Casimir-like drag forces in superfluids

Andrew Sykes,[1] Matthew Davis[1] and David Roberts[2]

[1] The University of Queensland, School of Mathematics and Physics, ARC Centre of Excellence for Quantum-Atom Optics, Qld 4072, Australia
[2] Theoretical Division and Center for Nonlinear Studies, Los Alamos national Laboratory, Los Alamos, New Mexico 87545, USA

Summary: We calculate the drag force acting on an impurity moving through a quasi-1D BEC using analytic solutions for the quantum fluctuations in the system [4]. We find a non-zero force arises for zero and finite temperature at all velocities due to an imbalance in the Doppler shift in the scattering of quantum and thermal fluctuations. The force is proportional to $\sqrt{mg/(\hbar^2 n)}$ and a cross-over from a thermal to a quantum regime occurs near $T \sim 10^{-3} gn_\infty$.

Introduction

One of the most remarkable properties of superfluidity is the emergence of a critical velocity, below which, an obstacle can move through a superfluid without experiencing any friction.

Landau's phenomenological explanation [1]: The excitation spectrum of the fluid may be such, that it only allows for the creation of quasiparticles above a critical velocity.

It is commonly believed that BEC lies at the heart of superfluidity. Experiments looking at BEC in dilute gases provide exciting opportunities to further our understanding of superfluidity.

Formalism

Hamiltonian in second-quantisation:

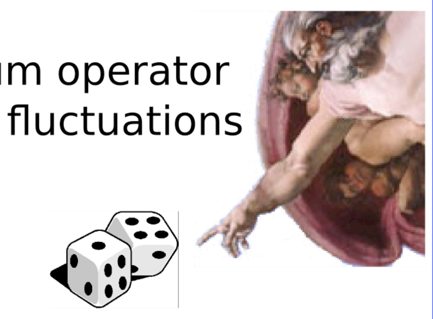
$$\hat{H} = \int dx\ \hat{\psi}^\dagger \left(-\frac{\hbar^2}{2m}\partial_x^2 + i\hbar v \partial_x + \eta\delta(x) + \frac{g}{2}\hat{\psi}^\dagger\hat{\psi} \right) \hat{\psi}.$$

where m is the mass of the atoms, v is the velocity of the impurity, η is the strength of the impurity, and g is the interaction strength between atoms.

Bogoliubov's perturbation expansion for a weakly interacting BEC:

$$\hat{\psi} = \psi + \hat{\delta}$$

condensate wavefunction — quantum operator for the fluctuations

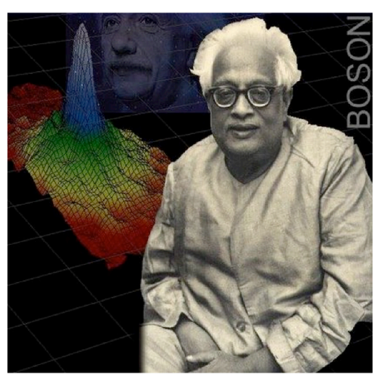


We wish to calculate the force acting on the impurity which is given by

$$F = -\langle \hat{\psi}^\dagger \nabla \left[\eta\delta(x)\right] \hat{\psi} \rangle$$

Gross-Pitaevskii equation

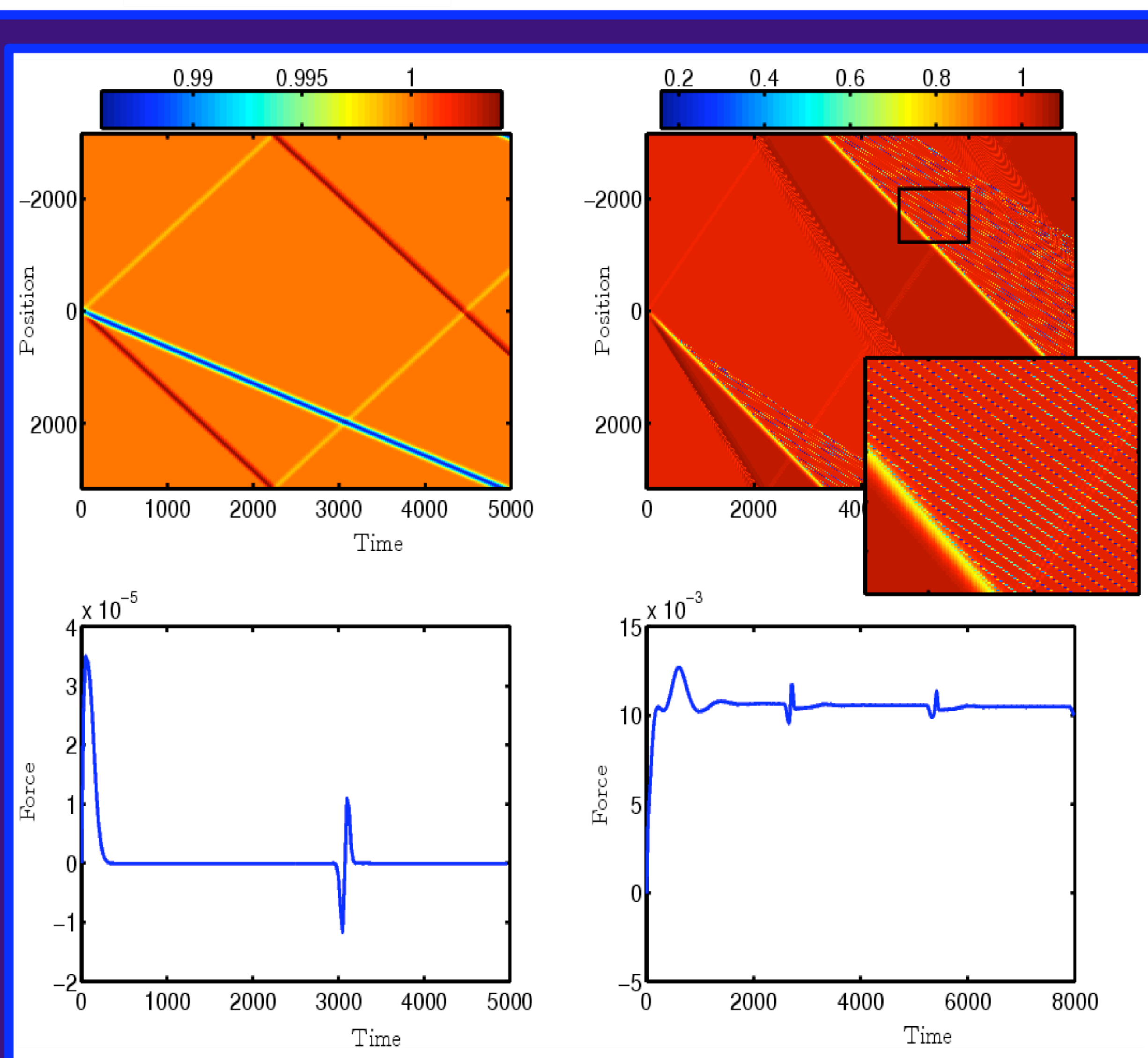
The condensate wavefunction usually contains the dominating effects in the system.



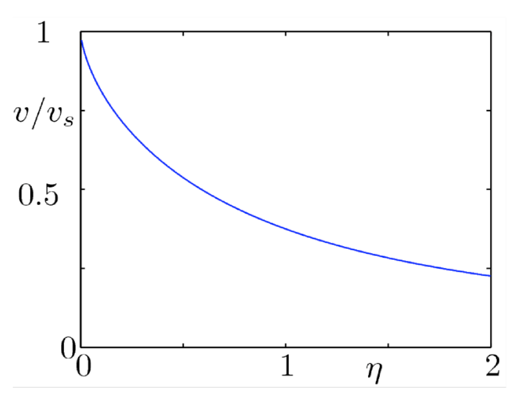
It is given by the Gross-Pitaevskii equation (also called the non-linear Schrodinger equation):

$$i\hbar\partial_t\psi_0 = \left[-\frac{\hbar^2}{2m}\partial_x^2 + iv\partial_x + \eta\delta(x) + g|\psi_0|^2 \right] \psi_0$$

The solution can be found analytically [2] for slow velocities, and numerically for faster velocities. The figure below shows the densities and forces acting on impurities moving through a condensate:



At the level of the Gross-Pitaevskii equation, the drag force is zero provided the velocity is less than a critical velocity (as shown in the left column above). Above the critical velocity one can observe soliton shedding in the wake of the impurity (as shown in the right column above). The critical velocity depends on the strength of the impurity, as shown in the figure below:



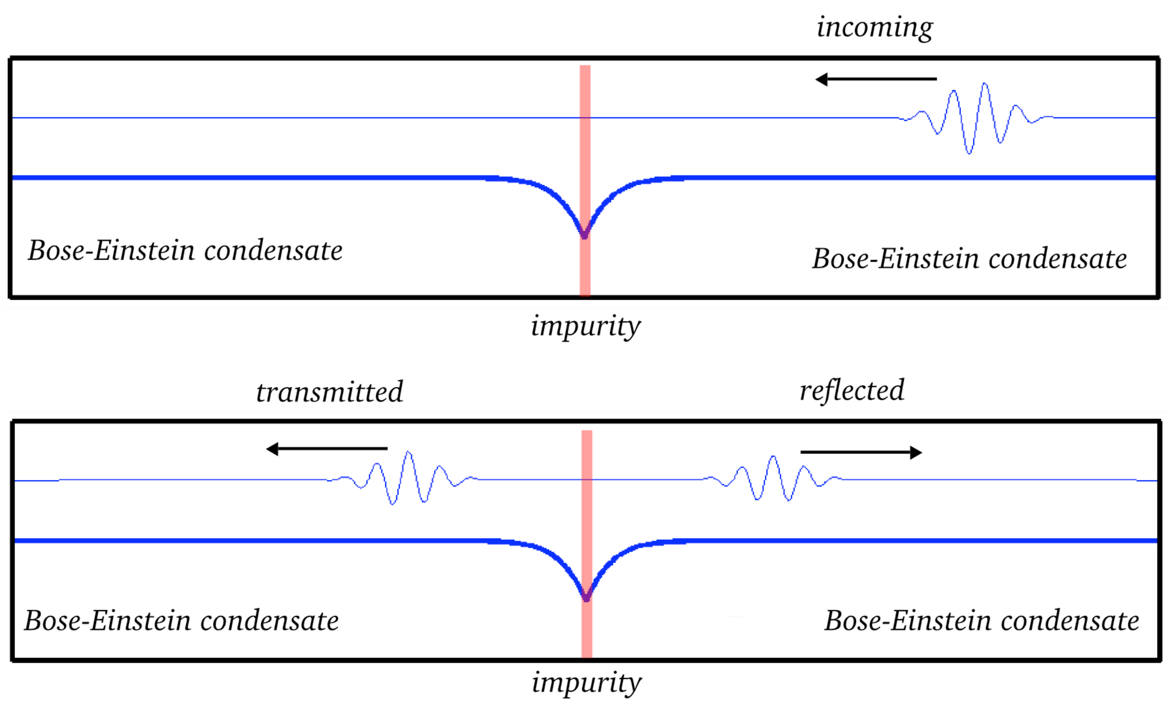
Bogoliubov-de Gennes equation

The fluctuations of the field gives the operator its quantum properties.

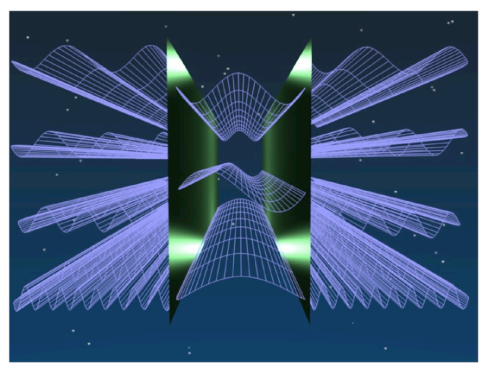
The equation governing these fluctuations is called the Bogoliubov-de Gennes equation.

$$i\hbar\partial_t\hat{\delta} = \left[-\frac{\hbar^2}{2m}\partial_x^2 + iv\partial_x + \eta\delta(x) + 2g|\psi_0|^2 - \mu_0 \right] \hat{\delta} + \psi_0^2\hat{\delta}^\dagger$$

The above equation defines a scattering problem [3]. Incoming excitations collide with the impurity and scatter into relected and transmitted modes.



Scattering occurs with incoming waves approaching from both the left and the right. When the impurity is moving relative to the condensate, there is an imbalance in the amount of momentum transferred to the impurity. This imbalance originates in the Doppler shifts of the reflected waves. The end result is: it generates a non-zero FORCE!



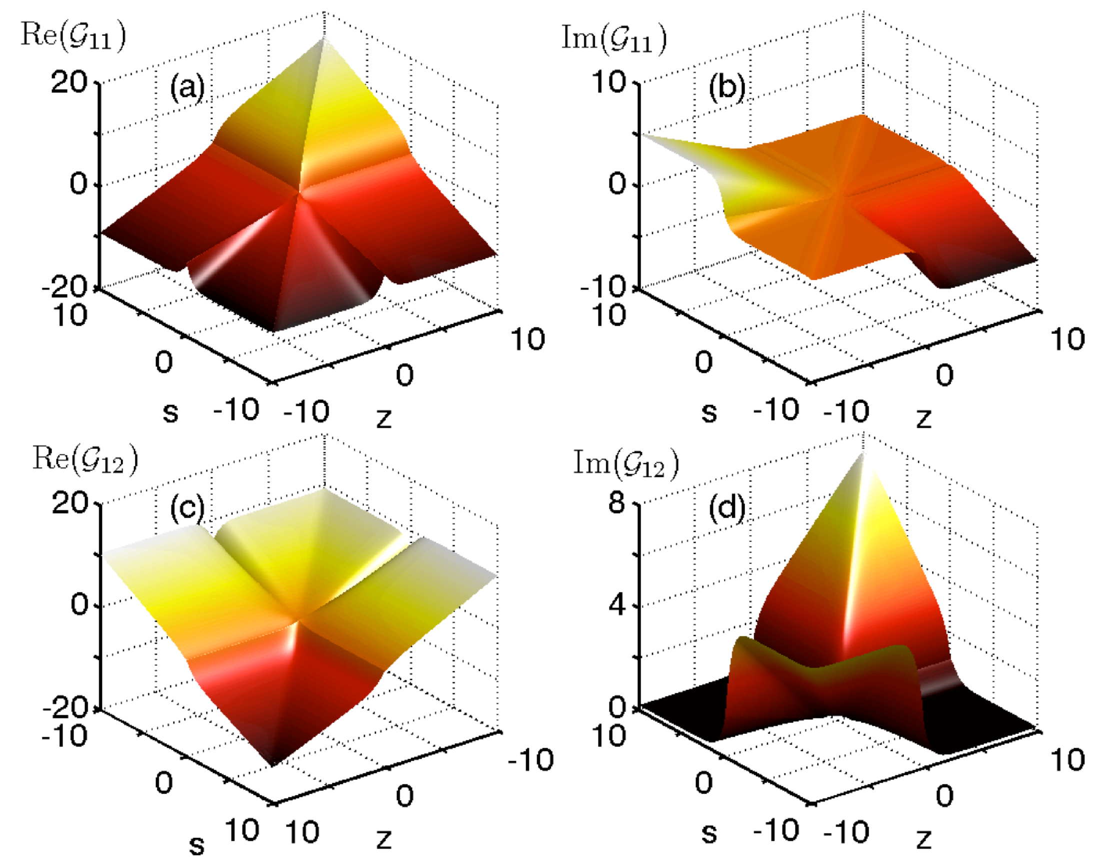
This is some what analagous to the casimir effect in the electromagnetic vacuum.

Generalised Gross-Pitaevskii equation

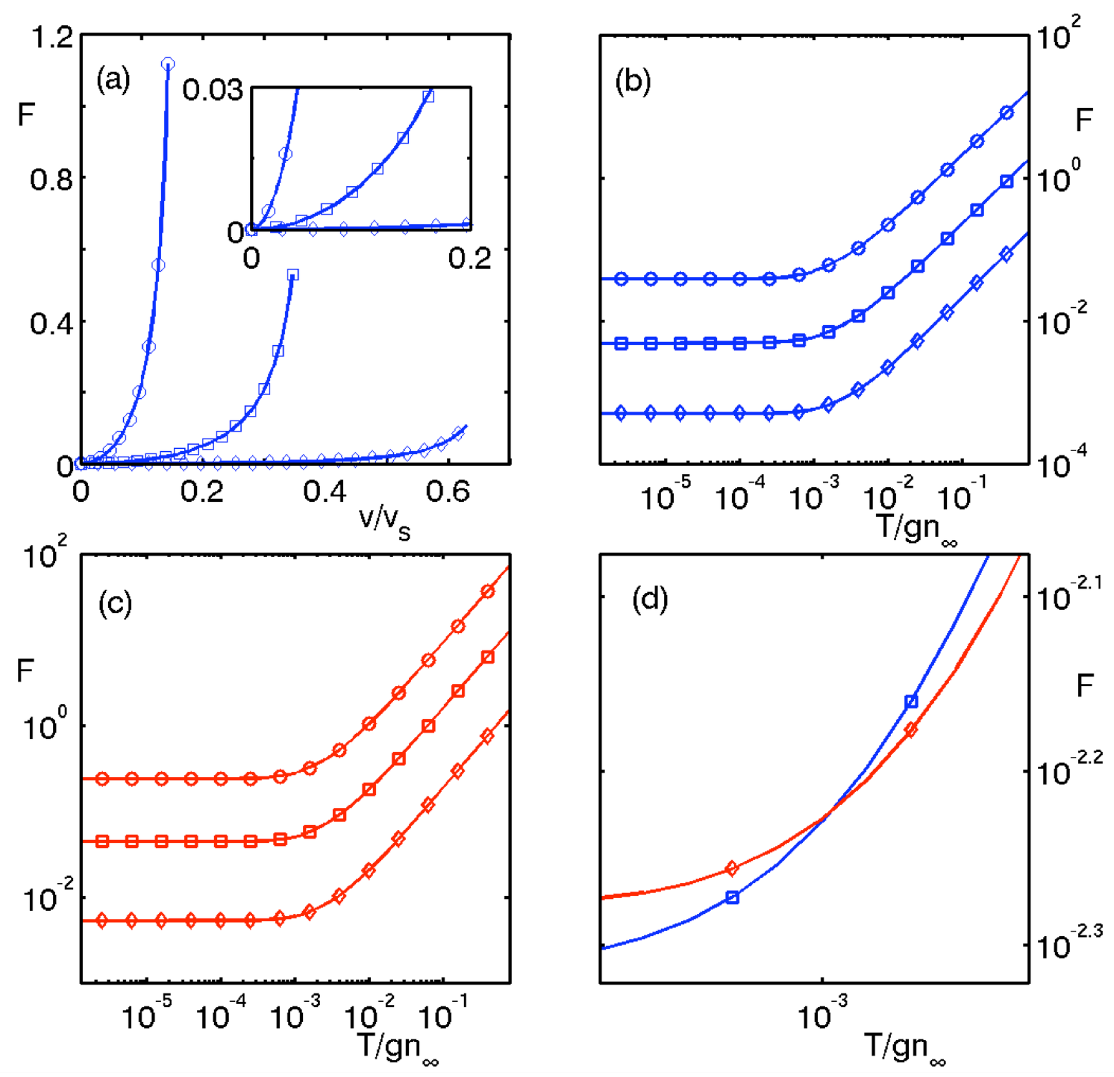
The final piece of the puzzle: What effect do the presence of the fluctuations have on the condensate wave-function? The answer lies in the so-called generalised-Gross-Pitaevskii equation:

$$\left[-\frac{\hbar^2}{2m}\partial_x^2 + iv\partial_x + \eta\delta(x) + 2g|\psi_0|^2 - \mu_0 \right] \psi_1 + g\psi_0^2\psi_1^* = g\langle\hat{\delta}\hat{\delta}\rangle\psi_0^* + 2g\langle\hat{\delta}^\dagger\hat{\delta}\rangle\psi_0$$

In order to solve this we derive a Greens matrix for the Bogoliubov-de Gennes equation. This involves finding all four zero eigenvalue solutions [4]. Difficult! ... but it looks quite pretty once you're done, and hopefully it will impress the girls.



Once the generalised GPE has been solved one can be quantitative about the size of the force, also, one can look at finite temperature fluctuations:



References:
[1] Landau, J. Phys. (Moscow) 5, 71 (1941)
[2] V. Hakim, Phys. Rev. E 55, 2835 (1997)
[3] X.-J Chen, et al. j. Phys. A 31, 6929 (1998)
[4] A. Sykes, M. Davis, and D. Roberts, Phys. Rev. Lett. 103, 0835302 (2009)