Bragg spectroscopy for measuring Casimir-Polder interactions with Bose-Einstein condensates above corrugated surfaces

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Abstract

We propose a method to probe dispersive atom-surface interactions by measuring via two-photon Bragg spectroscopy the dynamic structure factor of a Bose-Einstein condensate above corrugated surfaces. This method takes advantage of the condensate coherence to reveal the spatial Fourier components of the lateral Casimir-Polder interaction energy.

Casimir atom-surface interaction

Consider a ground-state atom at position $\mathbf{R}_A = (x_A, y_A, z_A)$ in front of a corrugated surface (with surface profile $h(x,y)$ measured with respect to the plane $z = 0$ (see figure). If

$$h(x) = \sum_j h_j \cos(jk_c x), \qquad (1)$$

then Casimir atom-surface interaction can be written as:

$$U(x,y,z) = U_N(z) + U_L(x,z), \qquad (2)$$

where first order approximation in h yields:

$$U_L^{(1)}(x_A, z_A) = \sum_{j=0}^{\infty} h_j \cos(jk_c x_A) g(jk_c, z_A). \qquad (3)$$

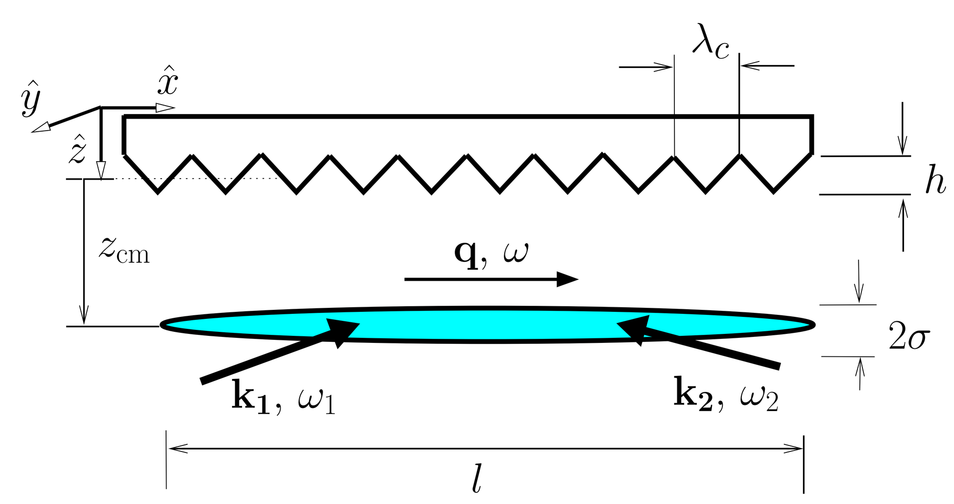
The response function $g(k,z)$ can be evaluated for a perfect conductor [1]:

$$g_{\rm CP}^{\rm perf}(k,z) = -\frac{3\hbar c\alpha(0)}{8\pi^2\epsilon_0 z^5} e^{-\mathcal{Z}}(1 + \mathcal{Z} + 16\mathcal{Z}^2/45 + \mathcal{Z}^3/45), \qquad (4)$$

where $\mathcal{Z} = kz$. The lateral term acting on a BEC is weak periodic potential.

Casimir-modified BEC energy spectrum

Consider an elongated BEC tightly confined



We describe it via Gross-Pitaevskii equation

$$i\hbar\partial_t\varphi = -(\hbar^2/2m)\nabla^2\varphi + [U_N(z) + U_L(x,z)]\varphi + (m/2)(\omega_r^2 r^2 + \omega_x^2 x^2)\varphi + g|\varphi|^2\varphi, \qquad (5)$$

We consider a strong trapping potential verifying

$$\mu - \hbar\omega_r \ll 8\hbar\omega_r. \qquad (6)$$

In this situation the order parameter dynamics is reduced from

$$\varphi = \sum_n f_n(r)\phi_n(x,t) \qquad (7)$$

into $f_0(r)\phi_0(x,t)$. The 1D problem has

$$V_{\rm eff}(x) = \hbar\omega_r + U_N(z_{\rm cm}) + U_L(x, z_{\rm cm}) \qquad (8)$$

and

$$g_{\rm eff} = g/2\pi\sigma^2 \qquad (9)$$

Thus the excitation spectrum of the quasi 1D BEC near a corrugated surface will be split into (Bloch-) bands.

To solve the problem we use TF approximation for the ground state and find linearized equations for the perturbations, namely:

$$\phi(x,t) = e^{-i\frac{\mu t}{\hbar}}[\phi_{\rm TF}(x) + \delta\phi(x,t)], \qquad (10)$$

where $\delta\phi(x,t)$ is a small disturbance and

$$\phi_{\rm TF}(x) = \{[\tilde{\mu} - U_L(x, z_{\rm cm})]/g_{\rm eff}\}^{1/2}. \qquad (11)$$

Perturbations can be expanded in plane waves, $\delta\phi(x,t) = u(x)e^{-i\frac{Et}{\hbar}} + v(x)e^{i\frac{Et}{\hbar}}$ where E should be determined from

$$Eu = -\frac{\hbar^2}{2m}\frac{d^2u}{dx^2} + (\tilde{\mu} - U_L(x, z_{\rm cm}))(u + v^*),$$
$$-Ev = -\frac{\hbar^2}{2m}\frac{d^2v}{dx^2} + (\tilde{\mu} - U_L(x, z_{\rm cm}))(u^* + v). \qquad (12)$$

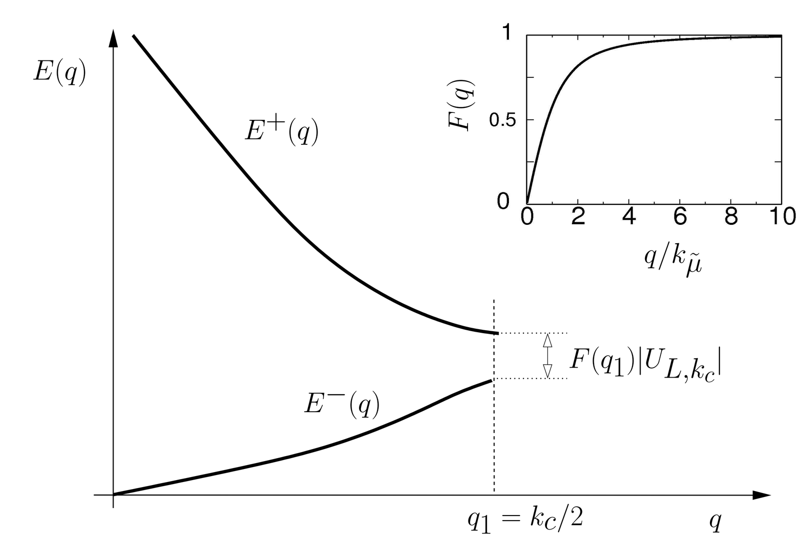
The lateral potential U_L is weak, so we can treat it perturbatively:

$$E(q) = E^{(0)}(q) + E^{(1)}(q) + \ldots. \qquad (13)$$

Where $E^{(0)}(q) = \sqrt{(\hbar^2 q^2/2m)(\hbar^2 q^2/2m + 2\tilde{\mu})}$. Solving this coupled problem yields first order gaps

$$\Delta E_{q_n} = |U_{L,nk_c}| \times F(q_n)$$
$$F(q) = T_q/E_q^{(0)} \qquad (14)$$

where T_q is the free kinetic energy $T_q = \hbar^2 q^2/2m$ and $q_n = nk_c/2$. The spectrum and the suppression factor are:



Bragg spectroscopy of the Casimir potential

The rate of momentum transferred to the BEC reads [2]:

$$\frac{dP_X}{dt} = -m\omega_x^2 X + \sum_{n,i} U_{L,nk_c}(nk_c)\langle\sin(nk_c x_i)\rangle + \frac{N\hbar q V_{\rm B}^2}{2}\int d\omega' \left[S(q,\omega') - S(-q,-\omega')\right] \times \frac{\sin([\omega-\omega']t)}{\omega-\omega'}. \qquad (15)$$

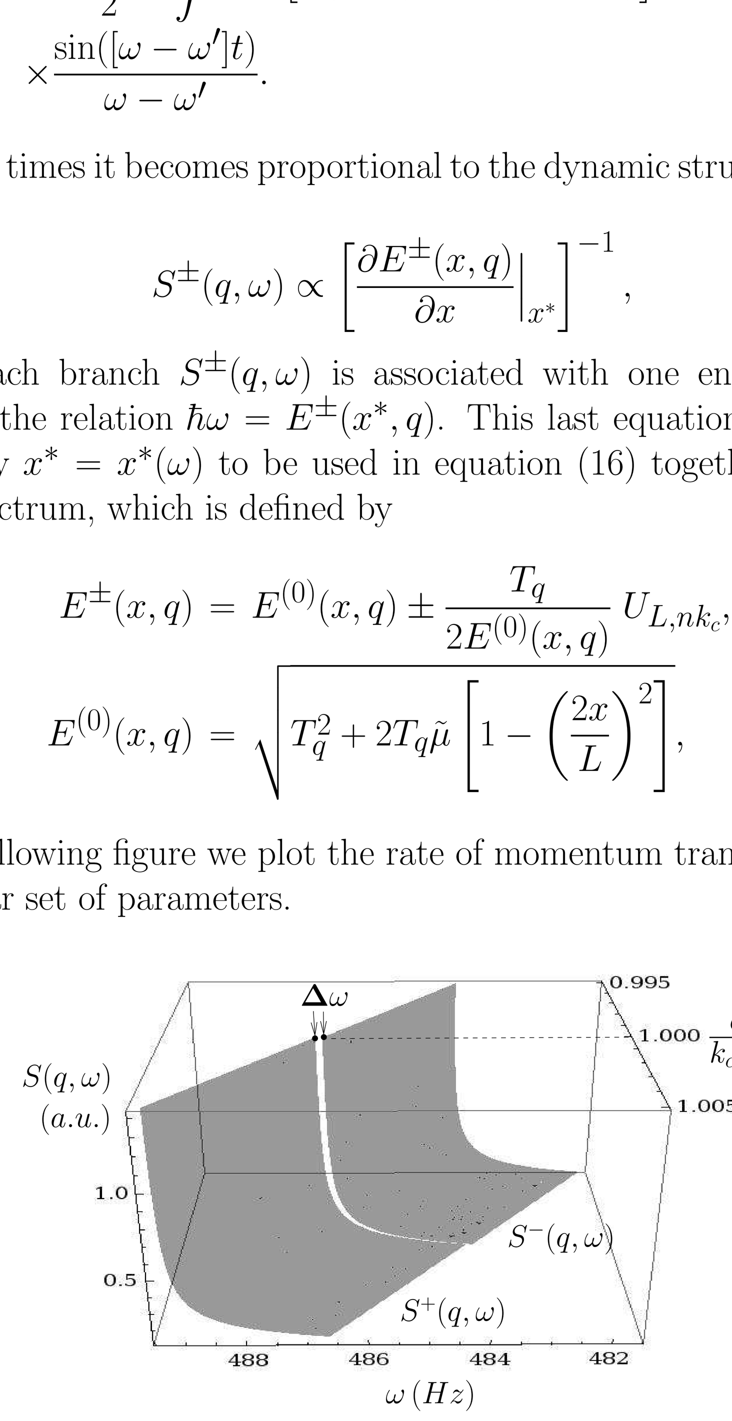
For large times it becomes proportional to the dynamic structure factor, namely

$$S^{\pm}(q,\omega) \propto \left[\frac{\partial E^{\pm}(x,q)}{\partial x}\Big|_{x^*}\right]^{-1}, \qquad (16)$$

where each branch $S^{\pm}(q,\omega)$ is associated with one energy branch through the relation $\hbar\omega = E^{\pm}(x^*, q)$. This last equation determines implicitly $x^* = x^*(\omega)$ to be used in equation (16) together with the local spectrum, which is defined by

$$E^{\pm}(x,q) = E^{(0)}(x,q) \pm \frac{T_q}{2E^{(0)}(x,q)} U_{L,nk_c},$$
$$E^{(0)}(x,q) = \sqrt{T_q^2 + 2T_q\tilde{\mu}\left[1 - \left(\frac{2x}{L}\right)^2\right]}, \qquad (17)$$

In the following figure we plot the rate of momentum transferred for a particular set of parameters.



Numerical estimations

Let us now evaluate the Casimir atom-surface lateral potential and the corresponding energy gaps. We choose a corrugation wavelength of $\lambda_c = 2\pi/k_c = 9.75\mu$m and corrugation amplitude $h = 1\mu$m separated by $z_{\rm cm} = 3\mu$m from a cigar-shaped ^{87}Rb condensate with $N = 10^4$ atoms. We assume that the BEC is trapped in an axially symmetric potential with trapping frequencies $\omega_x = 2\pi \times 0.83$ Hz and $\omega_r = 2\pi \times 2.7$kHz. For this trapping frequency the radius of the BEC is $\sigma = 0.2\mu$m. The typical size of the condensate and the chemical potential can be computed to be:

$$l/2 = (3g_{\rm eff}N/2m\omega_x^2)^{1/3} = 408\,\mu\text{m} \qquad (18)$$

and

$$\tilde{\mu} = (m\omega_x^2/8)^{1/3}(3g_{\rm eff}N/2)^{2/3} = \hbar 3.1\text{kHz}. \qquad (19)$$

The relevant kinetic energy is $T_{q_1=k_c/2} = \hbar 38\text{Hz} \ll \tilde{\mu}$. The typical Bogoliubov energy is $E_{\rm B}(q_1) = \hbar 485$Hz, and the suppression factor is $F(q_1) = 0.08$. The lateral Casimir-Polder potential (perfect conductor): $U_{L,k_c}^{(1)} = hg_{\rm CP}^{\rm perf}(k_c, z_{\rm cm})$, is approximately $\hbar\,1.4$Hz. This gives a signal of $F(q_1) \times 1.4\,Hz = 0.1\,Hz$. This high sensitivities have not been experimentally achieved yet. Other case can be studied, scaling the parameters given before to $z_{\rm cm} = 0.7\mu$m, $\lambda_c = 4\mu$m, and $h = 50$nm, results in a gap of 25Hz centered at $E = 1.2$kHz. Although this energy range has been experimentally demonstrated [3], the minimum distance of a BEC to the surface at present is limited to 2μm. Thermal effects are not important except for the coherence length of the BEC in the 1D configuration, yielding an upper bound on the temperature of the thermal cloud around the condensate, $T_{\rm BEC}$. It can be shown [4] that the typical decay length of the coherence is given by $2n_1\hbar^2/k_{\rm B}T_{\rm BEC}m$, where n_1 is the one-dimensional density. Using the above parameters one finds that the temperature of the BEC should be on the order of the nK to preserve the axial coherence up to scales on the order of the size of the sample.

Conclusions

Non-trivial geometry effects of the quantum vacuum, such as the lateral Casimir-Polder atom-surface interaction, modify the energy spectrum of a BEC in close proximity to a corrugated surface. The qualitative differences in the lowest energy (phonon-like) band were characterized in this context and a possible experimental set up for measuring the effect was discussed. As we have shown, using Bragg spectroscopy to measure this effect seems challenging with present day technology but could become feasible in the near future, opening a new window on the physics of the interaction between surfaces and coherent matter.

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