Physics of Repulsive Van der Waals forces.

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Van der Waals forces in vacuum

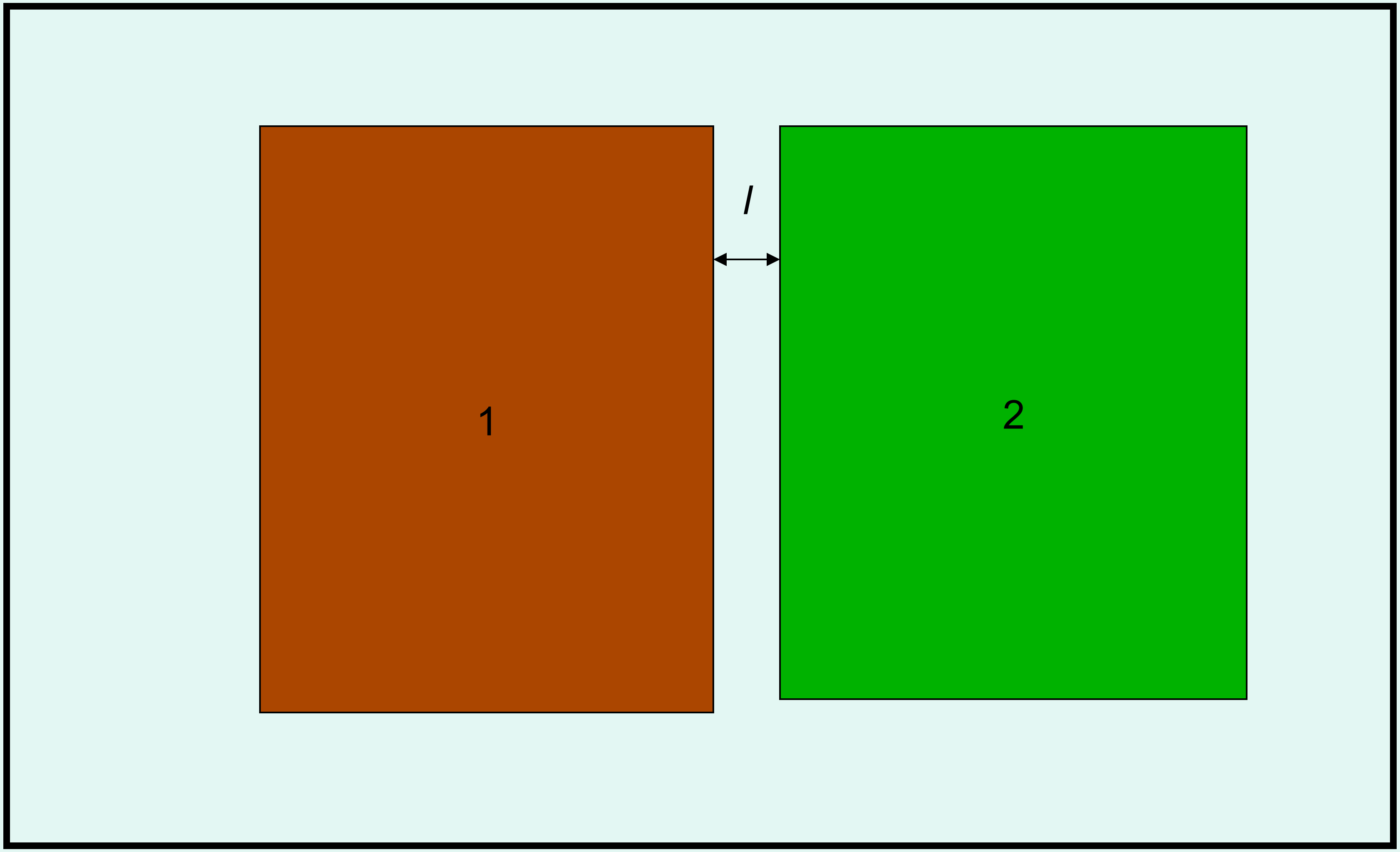
$$\text{Atom} - \text{atom interaction.}\ T = 0$$

$$F(l) = -\frac{18\hbar}{\pi l^7}\int_0^\infty \alpha_1(i\xi)\alpha_2(i\xi)\,d\xi$$

[**London (1930)** regime, $l << \lambda_0$]

Always attraction.

Interaction of bodies in vacuum



Interaction of bodies in vacuum

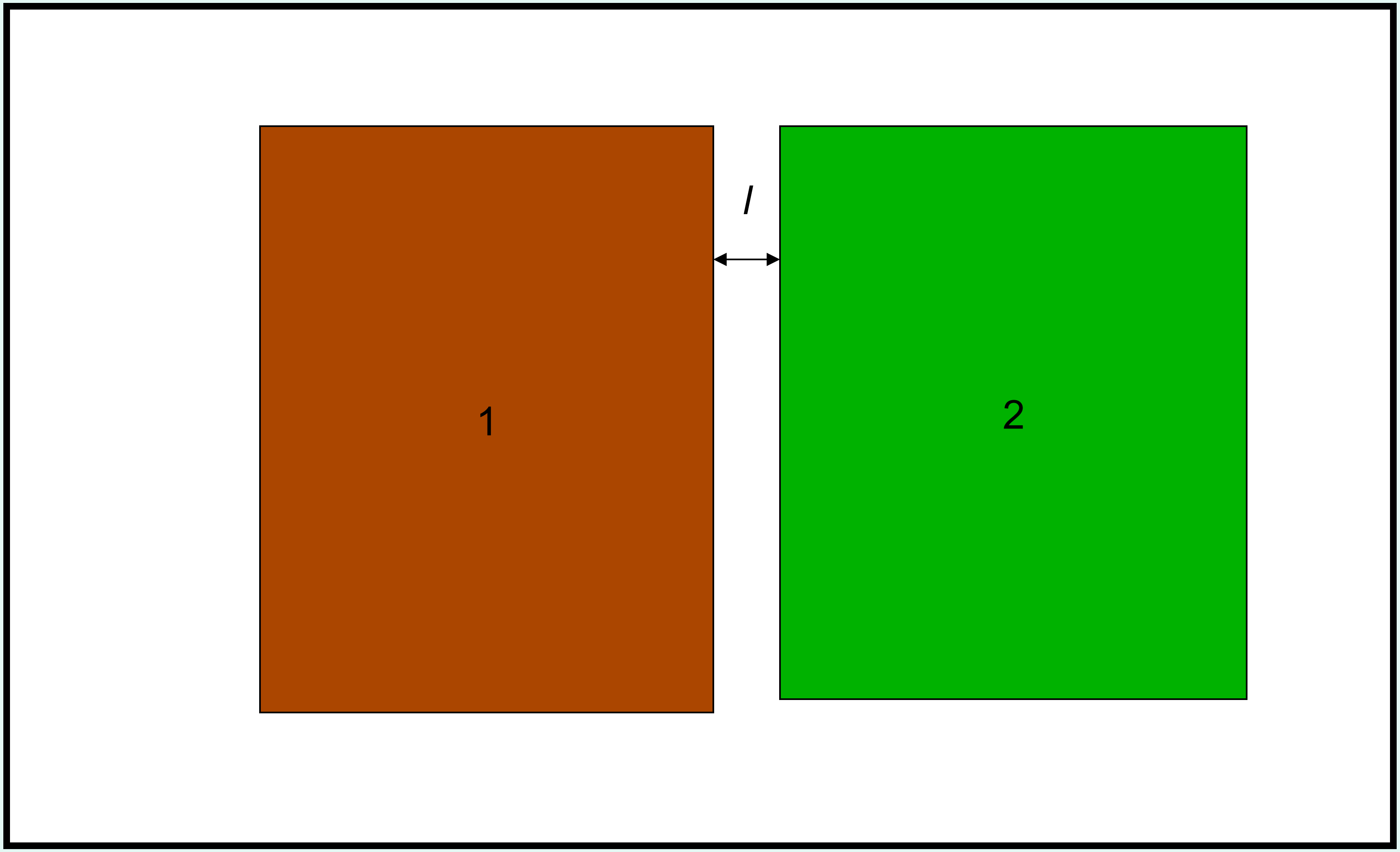
Lifshitz theory (1954):

$$F = -\frac{\hbar}{8\pi^2 l^3}\int_0^\infty \frac{(\varepsilon_1 - 1)(\varepsilon_2 - 1)}{(\varepsilon_1 + 1)(\varepsilon_2 + 1)} d\xi$$

$$\varepsilon_i = \varepsilon_i(i\xi)$$

Always attraction

Interaction of bodies immersed in a liquid



Interaction of bodies immersed in a liquid

$$F = -\frac{\hbar}{8\pi^2 l^3}\int_0^\infty \frac{(\varepsilon_1 - \varepsilon_3)(\varepsilon_2 - \varepsilon_3)}{(\varepsilon_1 + \varepsilon_3)(\varepsilon_2 + \varepsilon_3)} d\xi$$

$$\varepsilon_i = \varepsilon_i(i\xi)$$

$$\varepsilon_1 = \varepsilon_2 - \text{attraction}$$

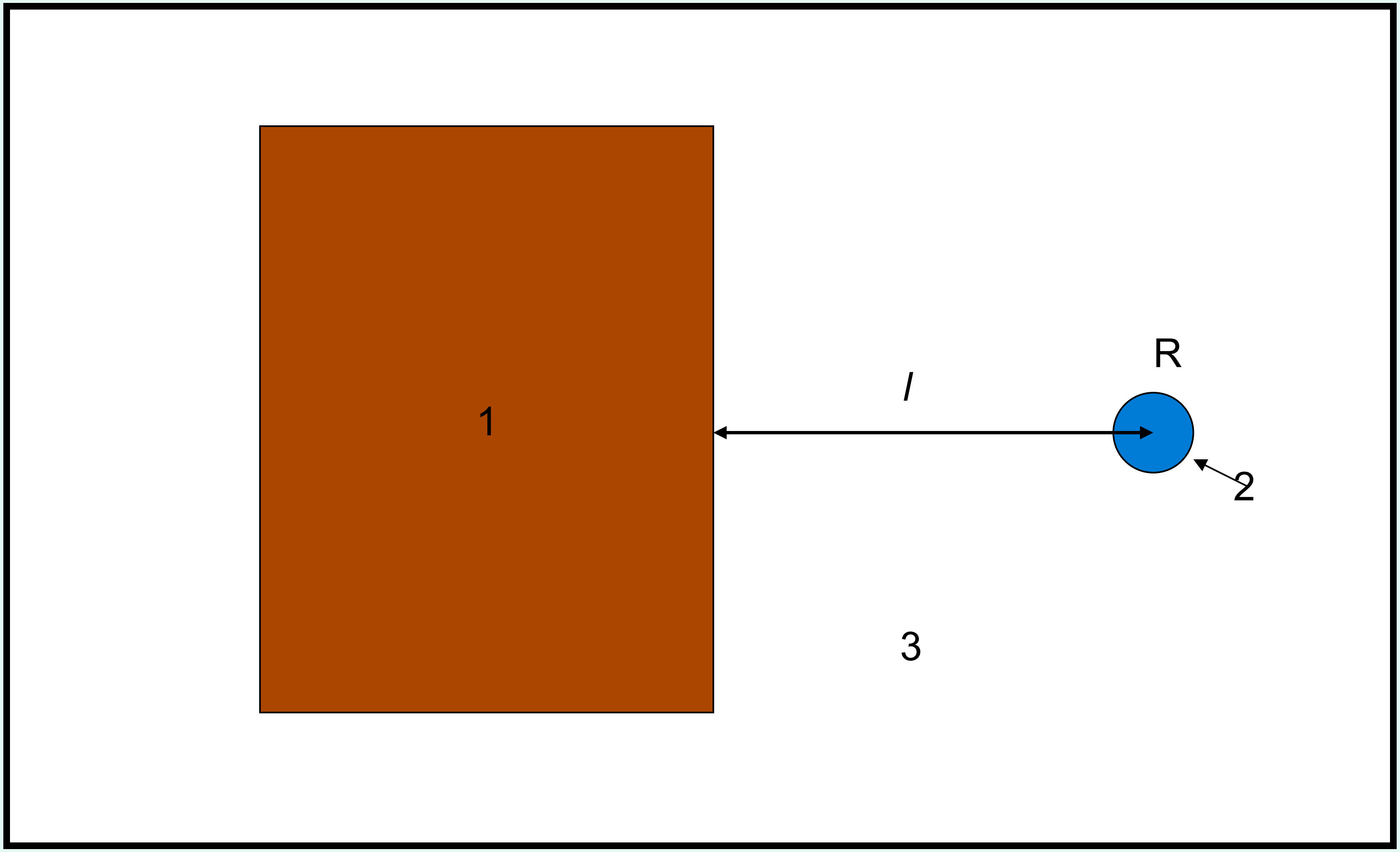
If at important frequencies $\xi \sim \xi_0$

$$\varepsilon_1(i\xi_0) > \varepsilon_3(i\xi_0) > \varepsilon_2(i\xi_0)$$

REPULSION

Dzyaloshinskii, Lifshitz and Pitaevskii (1959).

Interaction of a small sphere with a body immersed in a liquid



Interaction of a small sphere and a body immersed in a liquid

$$l >> R$$

$$F = -\frac{3\hbar R^3}{4\pi l^4}\int_0^\infty \frac{(\varepsilon_1 - \varepsilon_3)(\varepsilon_2 - \varepsilon_3)}{(\varepsilon_1 + \varepsilon_3)(\varepsilon_2 + 2\varepsilon_3)} d\xi$$

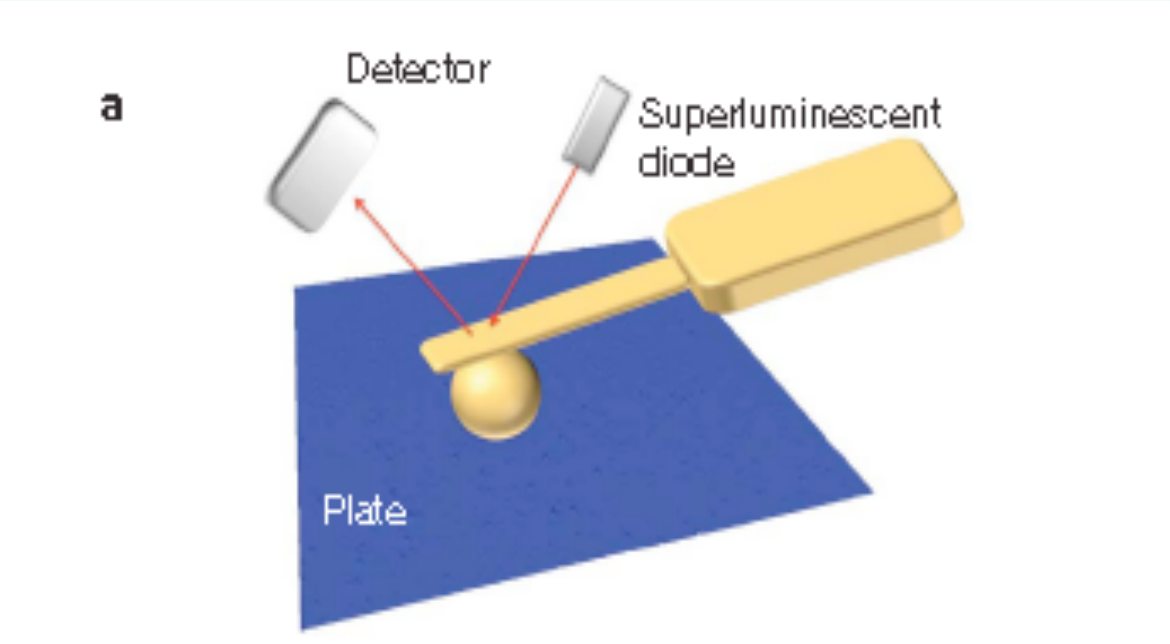
$$\varepsilon_i = \varepsilon_i(i\xi)$$

$$\text{Again: } \varepsilon_1 = \varepsilon_2 - \text{attraction}$$

$$\text{If } \varepsilon_1(i\xi_0) > \varepsilon_3(i\xi_0) > \varepsilon_2(i\xi_0)$$

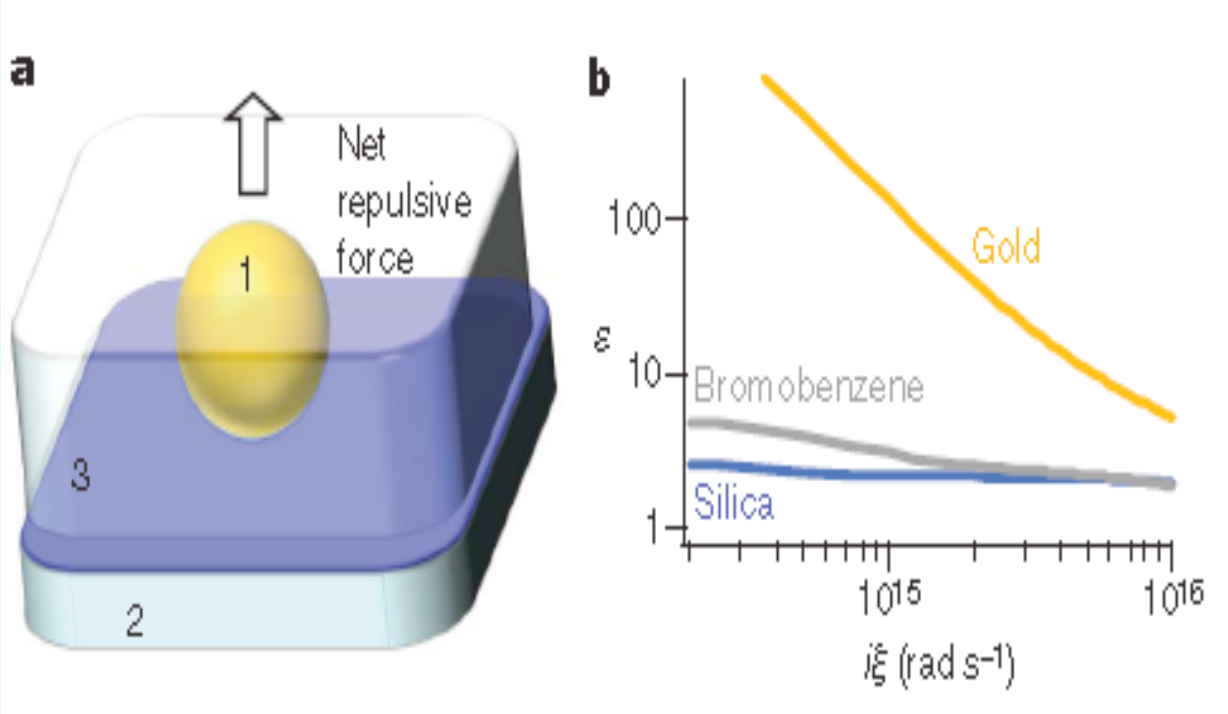
REPULSION

Experimental set up



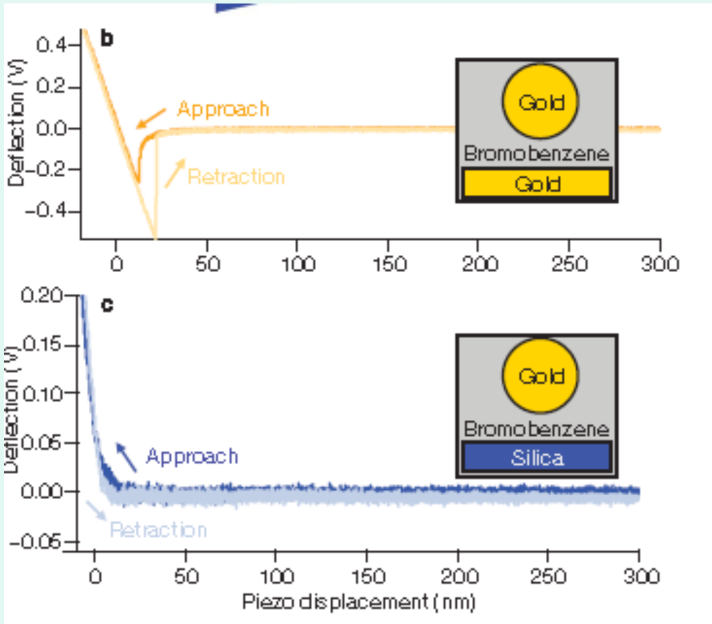
A sphere is attached to an atomic force cantilever, which is enclosed within a bromobenzene-filed cell for force measurements.
J. Munday, F. Capasso and A. Persian (2008)

Repulsive quantum electrodynamical forces can exist for two materials immersed in a fluid



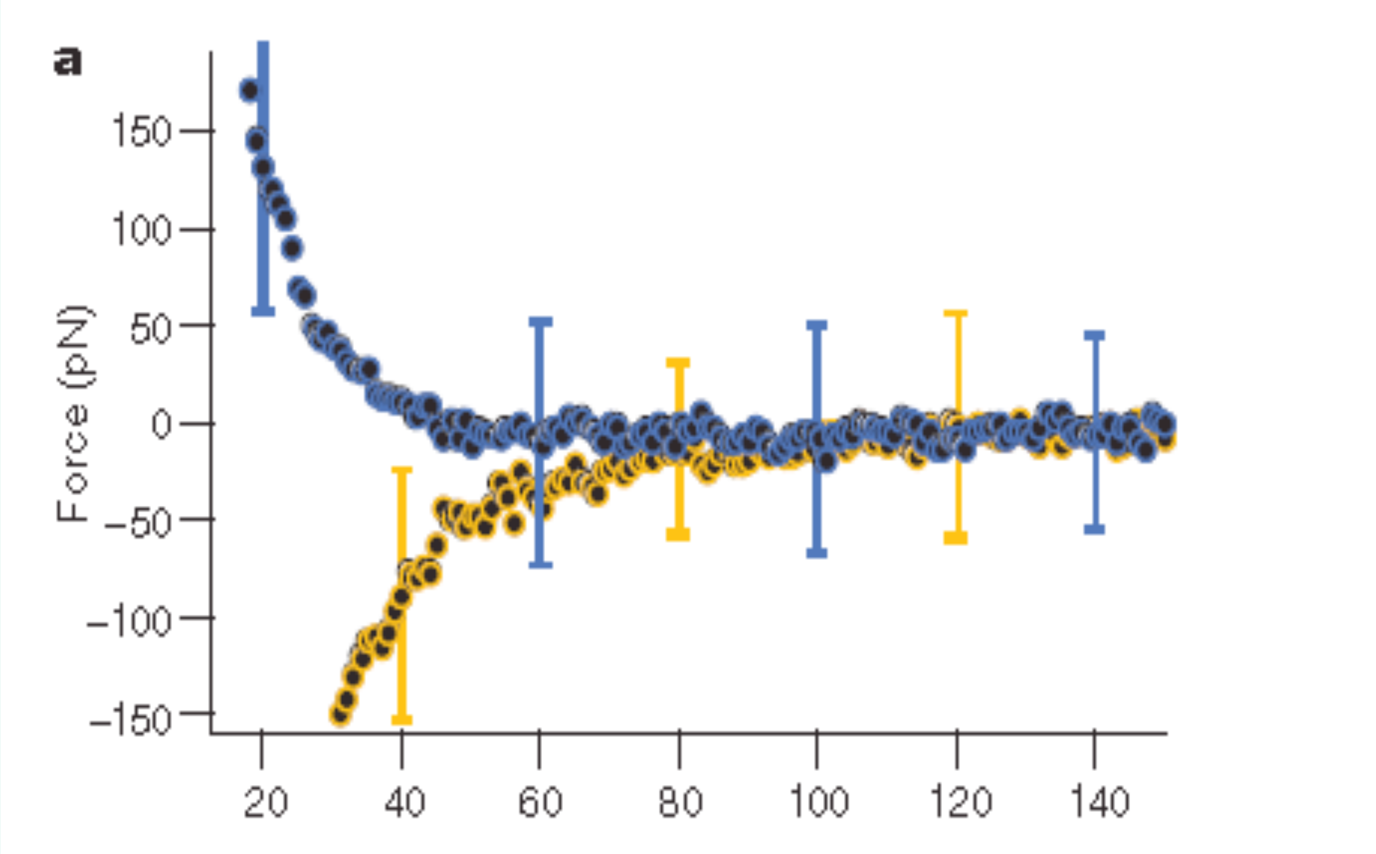
a, The interaction between material 1 and material 2 immersed in a fluid (material 3) is repulsive when $\varepsilon_1(i\xi) > \varepsilon_3(i\xi) > \varepsilon_2(i\xi)$.
b, The optical properties of gold (1), bromobenzene (3) and Silica (2) are such that this inequality is satisfied. This leads to a repulsive force between the gold and silica surfaces.

Experimental data



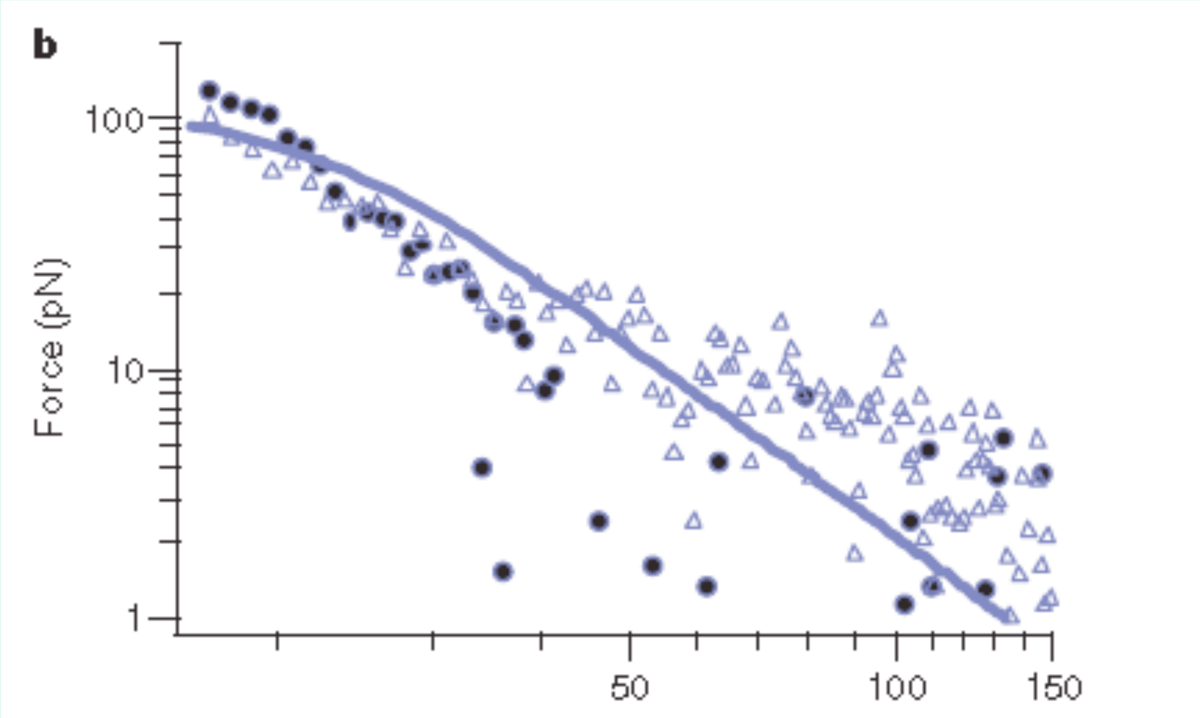
b, Deflection data showing attractive interactions between a gold sphere and a gold plate. c, For the case of the same gold sphere and a silica plate, deflection data show a repulsive interaction evident during both approach and retraction. Note that the deflection voltage signal is proportional to the bending of the cantilever.

Attractive and repulsive Casimir-Lifshitz force measurements, a.



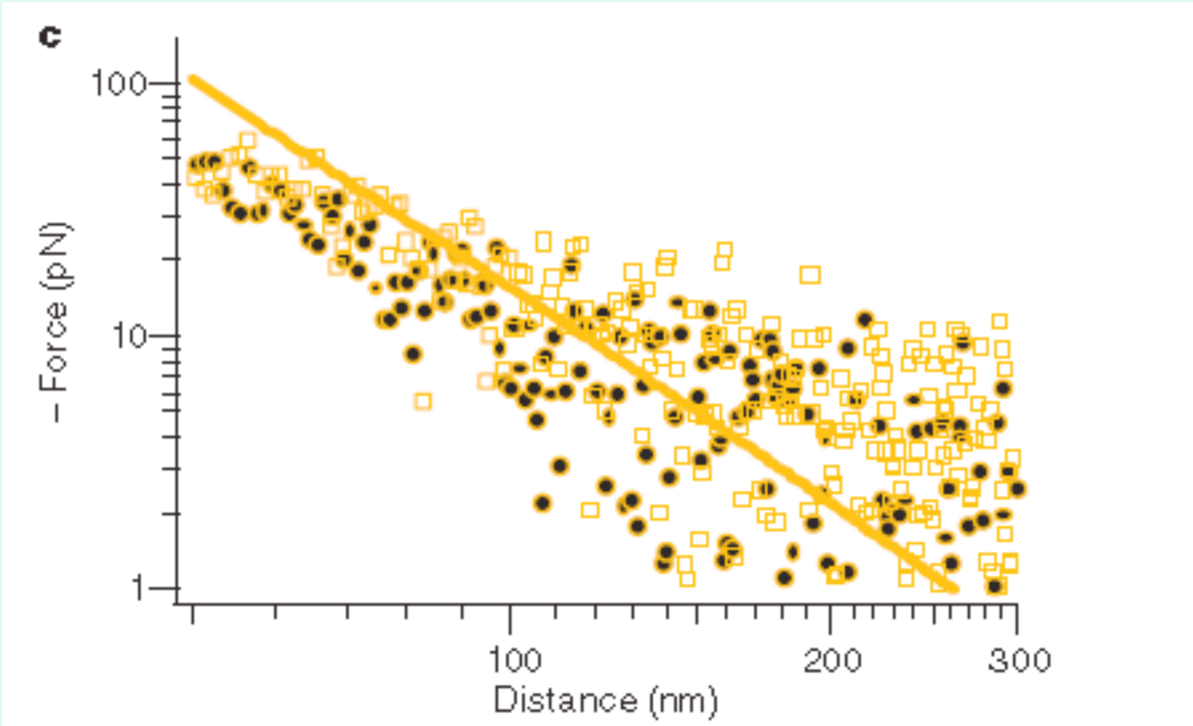
a, Blue (orange) circles represent the average of 50 data sets for the force between a gold sphere and a silica (gold) plate in bromobenzene.

Repulsive Casimir-Lifshitz force measurements, b.



b,Measured repulsive force between a gold sphere and a silica plate in bromobenzene on a log–log scale (blue circles) and calculated force using Lifshitz's theory (solid line) including corrections for the measured surface roughness of the sphere and the plate. Blue triangles are force data for another gold sphere/silica plate pair.

Attractive Casimir-Lifshitz force measurements, c.



c, Measured attractive force on a log–log scale for two gold sphere/plate pairs (circles and squares) in bromobenzene. The calculated force includes surface roughness corrections corresponding to the data represented by the circles.

What is the physical meaning of the effect?

Shielding by the medium 3 ?

Interaction of bodies immersed in a liquid

$$F = -\frac{\hbar}{8\pi^2 l^3}\int_0^\infty \frac{(\varepsilon_1 - \varepsilon_3)(\varepsilon_2 - \varepsilon_3)}{(\varepsilon_1 + \varepsilon_3)(\varepsilon_2 + \varepsilon_3)} d\xi$$

$$\varepsilon_i = \varepsilon_i(i\xi)$$

$$\varepsilon_1 = \varepsilon_2 - \text{attraction}$$

If at important frequencies $\xi \sim \xi_0$

$$\varepsilon_1(i\xi_0) > \varepsilon_3(i\xi_0) > \varepsilon_2(i\xi_0)$$

REPULSION

Dzyaloshinskii, Lifshitz and Pitaevskii (1959).

Limit of rarefied media

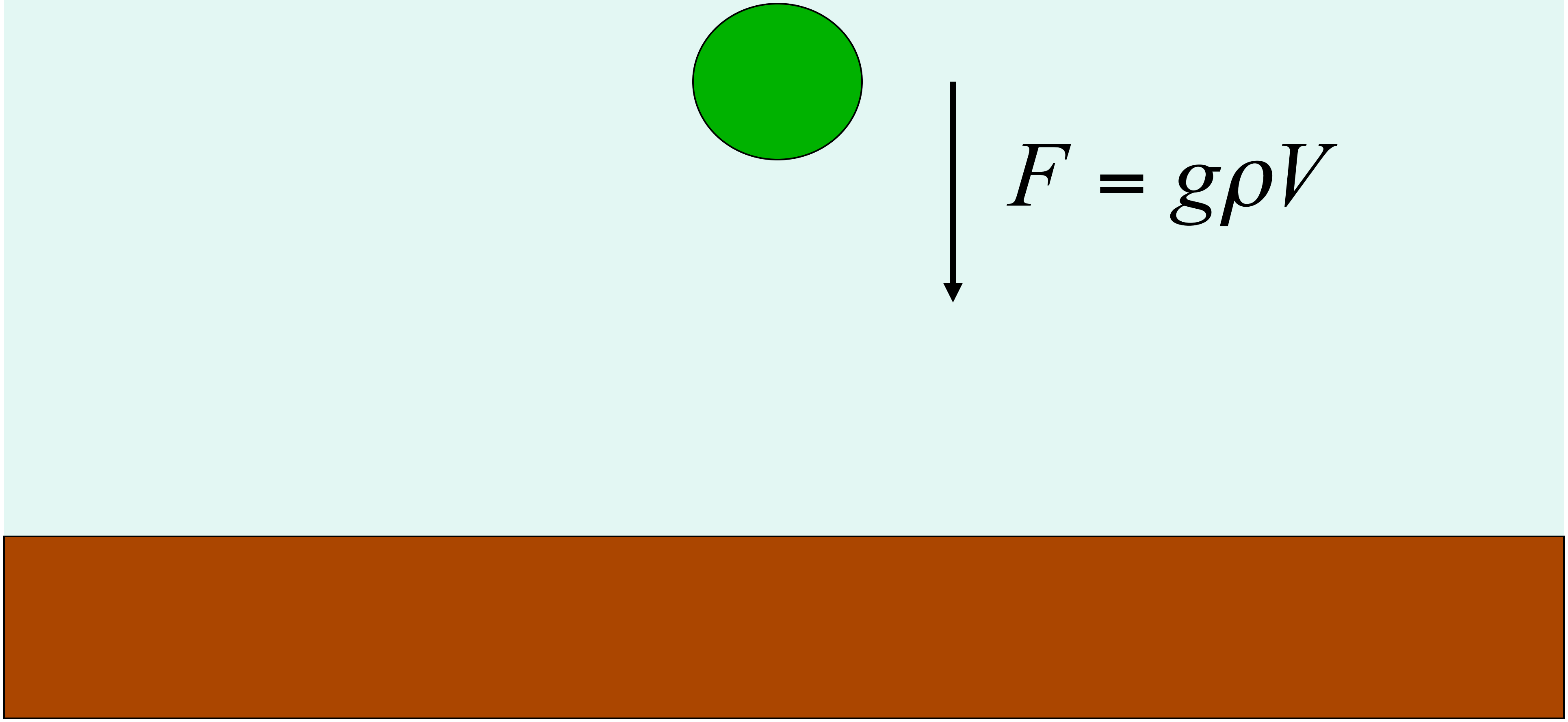
$$\alpha = 2,3: \quad \varepsilon_\alpha = 1 + \delta\varepsilon_\alpha, \left|\delta\varepsilon_\alpha\right| << 1$$

$$F = -\frac{\hbar}{16\pi^2 l^3}\int_0^\infty \frac{(\varepsilon_1 - 1)}{(\varepsilon_1 + 1)}(\delta\varepsilon_2 - \delta\varepsilon_3)d\xi$$

Forces are additive, no shielding,

but the effectstill exists.

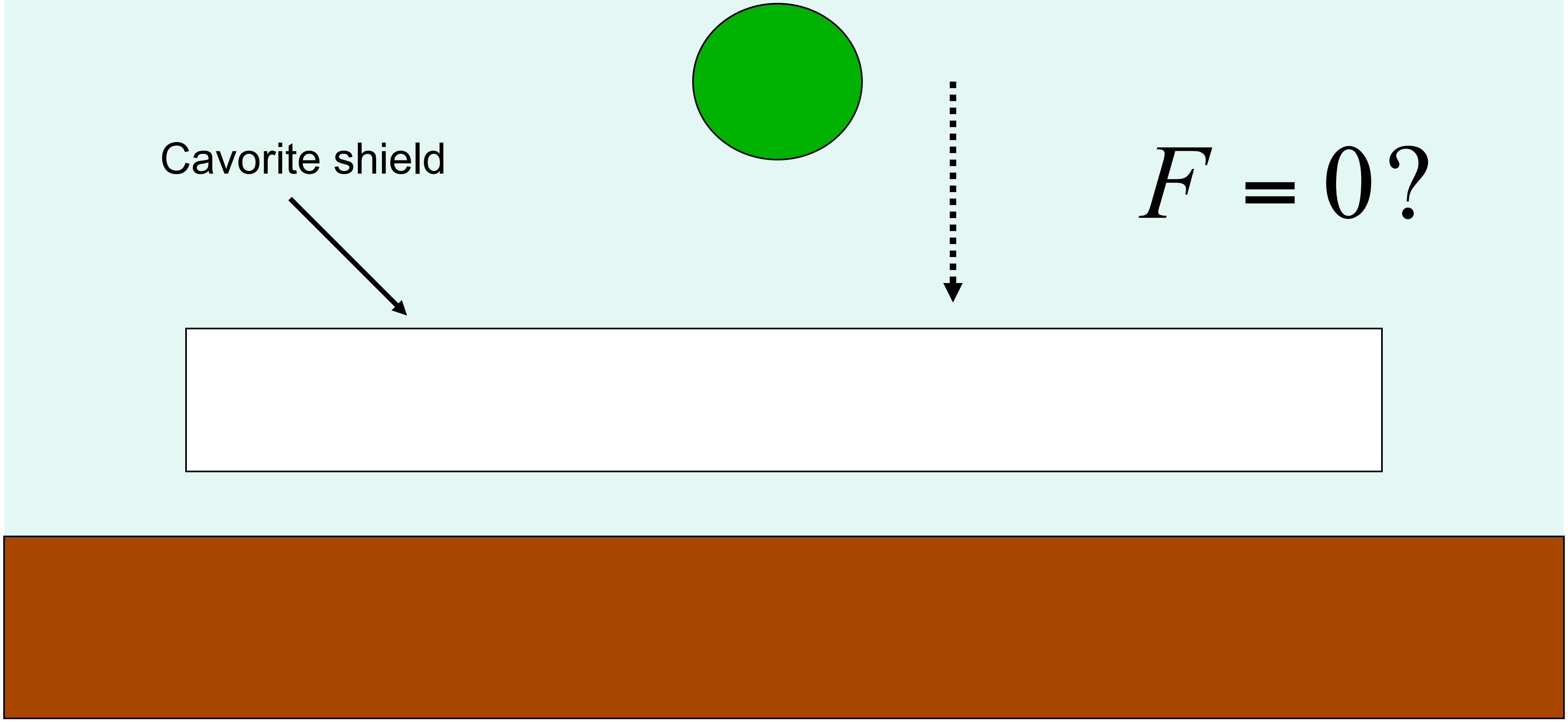
Instructive example: gravitation



$$F = g\rho V$$

Can we shield gravitation?

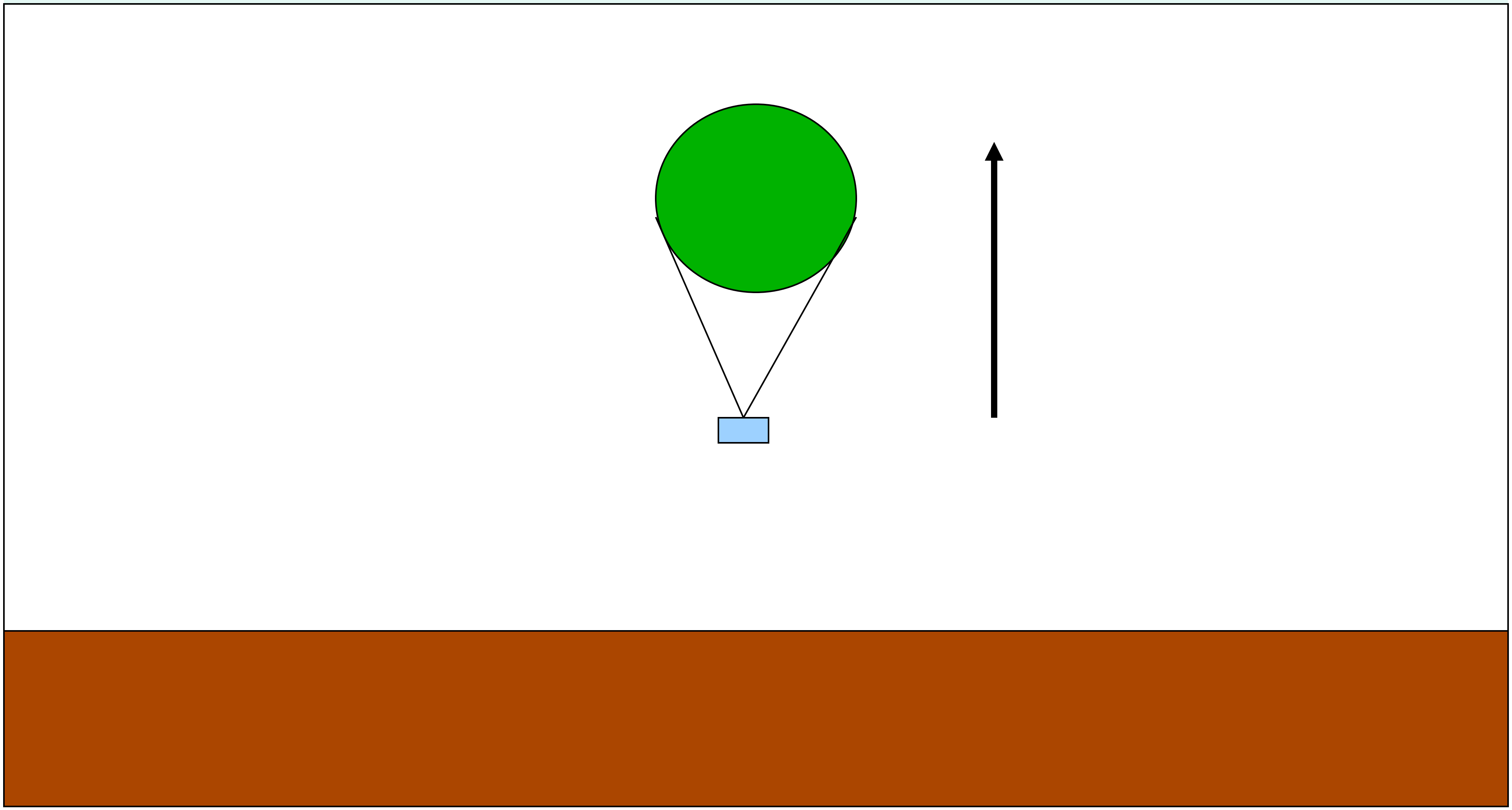
Idea from “The first men in the Moon”, by H.G. Wells



$$F = 0\,?$$

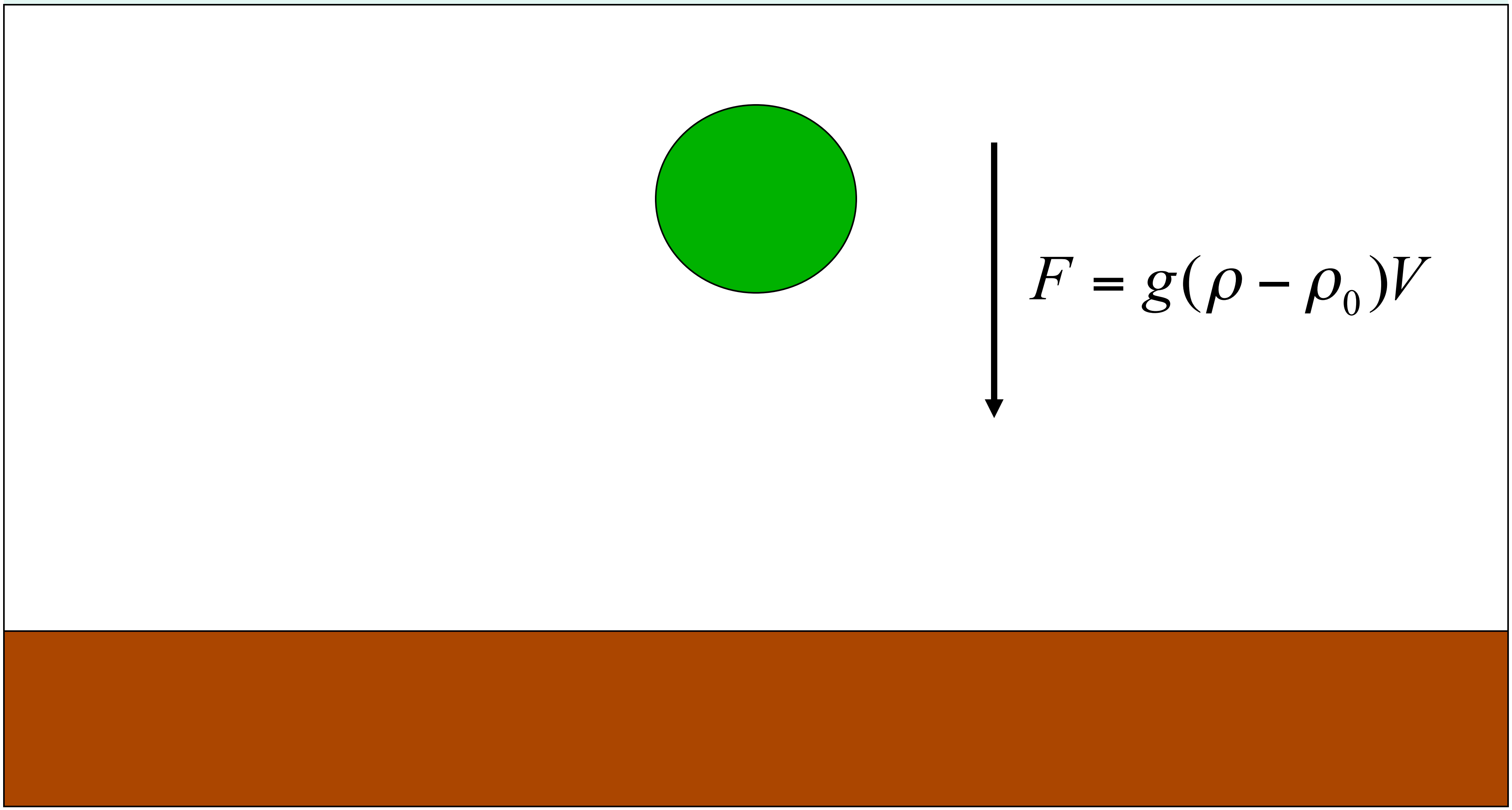
Cavorite space ship

However: a body immersed in a fluid can fly



Theory was given by Archimedes 290-212 B.C.

Archimedes law

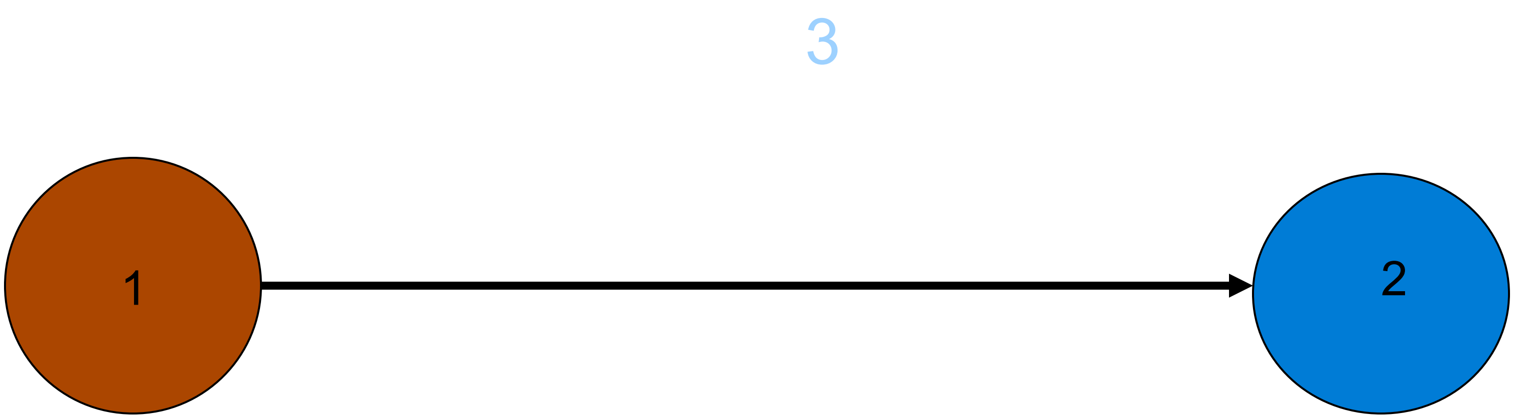


$$F = g(\rho - \rho_0)V$$

Origin of the Archimedes force

Force acting on the liquid results in re-distribution of the pressure which lifts the body

Archimedes law for body-body interaction



$$F = -\kappa \frac{(\rho_1 - \rho_3)(\rho_2 - \rho_3)}{l^2} V_1 V_2$$

The “Casimir-Lifshitz levitation” is mainly due to redistribution of pressure in the liquid, not due to any “shielding”. Direct mechanical contact of liquid with bodies is important. Effect of hydrostatics, not of electrodynamics.

Where the hydrostatics was involved?

Interaction through vacuum.

Maxwell stress tensor:

$$\sigma_{ik} = \frac{1}{4\pi} E_i E_k - \frac{1}{8\pi} E^2 \delta_{ik} + $$

$$+ [E_i \rightarrow B_i]$$

Lifshitz tensor: $\sigma^L{}_{ik} = \left\langle \sigma_{ik} \right\rangle_{Fl}$

$$= \frac{1}{4\pi} \left\langle E_i E_k \right\rangle_{Fl} - \frac{1}{8\pi} \left\langle E^2 \right\rangle_{Fl} \delta_{ik} + [E_i \rightarrow B_i]$$

Interaction through a dielectric liquid

The main difficulty:

Equation for the stress tensor for time-depending fields in an **absorbing** media does not exist. The solution is to consider **equilibrium** fluctuating fields from the very beginning.

Necessary conditions on stress tensor at thermodynamic equilibrium:

$$\sigma_{ik} = -P\delta_{ik} + \sigma'_{ik}$$

$$\sigma'_{ik} = \sigma'_{ki}$$

Symmetry insures angular momentum conservation

Mechanical equilibrium in fluid

$$\frac{\partial \sigma_{ik}}{\partial x_k} = \frac{\partial \sigma'_{ik}}{\partial x_k} - \frac{\partial P}{\partial x_i} = 0$$

$$(\mathrm{rot})_i \frac{\partial \sigma'_{ik}}{\partial x_k} = 0$$

$$\sigma'^{(1)}_{ik} n_k = \sigma'^{(2)}_{ik} n_k$$

Violation of one of these conditions would permit to build a *perpetuo mobile*.

Free energy variation

Static fields

$$\delta F = \delta F_0 - \int \frac{E^2}{8\pi} \delta\varepsilon_0(\mathbf{r}) dV$$

Equilibrium fluctuating fields

$$(T = 0)$$

$$\delta F = \delta F_0 - \int\int_0^\infty \frac{\langle E^2 \rangle_{i\xi}}{8\pi} \delta\varepsilon_0(\mathbf{r}, i\zeta) d\xi dV$$

Stress tensor of the equilibrium electromagnetic fluctuations in a liquid:

$$\sigma^{DP}{}_{ik} = -P(\rho, T)\delta_{ik} +$$

$$\int_0^\infty \frac{d\xi}{4\pi}\left[-\frac{1}{2}\delta_{ik}\left\langle E^2\right\rangle_{i\xi}\left(\varepsilon(i\xi) - \rho\frac{\partial\varepsilon(i\xi)}{\partial\rho}\right) + \varepsilon(i\xi)\left\langle E_i E_k\right\rangle_{i\xi} + (E \to H, \varepsilon \to \mu)\right]$$

Dzyaloshinskii and Pitaevskii, 1959.

Tensor σ^{DP} is a non-trivial result of many-body theory. Experimental confirmation is Important.

Hydrostatic equilibrium

Condition of hydrostatic equilibrium

$$\partial_k \sigma_{ik} = 0 \text{ gives}:$$

$$P(\rho, T) + \int_0^\infty \frac{d\xi}{4\pi}\left[\frac{1}{2}\left\langle E^2 \right\rangle_{i\xi} \rho \frac{\partial \varepsilon(i\xi)}{\partial \rho}\right] \equiv P_0 = const$$

Tensor actually used in DLP calculations:

$$\sigma'_{ik} = \sigma^{DP}{}_{ik} + P_0\delta_{ik} =$$

$$\int_0^\infty \frac{d\xi}{4\pi}\left[\varepsilon_3(i\xi)\left(-\frac{1}{2}\delta_{ik}\left\langle E^2\right\rangle_{i\xi} + \left\langle E_i E_k\right\rangle_{i\xi}\right) + \left(E \rightarrow H, \varepsilon \rightarrow 1\right)\right].$$

$$\partial_k \sigma'_{ik} \equiv 0$$

Condition of hydrostatic equilibrium

has been already imposed.

Can we obtain this tensor directly?

Variation at constant ρ implies mechanical equilibrium.

See J. Shwinger, L. DeRaad, and K. Milton (1978).

Conclusions

Appearance of repulsion in the problem of Casimir-Lifshitz interaction of bodies, immersed in a dielectric liquid, is due to the Archimedes-like effect of redistribution of the pressure of the liquid in the state of mechanical equilibrium. Direct mechanical contact of the liquid with the bodies is important.