Long Range Interactions in Cylindrical Structures

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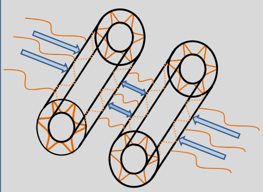
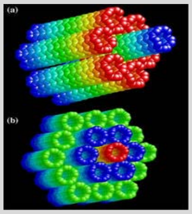
Abstract

We consider the interaction energy due to electromagnetic field fluctuations in various infinitely long cylindrical systems. The structures of interest are a single cylindrical layer with a finite thickness, *N* concentric infinitely thin shells, and two parallel full cylinders. In all cases, the mode summation method is applied to calculate the zero-point energy. The derived analytical expressions are used to investigate the energy dependence on the cylindrical radial curvature, size of the system, and the dielectric response properties of the involved objects and the medium. Of particular interest is the case of two parallel cylinders, for which we show that the interaction can be changed from attractive to repulsive by a suitable choice of the material composition of the cylinders and the environment. Our studies can serve as a test ground for future, more advanced theories of long ranged interactions in cylindrical systems. The presented results can also be viewed as a model of interactions due electromagnetic field fluctuations between tubular formations, such as nanotubes and nanowires.

Casimir Effect - Nanostructures

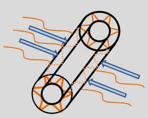




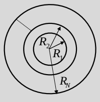
- Casimir force originates from vacuum fluctuations of the electromagnetic field and is a long-range dispersion force.
- Casimir force couples electrically neutral objects with no permanent electric and/or magnetic moments.
- Friction, adhesion, and wear are directly related to Casimir forces and become dominant at the nanoscale.
- Stability of carbon nanotube structures due to Casimir forces: bundles, ropes, multiwall tubes.
- Carbon nanotube applications due to Casimir force: nano-oscillator, rotor, energy storage, sensors, drug delivery.

[1] H. B. G. Casimir, Proc. K. Ned. Akad. Wet. ***51****, 793 (1948)*
[2] www.casimir.rl.ac.uk/measurements.htm
[3] www.physorg.com/news67008504.html
[4] K. Tatur, L.M. Woods, I.V. Bondarev, Phys. Rev. A 78 (2008) 012110; K. Tatur and L.M. Woods, Phys. Lett. A 372 (2008) 6705-67010.

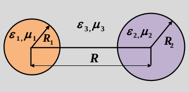
Cylindrical Structures



Cylindrical layer of finite thickness



Concentric metallic cylindrical shells of infinite thickness



Parallel solid cylinders

Models of different carbon nanotube systems

Mode Summation Method

$$E_C=\frac{\hbar}{2}\sum_{\{p\}}\left(\omega_p-\tilde{\omega}_p\right)$$

E_C - zero-point Casimir energy;

ω_p - the eigenfrequencies of the system;

$\tilde{\omega}_p$ - the eigenfrequencies with no boundaries present;

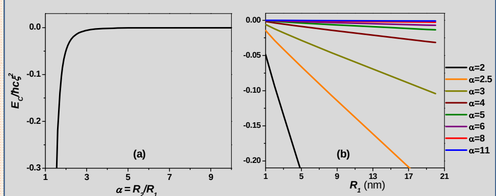
$\{p\}=(n,m,k_z)$ - quantum numbers for cylindrical geometry.

$$E_C(s)=\frac{\hbar c^{-s}}{4\sqrt{\pi}\,\Gamma\left(\frac{s}{2}\right)\Gamma\left(\frac{3-s}{2}\right)}\sum_{n=-\infty}^{\infty}\int_0^{\infty}dy\,y^{1-s}\frac{d}{dy}\ln\frac{f_n^{TE}(i\Re y)\,f_n^{TM}(i\Re y)}{f_n^{TE}(i\infty)\,f_n^{TM}(i\infty)}$$

$f_n^{TE,TM}(x)$ - dispersion relation for the transverse electric and magnetic modes.

Zero-Point Energy of a Cylindrical Layer

- Infinitely long cylindrical layer of finite thickness, with constant dielectric and magnetic properties (ε,μ) imbedded in an infinite medium with constant properties (ε_m,μ_m).
- Constant speed of light approximation: $\varepsilon\mu=\varepsilon_m\mu_m=c^{-2}$ (dielectric-diamagnetic system)



Interaction energy per unit length for the cylindrical dielectric layer as a function of **(a)** – the ratio of the outer and inner radii R_2/R_1, **(b)** – the inner radius R_1.

Discussions

- Successful in removing divergences for all models and obtained finite results for the zero-point energy.
- Casimir energy as a function of curvature, distance separation and dielectric properties.
- Repulsive interaction for two solid parallel cylinders can be achieved for specific choices of the dielectric constants- $\varepsilon_1(\varepsilon_2)>\varepsilon_3>\varepsilon_2(\varepsilon_1)$

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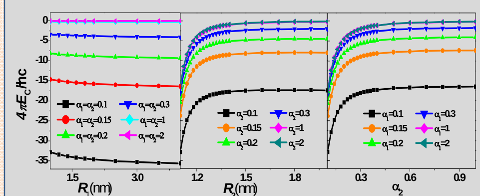
Zero-Point Energy of *N* concentric shells

- *N* concentric infinitely long, infinitely thin, perfectly conducting cylindrical shells in an infinite medium.

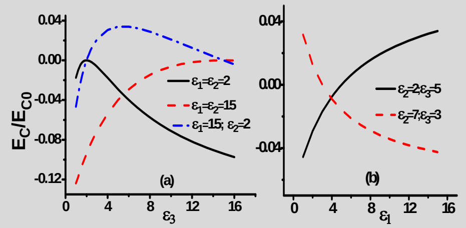


Interaction energy per unit length for the case of *N=3* shells: (a) as a function of the inner radius R_1; (b) as a function of the radius of the second shell R_2; and (c) as a function of separation between the two outer shells.

Here $\alpha_1=R_2/R_1$ and $\alpha_2=R_3/R_2$

Zero-Point Energy of two solid parallel cylinders

- Two infinitely long solid parallel cylinders, with constant dielectric and magnetic properties (ε_1,μ_1) and (ε_2,μ_2) imbedded in an infinite medium with properties (ε_3,μ_3).
- Constant speed of light approximation: $\varepsilon_1\mu_1=\varepsilon_2\mu_2=\varepsilon_3\mu_3=c^{-2}$ (dielectric-diamagnetic system)



Dimensionless interaction energy as a function of the a) dielectric function of the medium, and b) dielectric function of one cylinder. E_{C0} is defined as $\hbar c l/\pi R_1^2$. The cylinders have equal radii, $R_1=R_2=1\text{nm}$ and center-to-center separation is $R=2.2\text{nm}$.