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Repulsive Casimir Force in Chiral Metamaterials

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Background

- **Casimir Force:** When two neutral conducting surfaces separated by a vacuum are very close to each other, they generate an attractive force between them [1]. It arises from the quantum fluctuations of the vacuum field.
- **Three approaches to get Repulsive Casmir Force:**
  - Dzyaloshinskii's Casimir Repulsion [2, 3]:
    $\varepsilon_1(i\xi)$ | $\varepsilon_3(i\xi)$ | $\varepsilon_2(i\xi)$; ($\varepsilon_1(i\xi)<\varepsilon_3(i\xi)<\varepsilon_2(i\xi)$)
    *This system still has friction because of the existence of the liquid $\varepsilon_3(i\xi)$.
  - Boyer's Casimir repulsion [4, 5]:
    mainly (purely) electric | vacuum | mainly (purely) magnetic
    *Nontrivial magnetic materials in the optical regime do not exist in nature.
  - Leonhardt's Casimir repulsion [6]:
    plate | perfect lens + vacuum | plate
    *It is extremely difficult to obtain a perfect lens in a broadband range, and impossible at all frequencies.

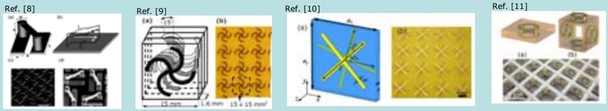
New approach

We obtain repulsive Casimir forces by using chiral metamaterials (CMMs) [7].

CMM | Vacuum | **CMM**

Chiral Metamaterials

Chiral metamaterials: The material lacks any planes of mirror symmetry.



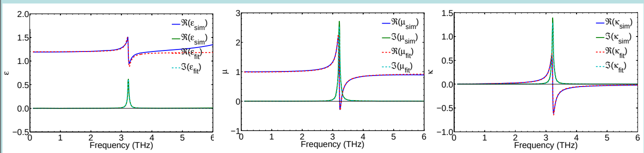
Constitutive equations: a cross coupling between the electric and the magnetic fields along the same direction

$$\begin{pmatrix}\mathbf{D}\\ \mathbf{B}\end{pmatrix}=\begin{pmatrix}\varepsilon_0\varepsilon & i\kappa/c_0\\ -i\kappa/c_0 & \mu_0\mu\end{pmatrix}\begin{pmatrix}\mathbf{E}\\ \mathbf{H}\end{pmatrix}$$

Physical Realistic Constitutive parameters:

$$\varepsilon(\omega)=1-\frac{\omega_{pl}^2}{\omega^2+i\gamma_{pl}\omega}-\frac{\omega_e^2}{\omega^2-\omega_R^2+i\gamma_R\omega},\ \mu(\omega)=1+\alpha-\frac{A\omega^2}{\omega^2-\omega_m^2+i\gamma_m\omega},\ \kappa(\omega)=\frac{\omega_\kappa\omega}{\omega^2-\omega_{\kappa R}^2+i\gamma_\kappa\omega}.$$

- The ω^2 dependence of $\mu(\omega)$ and the linear ω dependence of $\kappa(\omega)$ are derived from the equivalent circuit approach [12]. The linear ω dependence of $\kappa(\omega)$ is analogy to the Condon model for homogenous chiral molecular media [13]; the linear frequency dependence in low frequencies is also a general feature of natural optically active materials [14].
- One example of a real chiral metamaterials [12]:



Physical requirement:

$$\varepsilon(\infty)\to 1\ ;\quad \begin{cases}\mu(\infty)=1\Rightarrow\alpha=A;\\ \mu(0)\approx 1\Rightarrow\alpha\text{ is very small}\end{cases}$$

Theory

Casimir energy per unit area:

$$\frac{E(d)}{A}=\frac{\hbar}{2\pi}\int_0^\infty d\xi\int\frac{d^2\mathbf{k}_\parallel}{(2\pi)^2}\ln\det(1-\mathbf{R}_1\cdot\mathbf{R}_2e^{-2Kd}),\quad \mathbf{R}_j=\begin{vmatrix}r_j^{ss} & r_j^{sp}\\ r_j^{ps} & r_j^{pp}\end{vmatrix},\quad K=\sqrt{\mathbf{k}_\parallel^2+\xi^2/c^2}.$$

This integration is over imaginary frequency $(\xi=-i\omega)$.

Reflection elements:

$$r_j^{ss}=\frac{-\Gamma_-(\chi_++\chi_-)-(\chi_+\chi_--1)}{\Gamma_+(\chi_++\chi_-)+(\chi_+\chi_-+1)},\ r_j^{pp}=\frac{\Gamma_-(\chi_++\chi_-)-(\chi_+\chi_--1)}{\Gamma_+(\chi_++\chi_-)+(\chi_+\chi_-+1)},$$

$$r_j^{sp}=\frac{i(\chi_+-\chi_-)}{\Gamma_+(\chi_++\chi_-)+(\chi_+\chi_-+1)},\quad r_j^{ps}=-r_j^{sp}$$

$$\chi_\pm=\frac{K_\pm}{n_\pm K},\ \Gamma_\pm=\frac{\eta_0^2\pm\eta_j^2}{2\eta_0\eta_j},\ K_\pm=\sqrt{k_\parallel^2+n_\pm^2\xi^2/c^2},\ n_\pm=\sqrt{\varepsilon\mu}\pm\kappa,$$

$$\eta_0=\sqrt{\mu_0/\varepsilon_0},\ \eta_j=\sqrt{\mu_0\mu_j/\varepsilon_0\varepsilon_j}.$$

Even $n_\pm(i\xi)$ are complex, the reflection elements are still purely real because $\chi_+=\chi_-^*$.

For simplicity, consider a special case with two identical CMM plates:

$$\varepsilon_1=\varepsilon_2=\varepsilon,\ \mu_1=\mu_2=\mu,\ \kappa_1=\kappa_2=\kappa.$$

A negative value of the Casimir force is favored by making the quantity:

$$I\equiv\mathrm{Tr}\left[\frac{\mathbf{R}_1\cdot\mathbf{R}_2e^{-2Kd}}{1-\mathbf{R}_1\cdot\mathbf{R}_2e^{-2Kd}}\right]$$

as negative as possible over as broad a range as possible. The quantity I has the same sign as the quantity:

$$J=\frac{(r_{ss}^2+r_{pp}^2-2r_{sp}^2)e^{-2Kd}-2(r_{sp}^2+r_{ss}r_{pp})^2e^{-4Kd}}{1-(r_{ss}^2+r_{pp}^2-2r_{sp}^2)e^{-2Kd}+(r_{sp}^2+r_{ss}r_{pp})^2e^{-4Kd}}$$

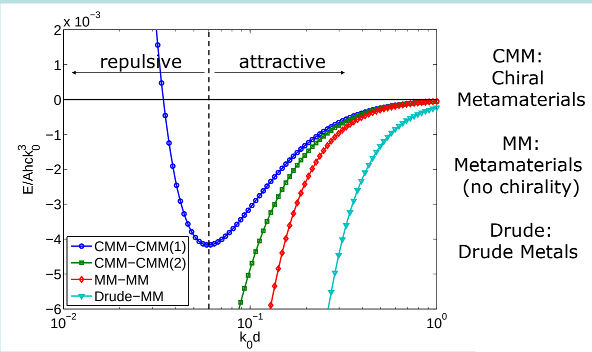
If r_{sp} is large enough, we can get $J<0$.
For example:

$$2r_{sp}^2>r_{ss}^2+r_{pp}^2\Rightarrow J<0$$

Therefore, we need a **large enough chirality κ.**

Realistic Example

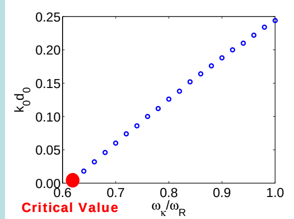
Casimir energy VS the distance between two plates



Casimir interaction energy per unit area E/A (in units of hck_0^3) versus k_0d; $k_0=\omega_R/c$. The triangle curve corresponds to $\alpha=A=0.001$, $\kappa=0$ (no chirality), $\omega_m=\omega_R$, $\omega_{pl}=0$, $\omega_e=\omega_R$ for material 1, while $\alpha=A=0$, $\omega_{pl}=10\omega_R$, $\omega_e=0$ for material 2. The diamond curve is the case with $\alpha=A=0.001$, $\kappa=0$, $\omega_m=\omega_R$, $\omega_{pl}=0$, $\omega_e=\omega_R$. The squares curve is the case with $\alpha=A=0.001$, $\omega_{\kappa1}=\omega_{\kappa2}=0.6\omega_R$, $\omega_m=\omega_{\kappa R}=\omega_R$, $\omega_{pl}=0$, $\omega_e=\omega_R$. Finally, the circle curve shows repulsion for $k_0d<0.0586$ and a stable equilibrium point at $k_0d=0.0586$; the parameters are the same as for the square curve except for $\omega_{\kappa1}=\omega_{\kappa2}=0.7\omega_R$. All the γ's equal $0.05\omega_R$ except $\gamma_{pl}=0.05\,\omega_{pl}$.

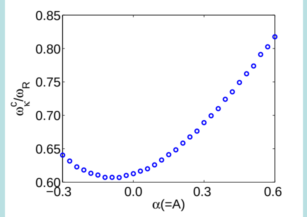
Critical Value of Chirality

Equilibrium Distance VS Chirality Strength



(a) The equilibrium distance k_0d_0 versus ω_κ for $\alpha=A=10^{-3}$. The critical value of chirality is $\omega_\kappa{}^c=0.612\omega_R$.

Critical Chirality VS Magnetic Resonance Strength



(b) The critical value of chirality $\omega_\kappa{}^c$ versus $\alpha(=A)$, for two identical CMM plates.

Therefore, we can get repulsive Casimir force
if the chirality exceeds the critical value

Conclusion and Future

In this work we have extended the Lifshitz theory to calculate the Casimir force by including chirality terms for the first time. We have shown that the chirality, if strong enough, is of critical importance in producing nanolevitations under realistic frequency dependence. The CMMs might possibly be the main candidates to achieve experimentally the goal of Casimir repulsion, which might open up many opportunities for application.

In the future, we will pursue new CMM designs to raise the value of ω_κ possibly above the critical value.

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