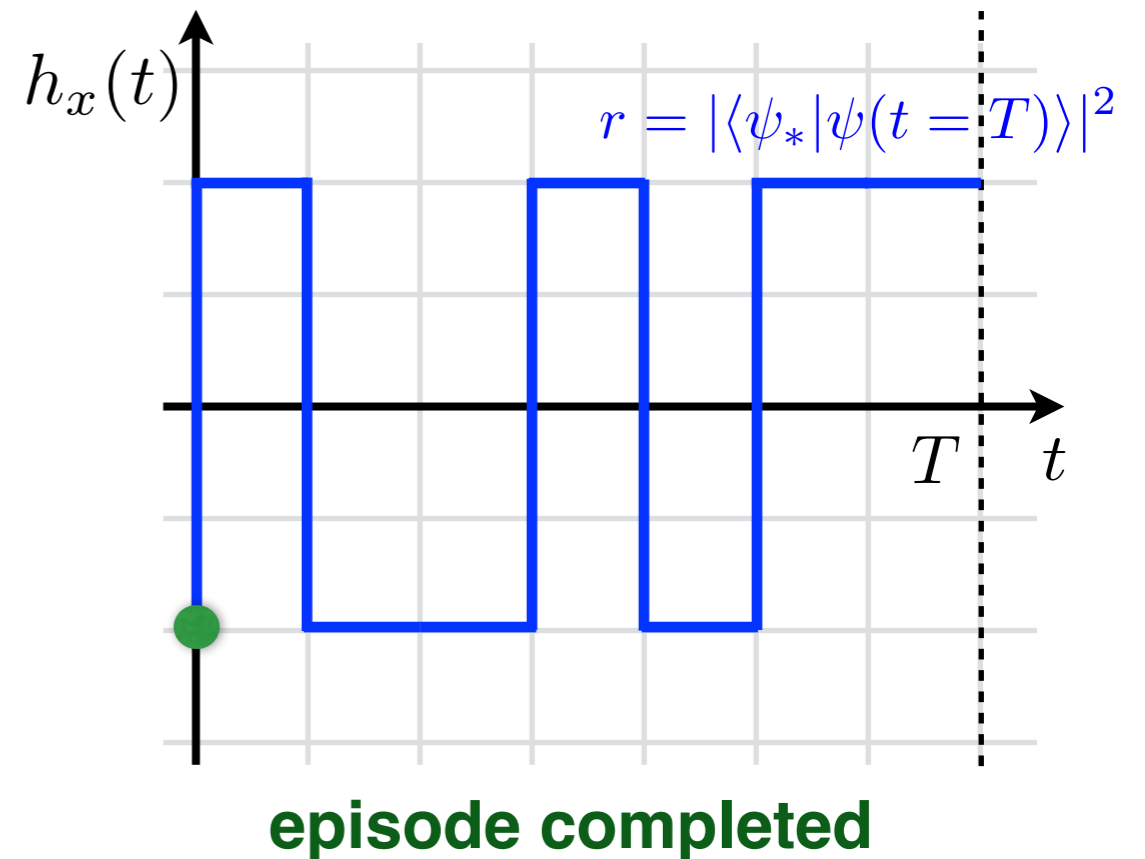
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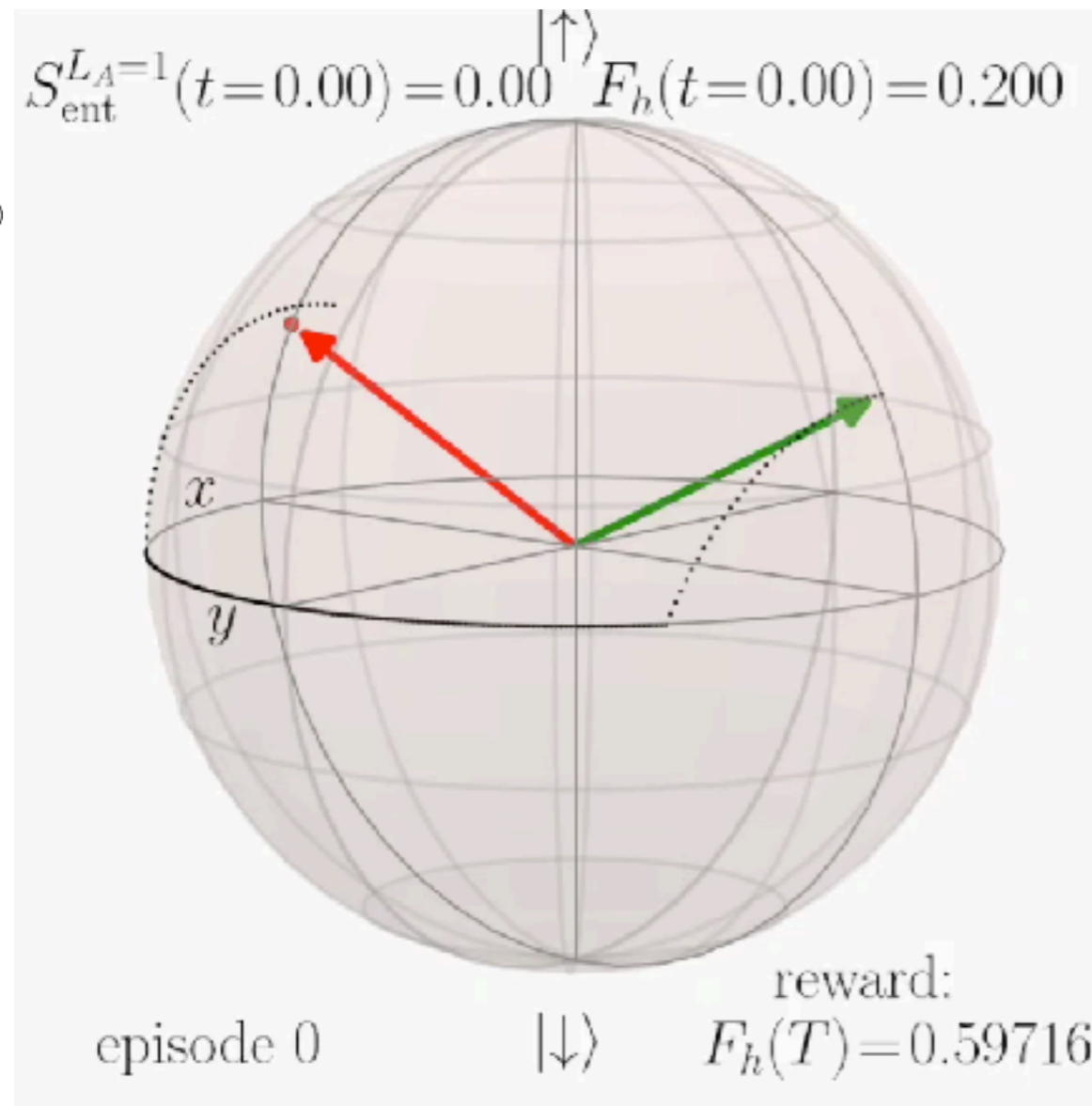
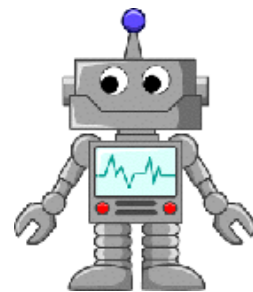
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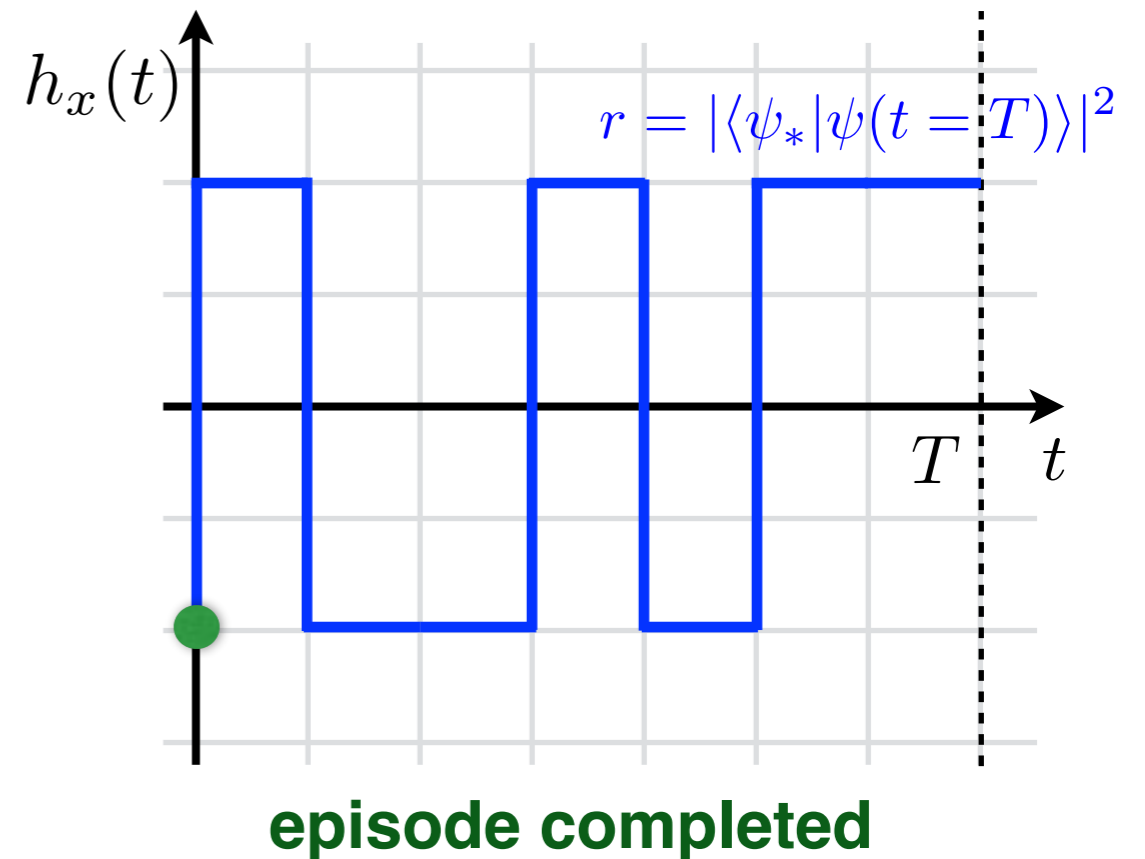
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RL-inspired Discovery: Phase Diagram of Quantum Control

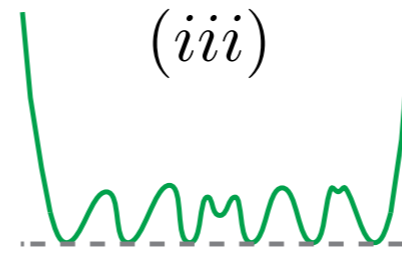
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bang-bang protocols

$$h \mapsto 1 - F_h(T)$$

RL-inspired Discovery: Phase Diagram of Quantum Control

infidelity landscape (schematic)



$$H(t) = -S^z - h_x(t)S^x$$

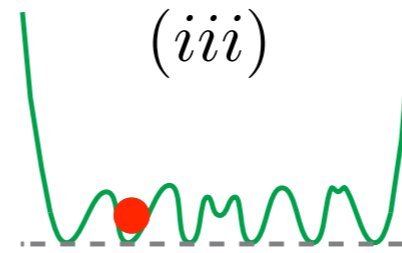
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$$h \mapsto 1 - F_h(T)$$

infidelity landscape
minima: $\{h^\alpha\}$

RL-inspired Discovery: Phase Diagram of Quantum Control

infidelity landscape (schematic)



$$H(t) = -S^z - h_x(t)S^x$$

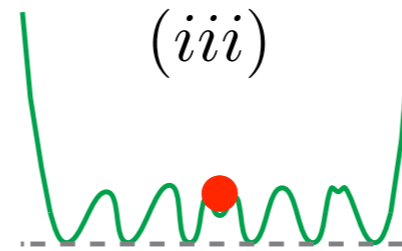
bang-bang protocols

$$h \mapsto 1 - F_h(T)$$

*infidelity landscape
minima: $\{h^\alpha\}$*

RL-inspired Discovery: Phase Diagram of Quantum Control

infidelity landscape (schematic)



$$H(t) = -S^z - h_x(t)S^x$$

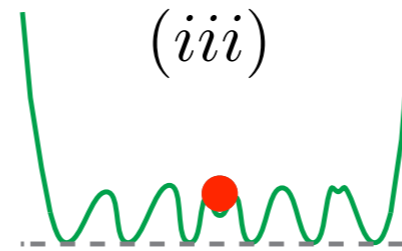
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$$\bar{h}(t) = \frac{1}{\#\text{real}} \sum_{\alpha} h^{\alpha}(t)$$

*Edwards-Anderson-like
order parameter:*

$$q(T) \sim \overline{\sum_t \{h(t) - \bar{h}(t)\}^2}$$

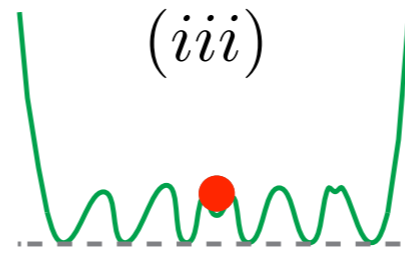
RL-inspired Discovery: Phase Diagram of Quantum Control

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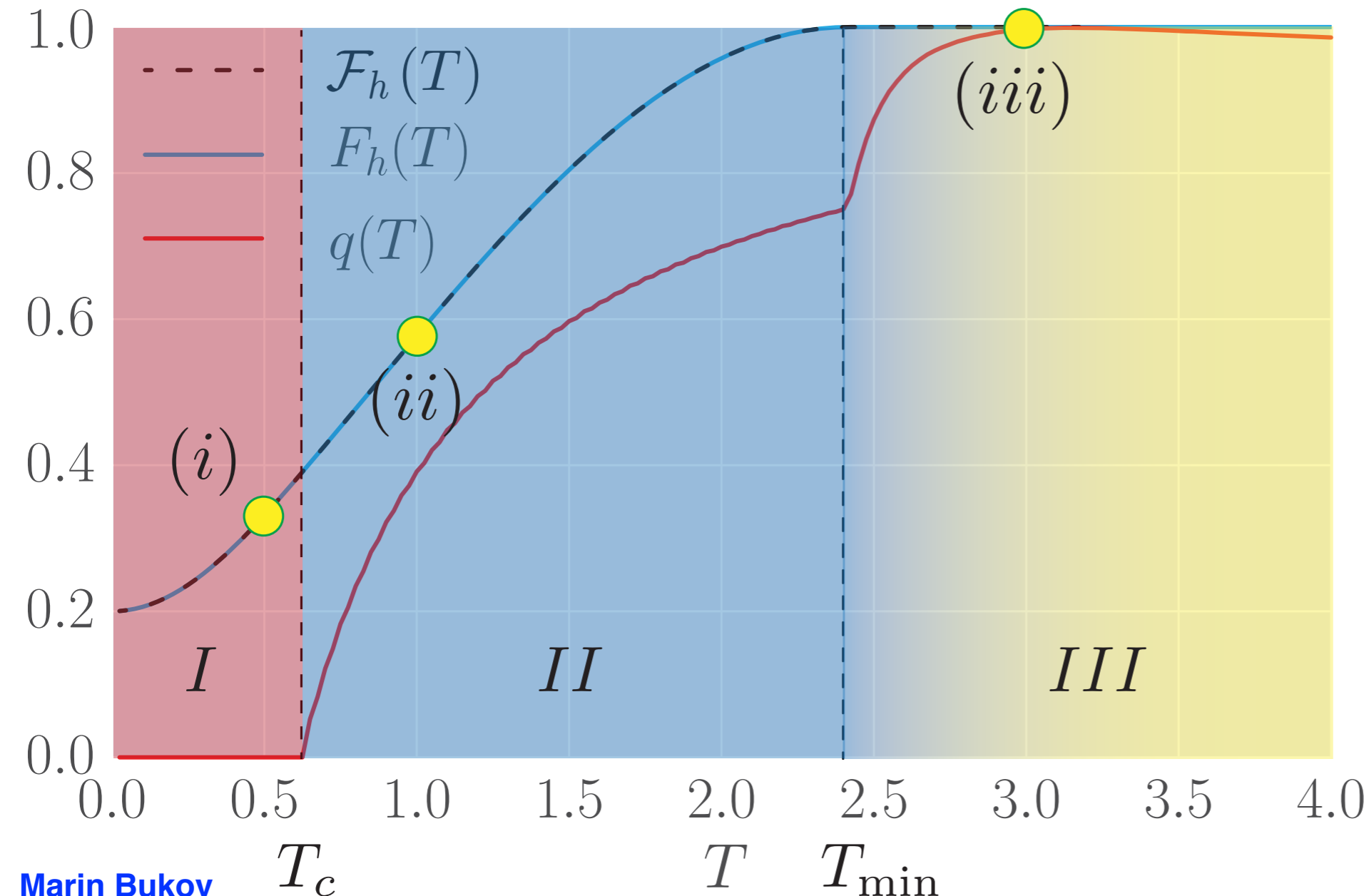


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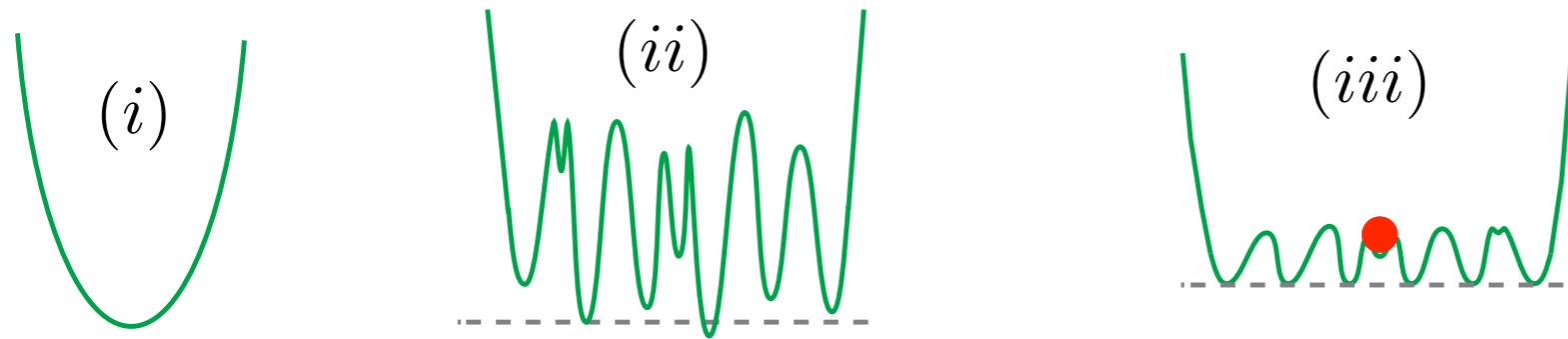


RL-inspired Discovery: Phase Diagram of Quantum Control

infidelity landscape (schematic)

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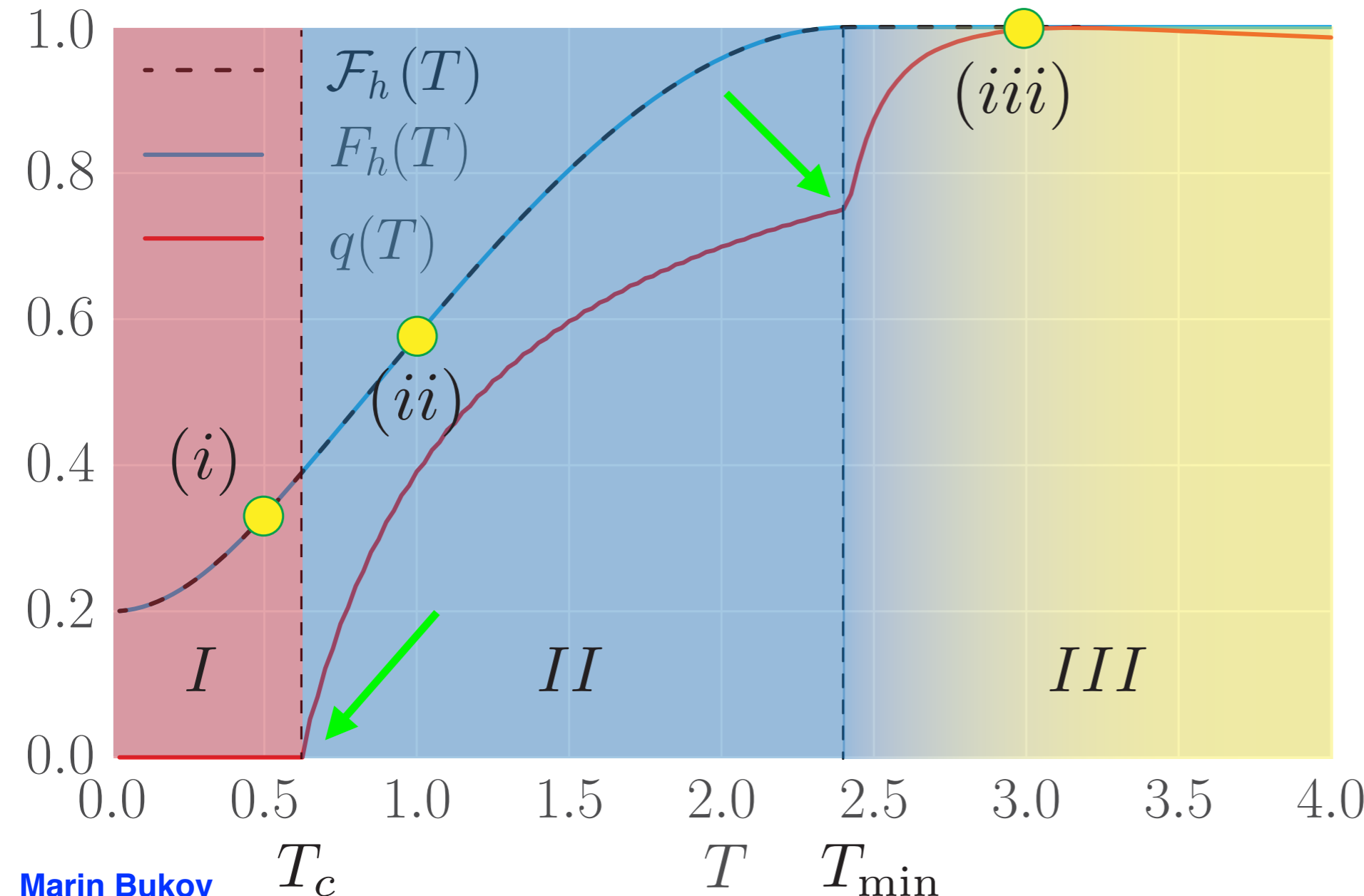
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infidelity landscape minima: $\{h^\alpha\}$

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Edwards-Anderson-like order parameter:

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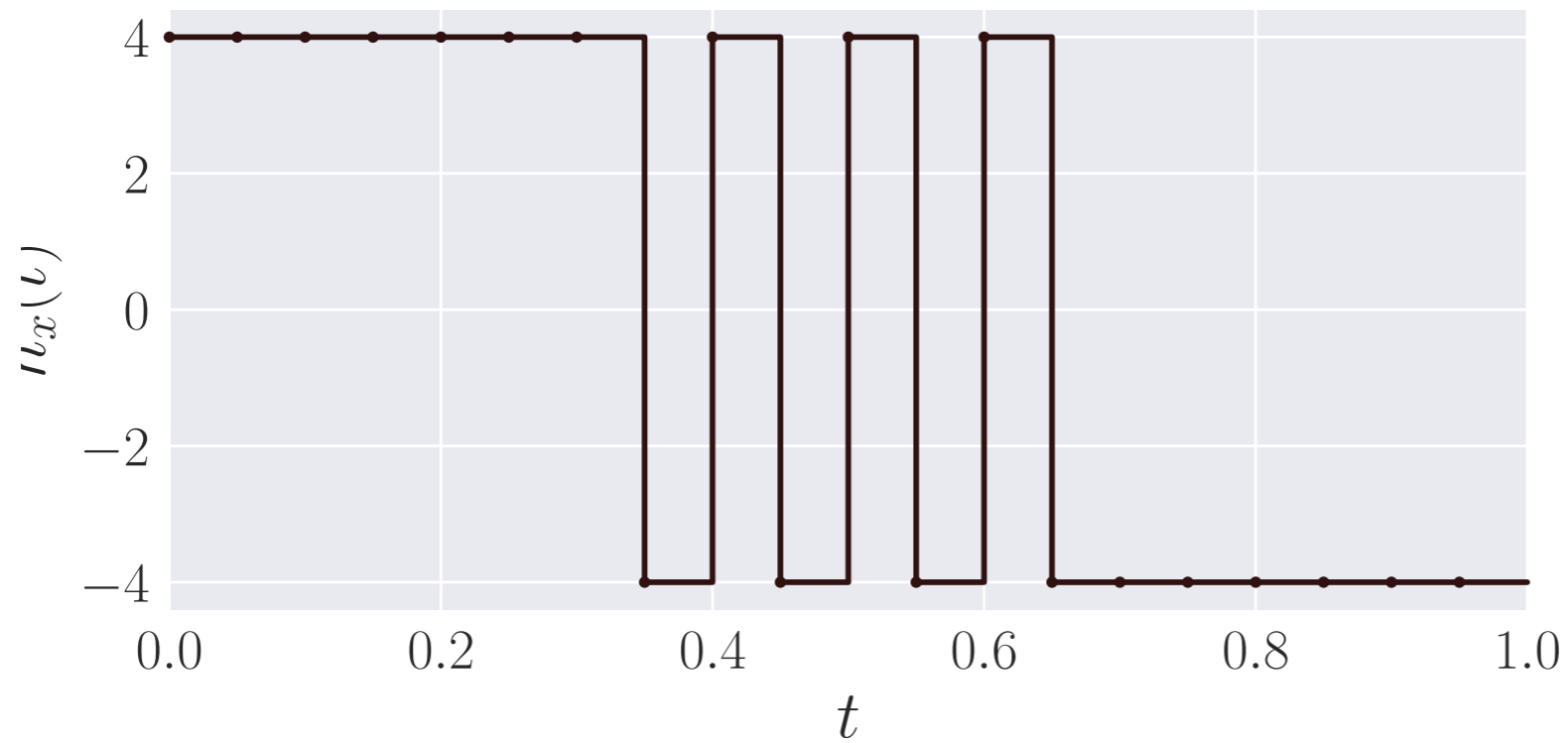


Nature of Control Phase Transitions

$$H(t) = -S^z - h_x(t)S^x$$

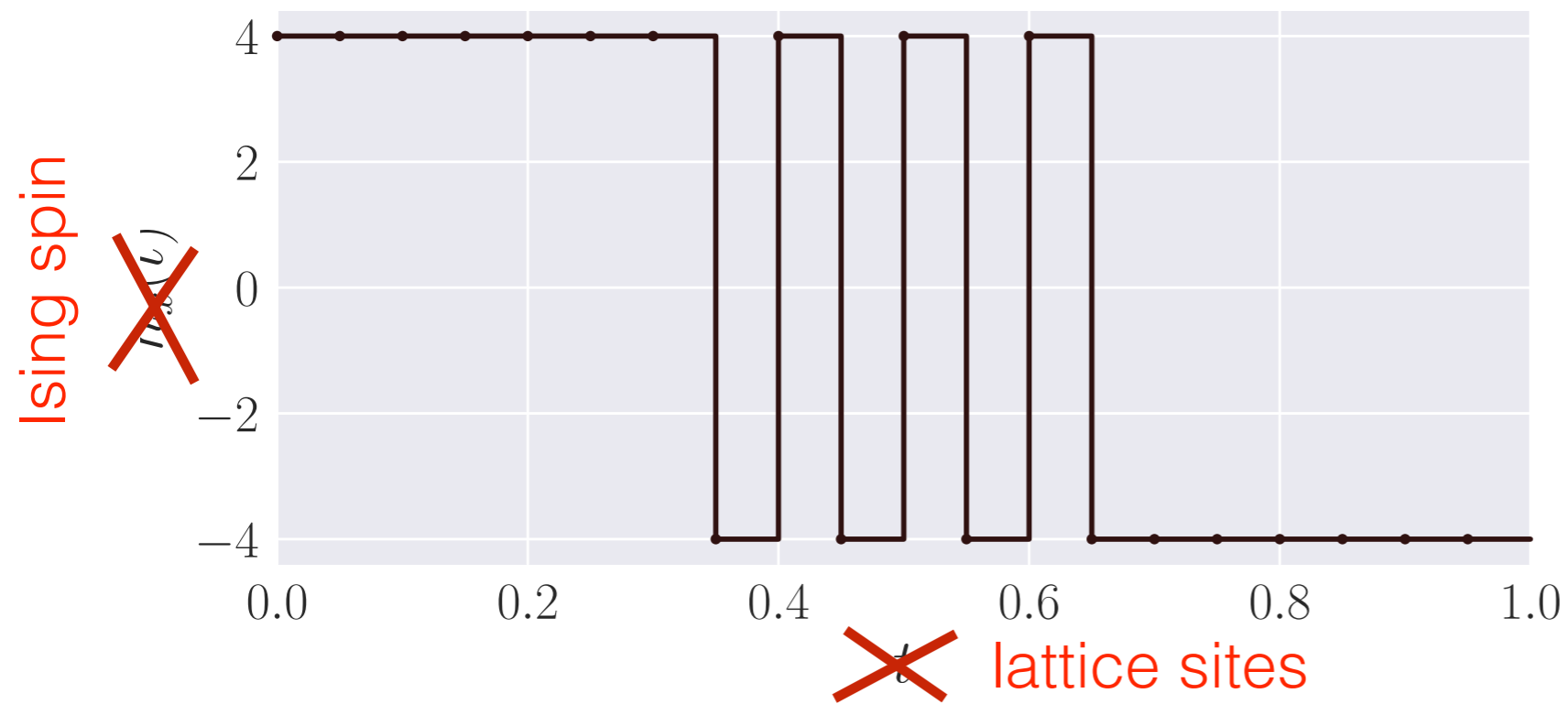
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Nature of Control Phase Transitions

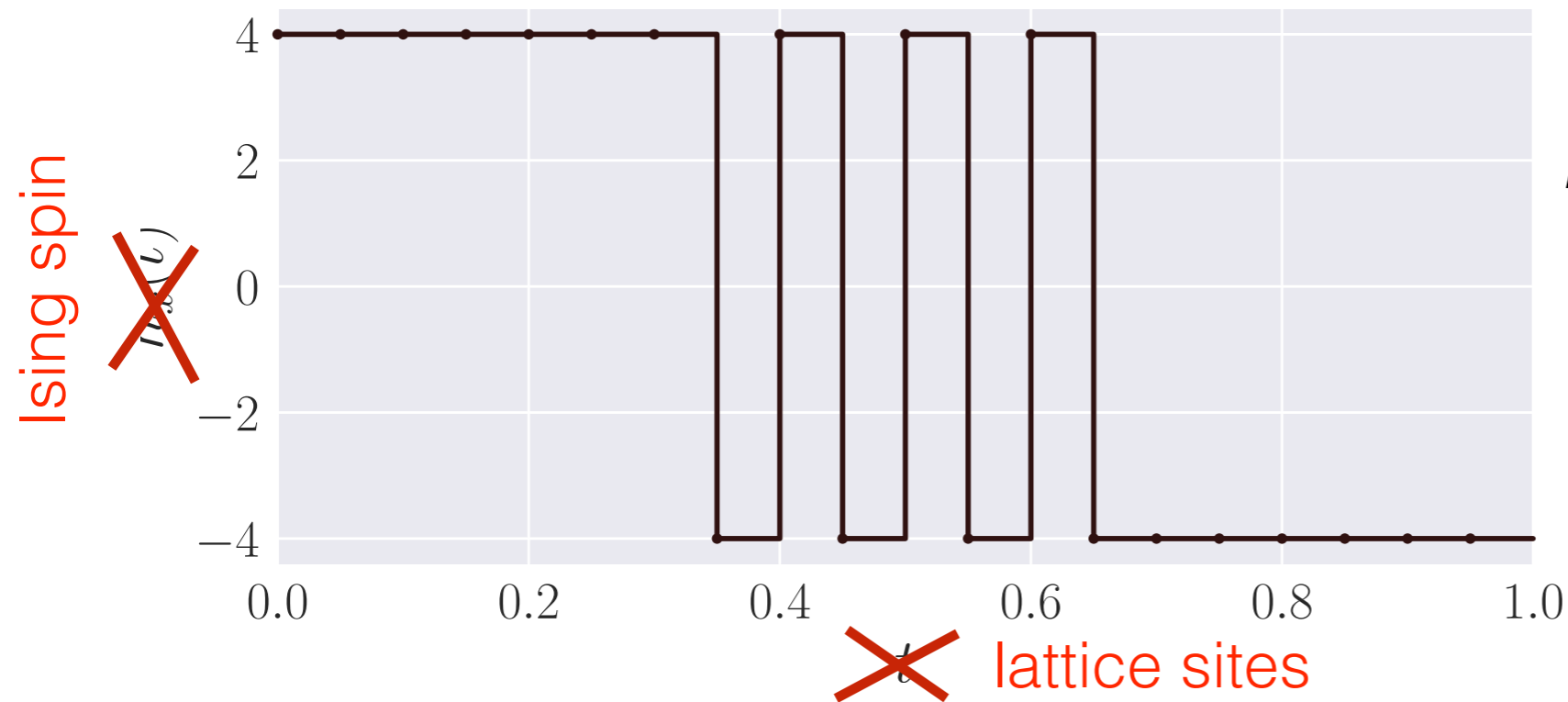
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Nature of Control Phase Transitions

$$H(t) = -S^z - h_x(t)S^x$$

→ one-to-one correspondence:



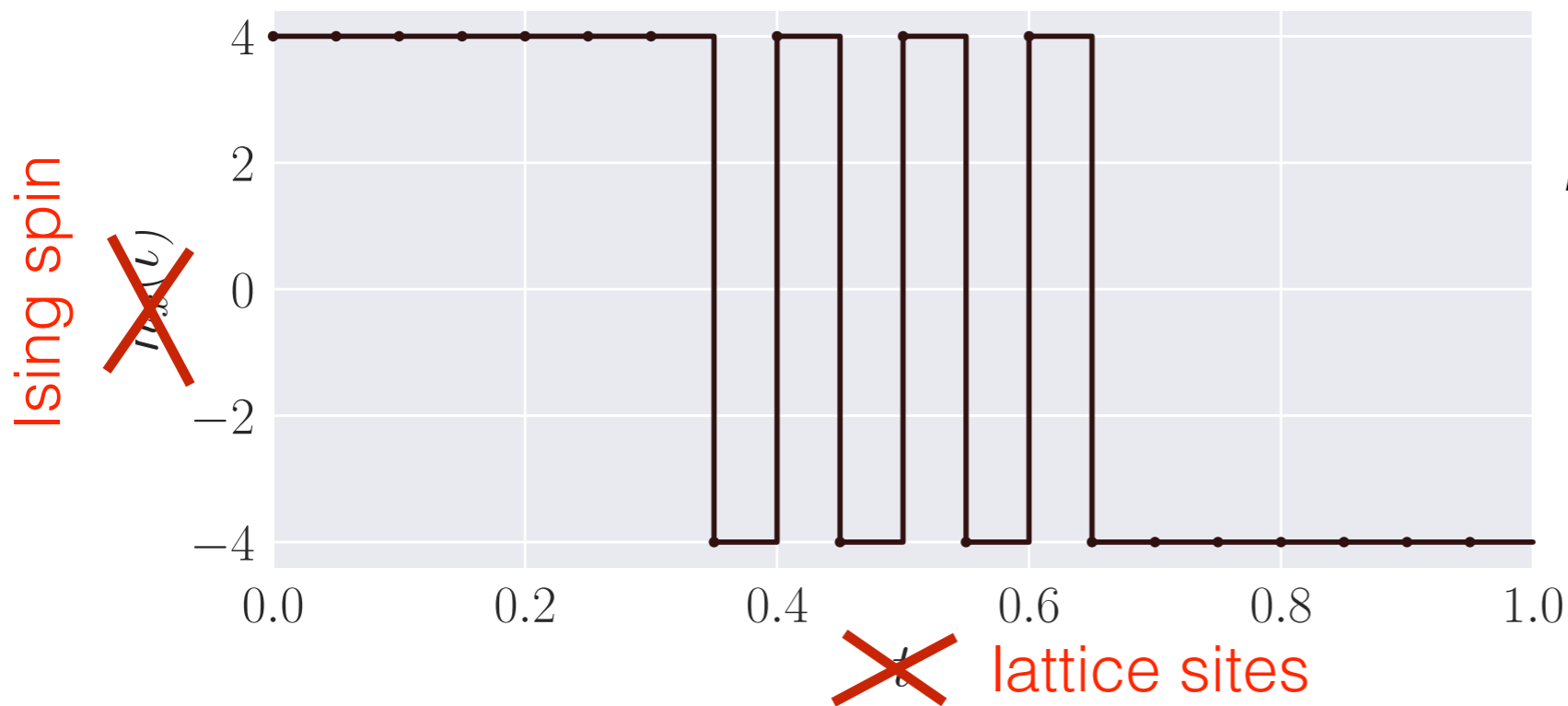
bang-bang protocol ↔ **classical spin state**

infidelity ↔ *energy*

Nature of Control Phase Transitions

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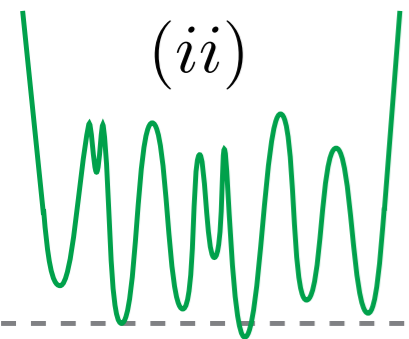
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bang-bang protocol ↔ **classical spin state**

infidelity ↔ *energy*

→ effective *classical* energy function governs control phase transitions



(ii)

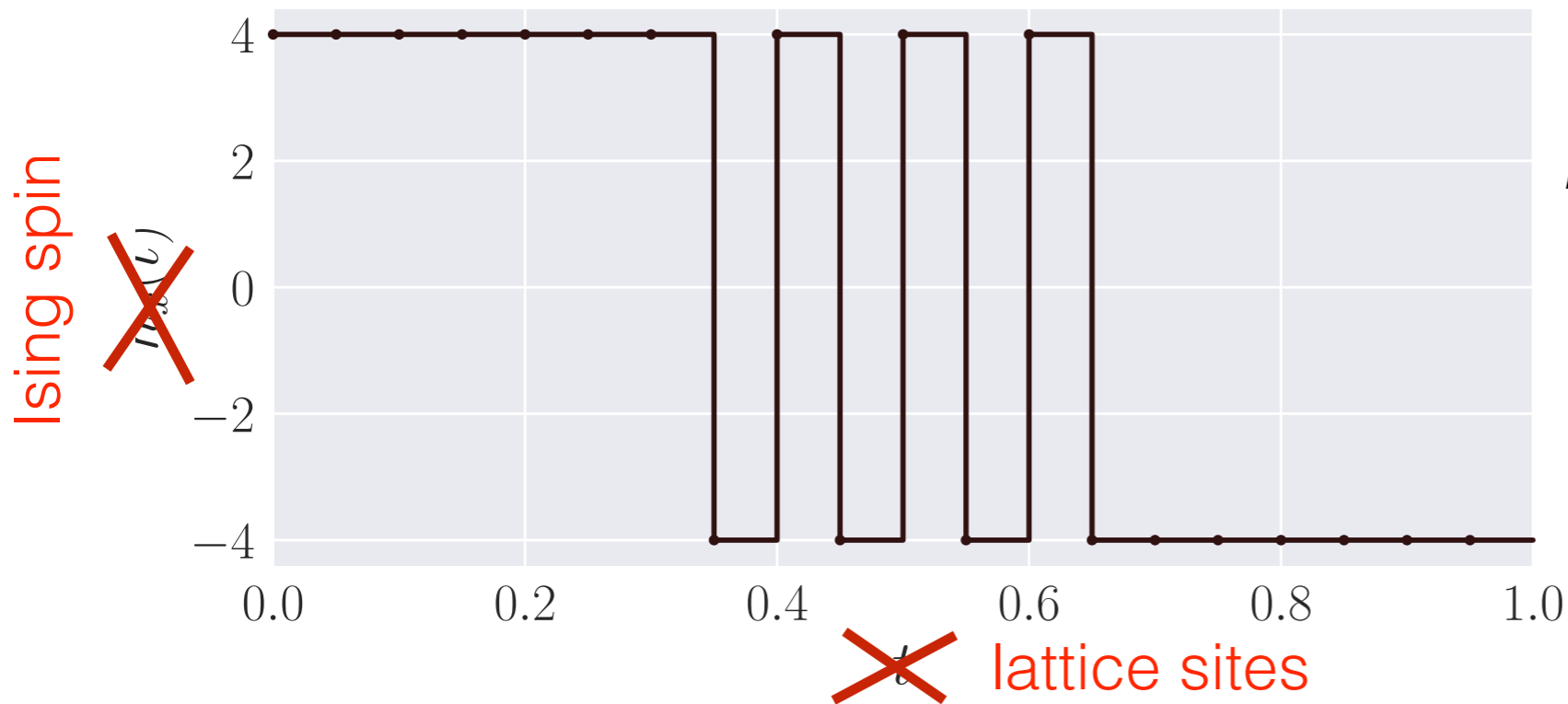
$$\mathcal{H}_{\text{eff}}(T) = I(T) + \sum_j G_j(T)h_j + \sum_{ij} J_{ij}(T)h_ih_j + \sum_{ijk} K_{ijk}(T)h_ih_jh_k + \dots$$

j : sites on time lattice

Nature of Control Phase Transitions

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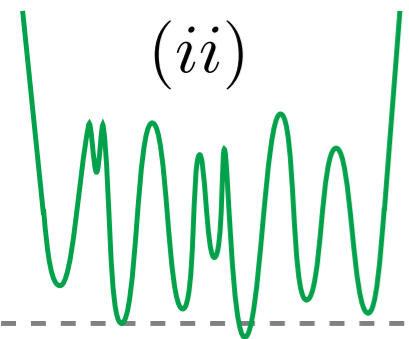
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→ control phase transitions: *classical (?)*, **non-equilibrium**

Outlook

- one can teach a reinforcement learning agent to prepare quantum states at short times with high fidelity
- finding optimal driving protocol as hard as searching for absolute GS of a spin glass (even if system is disorder-free)
- quantum control problems have extremely rich phase diagrams with overconstrained, controllable, correlated and glassy phases → POSTER (Alex Day)
exhibit symmetry breaking → POSTER (MB)
- control phase transitions: classical & nonequilibrium, generic?



Outlook



QuSpin: weinbe58.github.io/QuSpin/

open-source Python package for ED and quantum dynamics of arbitrary boson, fermion and spin many-body systems, supporting various (user-defined) symmetries and time evolution.

SciPost Phys. 2, 003 (2017)

arXiv: 1705.00565 (2017)

arXiv: 1711.09109 (2017)

web: mgbukov.github.io