Beyond the proximity force approximation

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J. Chevrier (I. Néel), C. Genet (ISIS),
V. Klimov (Moscow), G. Ingold (Augsburg) …

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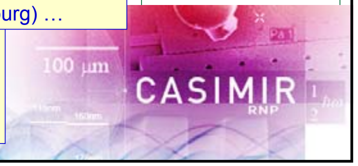
Discussions with a number of other people, in particular within the ESF network "CASIMIR"
http://www.casimir-network.com



The many facets of the Casimir effect

- Casimir effect
  - an observable "mesoscopic" effect of quantum fluctuations
- A fascinating interface with other problems in fundamental physics
  - gravity : "vacuum energy" problem
  - non trivial effects of geometry :
    beyond the "Proximity Force Approximation"
  - "principle of relativity of motion" : dynamical Casimir-like effects
  - "new physics" : search for hypothetical new short-range forces expected to lie "beyond the standard model"
- A dominant force in the mesoscopic world : strong connections with
  - atomic and molecular physics, quantum optics
  - condensed matter physics, surface physics
  - chemical physics and biological physics
  - micro- and nano-technology : potential applications in new solutions for actuating or controlling micro/nanosystems …

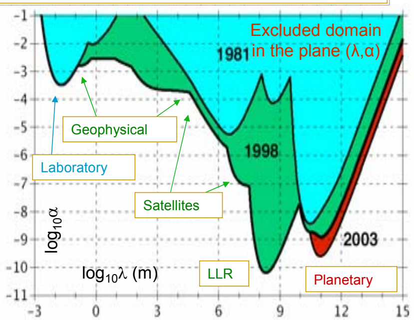
Search for scale dependent modifications of the gravity force law ("fifth-force experiments")

The exclusion plot for deviations with a Yukawa form

$$V(r) = -\frac{GMm}{r}\left(1+\alpha e^{-\frac{r}{\lambda}}\right)$$

Windows remain open for deviations at short ranges $\lambda < 1\,\mathrm{mm}$

or long ranges $\lambda > 10^{16}\,\mathrm{m}$



Courtesy : J. Coy, E. Fischbach, R. Hellings, C. Talmadge & E. M. Standish (2003) ; see M.T. Jaekel & S. Reynaud IJMP **A20** (2005)

The Search for Non-Newtonian Gravity, E. Fischbach & C. Talmadge (1998)

Constraints at sub-mm scales

- Best result to date :
  Eöt-wash group
  (U. Washington, Seattle)
- Cavendish-type experiments with torsion pendulum
- At 95% confidence, a Yukawa interaction with gravitational strength $\alpha > 1$ must have a range $\lambda < 56\mu m$

* D.J. Kapner *et al*
  PRL **98** (2007) 021101
* E.G. Adelberger et al
  PRL **98** (2007) 131104

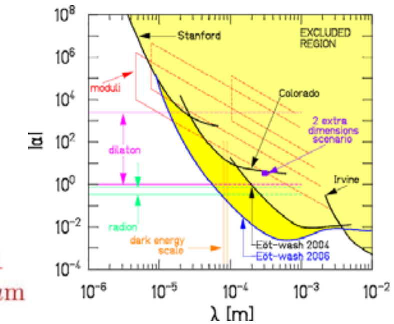
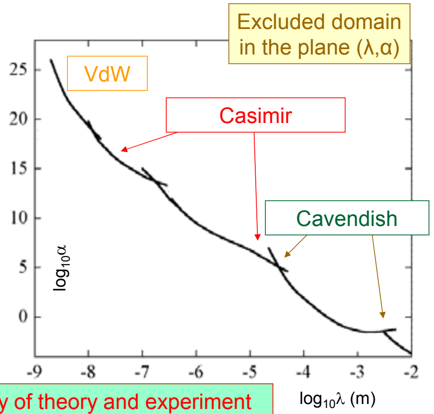


FIG. 6: Constraints on Yukawa violations of the gravitational 1/r² law. The shaded region is excluded at the 95% confidence level. Heavy lines labeled Eöt-Wash 2006, Eöt-Wash 2004, Irvine, Colorado and Stanford show experimental constraints from this work, Refs. [11], [14], [15] and [16, 17], respectively. Lighter lines show various theoretical expectations summarized in Ref. [9].

Constraints at shorter scales

- At (~) µm scales, comparison with theory of Casimir measurements
- Also interesting with Van der Waals and nuclear* forces at shorter scales



- The hypothetical new force would be seen as a difference between experiment and theory

$$F_{\text{new}} \equiv F_{\text{exp}} - F_{\text{th}}$$

- The accuracy and reliability of theory and experiment have to be assessed <u>independently</u>

(*) V. Nesvizhevsky *et al*, Phys. Rev. **D77** (2008) 034020

Casimir expt/theory comparison ..

Precise experimental results deviate from theoretical expectations

BRIEF REPORTS PHYSICAL REVIEW D 75, 077101 (2007)

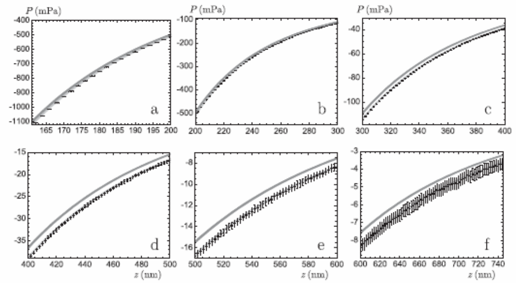


FIG. 1. Experimental data for the Casimir pressure as a function of separation z. Absolute errors are shown by black crosses in different separation regions (a–f). The light- and dark-gray bands represent the theoretical predictions of the impedance and Drude model approaches, respectively. The vertical width of the bands is equal to the theoretical error, and all crosses are shown in true scale.

Talk R. Decca

The difference does not have a Yukawa form !

R.S. Decca, D. Lopez, E. Fischbach *et al*, Phys. Rev. **D75** (2007) 077101

.. Casimir expt/theory comparison

- A problem with the description of metallic mirrors ?
  - Lifshitz formulation of the Casimir force
    - interaction between two plane mirrors
  - Plasma models for the dielectric functions
    - conduction electrons in the metals with relaxation accounted for (Drude model)
  - Gold has a finite conductivity
    - γ≠0 is certainly a better description than γ=0

$$\varepsilon(i\xi) = \bar{\varepsilon}(i\xi) + \frac{\sigma(i\xi)}{\xi}$$

$$\sigma(i\xi) = \frac{\omega_P^2}{\xi + \gamma}$$

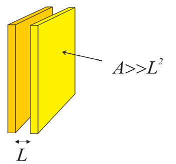
$$\sigma(0) = \frac{\omega_P^2}{\gamma}$$

- <u>BUT</u> experiments agree better with γ =0 than with the expected γ≠0
  - Artifact in the experiments ?
  - Inaccuracy in the theoretical evaluations ?
  - Difference between the situation studied in theory and the experimental realization ?

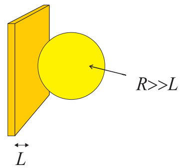
Many talks at QFEXT'09

The role of geometry

- The geometry of the precise experiments is not the geometry of the precise calculations !



- Calculations in the plane-sphere geometry usually performed by using the proximity force approximation (PFA)
  - This amounts to a mere averaging of the plane-plane result over the distances !

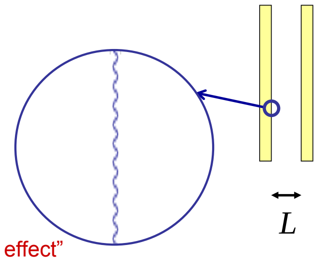


- ✓ The PFA is expected to be accurate at the limit of large spheres
- But what is the accuracy for a given value of L/R ?
- Does this accuracy depend on the material of the mirror ?

A. Canaguier-Durand, P.A. Maia Neto, I. Cavero-Pelaez, A. Lambrecht, S. Reynaud PRL **102** (2009) 230404

Roughness and corrugations

- For a static surface, frequency is unchanged $\Delta\omega = 0$
- For a non flat surface, lateral wavevector is changed $\Delta k = \frac{2\pi}{\lambda_C}$



- The PFA disregards this "diffraction effect"
  - it can be valid at the PFA limit $\Delta k \to 0$
  - it cannot remain accurate for large wavevectors
  - effect "beyond PFA" hard to see on the effect of roughness
  - may be visible on the lateral force between corrugated plates

P. Maia Neto, A. Lambrecht, S. Reynaud, EPL **69** (2005); PRA **72** (2005)

R.B. Rodrigues, P.A. Maia Neto, A. Lambrecht, S. Reynaud PRL **96** (2006) 100402

Going beyond the PFA : The non-specular scattering formula

- In an arbitrary geometry with two scatterers in vacuum, the Casimir energy can be deduced from the scattering formula

$$E = \hbar \int_0^\infty \frac{d\xi}{2\pi} \mathrm{Tr} \ln \mathcal{D} \quad , \quad \mathcal{D} \equiv 1 - \mathcal{R}_1 e^{-\mathcal{K}L} \mathcal{R}_2 e^{-\mathcal{K}L}$$

- $\mathcal{R}_1, \mathcal{R}_2$ are reflection matrices (at imaginary frequencies) $\omega \equiv i\xi$
  - They are defined in the absence of the other object (long-range)
  - They describe non-specular couplings between all modes with the same frequency but different wavevectors and polarizations
- $e^{-\mathcal{K}L}$ are free propagators (diagonal in a plane wave basis)

A. Lambrecht, P. Maia Neto, S. Reynaud, New J. Physics **8** (2006) 243

See also Balian & Duplantier - Emig, Kardar, Jaffe, et al - Kenneth, Klich et al, Milton et al - Bordag et al - Wirzba et al - Gies et al – Johnson et al …

The particular case of plane mirrors

- With plane and parallel mirrors, the reflection matrices are also diagonal in the plane wave basis (specular scattering)
- This greatly simplifies the expression of the Casimir energy (here written at null temperature)

$$E = \hbar A \sum_p \int \frac{dk_x dk_y}{4\pi^2} \int \frac{d\xi}{2\pi} \ln d^p(i\xi)$$

$$d^p(i\xi) \equiv 1 - r_1^p r_2^p e^{-2\kappa L}$$

$$\kappa \equiv \sqrt{k_x^2 + k_y^2 + \frac{\xi^2}{c^2}}$$

Reflection amplitudes have not yet been specified, but for general properties : unitarity, causality and high-ω transparency

M. Jaekel & S. Reynaud, J. Physique **I-1** (1991) 1395 *arXiv:quant-ph/0101067*

The Lifshitz formula

- The Lifshitz formula is reproduced
  - for semi-infinite bulk mirrors characterized by local dielectric response functions $\varepsilon(\omega)$
  - with the reflection amplitudes given by Fresnel laws

$$r^{TE} = \frac{\kappa - K}{\kappa + K}$$

$$r^{TM} = \frac{K - \varepsilon\kappa}{K - \varepsilon\kappa}$$

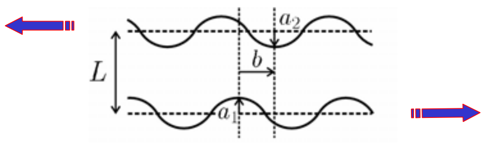
$$K \equiv \sqrt{k_x^2 + k_y^2 + \varepsilon\frac{\xi^2}{c^2}}$$

- The scattering formula accommodates more general expressions for the reflection amplitudes :
  - finite thickness
  - multilayer structure
  - non local dielectric response, …
- It can be extended for
  - non isotropic response, chiral materials, …

Many talks at QFEXT'09

The scattering formula is valid and regular for any realistic model of reflection amplitudes

Lateral force between corrugated plates



- Symmetry broken for transverse translations → Lateral forces
- Except at the PFA limit, the energy has to be obtained from the non-specular scattering formula

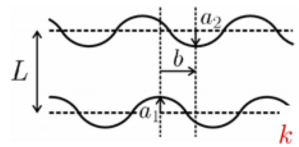
- At lowest order in the corrugation amplitudes, the lateral effect varies with the variations of the two reflection matrices

$$\delta E^{\mathrm{corrug}} = -\hbar \int_0^\infty \frac{d\xi}{2\pi} \mathrm{Tr}\left(\delta\mathcal{R}_1 \frac{e^{-\mathcal{K}L}}{\mathcal{D}_0} \delta\mathcal{R}_2 \frac{e^{-\mathcal{K}L}}{\mathcal{D}_0}\right)$$

R. Rodriguez, P. Maia Neto, A. Lambrecht & S. Reynaud, PRA 75 (2007) 062108

Spectral sensitivity

- After explicit calculations, the dependence of the energy on the corrugation is described by a spectral sensitivity



$$k = \frac{2\pi}{\lambda_c}$$

$$\delta E^{\mathrm{corrug}} = a_1 a_2 \cos(kb) G_c(k)$$

This is the PF Theorem

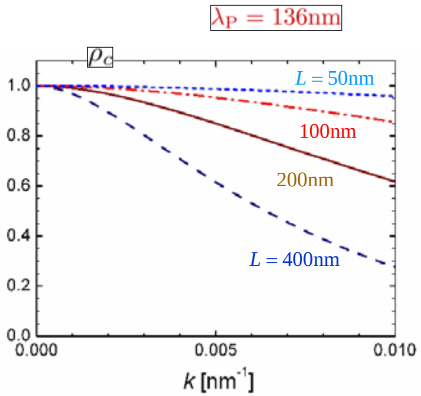
- PFA is recovered as the limit $k \to 0$
- Deviation from PFA is measured by the factor $\rho_c(k) \equiv \frac{G_c(k)}{G_c(0)}$

➧ Non trivial effect of geometry to be seen when $\rho_c(k) \neq 1$

R. Rodriguez, P. Maia Neto, A. Lambrecht & S. Reynaud, PRL **96** (2006) 100402

Non trivial effects of geometry

- Simple microscopic model
  - Bulk metallic mirrors described by the plasma model
  - Non specular reflection amplitudes in the Rayleigh approximation



- The force is reduced with respect to PFA

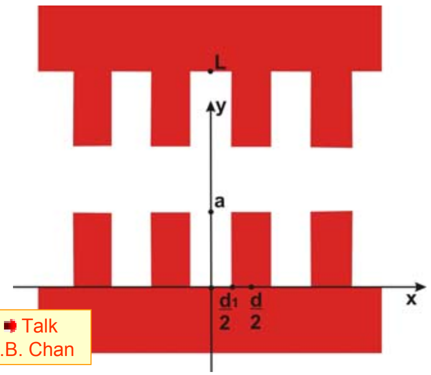
$$\rho_c(k) \equiv \frac{G_c(k)}{G_c(0)} < 1$$

R. Rodriguez, P. Maia Neto, A. Lambrecht & S. Reynaud, PRL **96** (2006) 100402

R. Rodriguez, P. Maia Neto, A. Lambrecht & S. Reynaud, PRL **98** (2007) 068902

Calculating deep nanostructures

- Calculation also done for the force between two nanostructured surfaces made of real materials with arbitrary corrugation depth a, corrugation width d and distance L
- Confirms the deviation from PFA seen in the U. Florida experiment though agreement is not immediate …



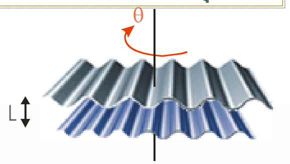
➧ Talk H.B. Chan

H.B. Chan et al, PRL **101** (2008) 030401

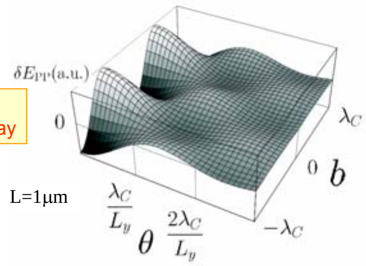
A. Lambrecht and V.N. Marachevsky, PRL **101** (2008) 160403
A. Lambrecht, Nature **454** (2008) 836

New applications : Vacuum-induced torques

- Symmetry broken for rotations
  → vacuum fluctuations induce a torque which tends to align two plates whose corrugations would not be parallel



➧ Casimir torque



- Larger effect than with birefringent plates

➧ Talk J. Munday

- But the two motions of rotation [θ] and lateral translation [b] are coupled

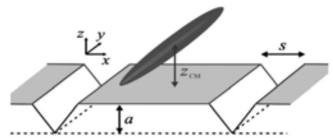
R. Rodrigues, P. Maia Neto, A. Lambrecht, S. Reynaud, EPL **76** (2006) 822

Non trivial effects with atoms

- Atoms can be used as local weakly-perturbing probes of vacuum
- Large effect appearing at the first order in corrugation amplitudes
- One can use the novel technological possibilities offered by cold atoms

➧ Poster R. Messina

Example : a Bose-Einstein condensate above a nano-grooved plate



➧ Poster G. Moreno

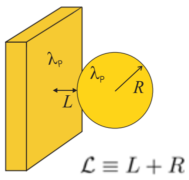
Non trivial effects of geometry should also be visible with atoms

D. Dalvit, P. Maia Neto, A. Lambrecht, S. Reynaud, PRL **100** (2008) 040405

The plane-sphere geometry beyond PFA

- We start from the non-specular scattering formula

$$E = \hbar \int_0^\infty \frac{d\xi}{2\pi} \mathrm{Tr} \ln \left(1 - \mathcal{R}_P e^{-\mathcal{K}\mathcal{L}} \mathcal{R}_S e^{-\mathcal{K}\mathcal{L}}\right)$$



$\mathcal{L} \equiv L + R$

  - Then we write the reflection matrices, $\mathcal{R}_P$ for plane waves on the plane mirror, $\mathcal{R}_S$ for spherical waves on the sphere (Mie amplitudes) …
  - … and the transformation from plane to spherical waves
  - All calculations are performed for electromagnetic fields

- We obtain an "exact" multipolar expansion of the energy

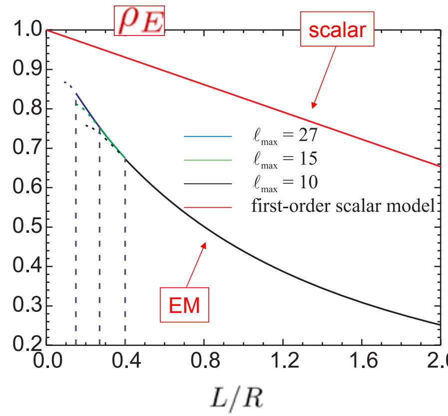
$x \equiv \frac{L}{R}$

  - Spherical waves are labeled by ℓ and m with $|m| \leq \ell$
  - Sums are truncated at some $\ell_{\max}$ for doing the numerics
  - The results are accurate for $x > x_{\min}$ with $x_{\min} \propto \frac{1}{\ell_{\max}}$

P. Maia Neto, A. Lambrecht, S. Reynaud, Phys. Rev. A **78** (2008) 012115

Calculation for perfect mirrors

- Perfect mirrors described by the plasma model at the limit $\lambda_P \to 0$
- Energy is found to be reduced with respect to PFA
  $\rho_E \equiv \frac{E_{PS}}{E_{PS}^{PFA}} < 1$



- Electromagnetic result departs from PFA more rapidly than scalar calculation

P. Maia Neto, A. Lambrecht, S. Reynaud, Phys. Rev. A **78** (2008) 012115

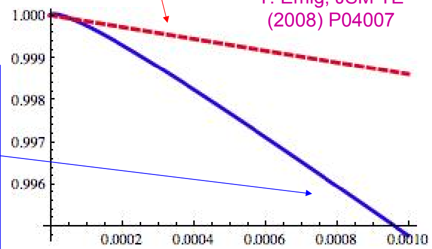
Comparison with other results

- Hints on values at low values of x can be found through a polynomial interpolation $\rho_E(x) \simeq 1 + \beta_E x + \ldots$
- Slope ~8 times larger than the value found in scalar calculations $\beta_E \simeq -1.4$

Agreement with Emig's results

T. Emig, JSM TE (2008) P04007

- The small-x expansion presented by M.Bordag at QFEXT'09 departs even more rapidly from PFA in the range of experiments



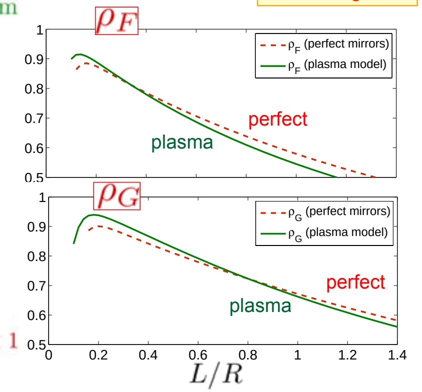
P. Maia Neto, A. Lambrecht, S. Reynaud, Phys. Rev. A **78** (2008) 012115

Plane and spherical metallic surfaces

Poster A. Canaguier

- Metallic mirrors described by the plasma model $\lambda_P = 136\text{nm}$
- Results close to the limit of perfect reflection for large spheres
- Differences seen for smaller spheres (R=100nm on the plots)
- Force F and gradient G found to be reduced with respect to PFA

$$\rho_F \equiv \frac{F_{PS}}{F_{PS}^{PFA}} < 1, \ \rho_G \equiv \frac{G_{PS}}{G_{PS}^{PFA}} < 1$$



A. Canaguier-Durand, P.A. Maia Neto, I. Cavero-Pelaez, A. Lambrecht, S. Reynaud PRL **102** (2009) 230404

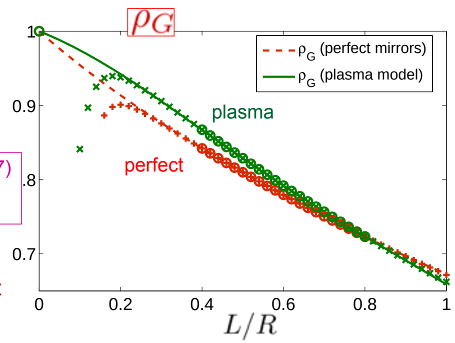
Interplay between geometry and material

Poster A. Canaguier

- Hints on small values of x through an interpolation $\rho_G \simeq 1 + \beta_G x + \ldots$
- Slope found for the plasma model could be compatible with the Purdue* experiment $\beta_G \sim -0.21$

(*) D. Krause et al, PRL (2007) $|\beta_G| \lesssim 0.4$

- Whereas slope found for perfect mirrors would not $\beta_G \sim -0.48$



A. Canaguier-Durand, P.A. Maia Neto, I. Cavero-Pelaez, A. Lambrecht, S. Reynaud PRL **102** (2009) 230404

Thank for your attention

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