# Thermal effects in the magnetic Casimir-Polder interaction

Harald Haakh, Francesco Intravaia, Carsten Henkel. Institut für Physik und Astronomie, Universität Potsdam, Karl-Liebknecht-Str. 24/25, D-14476 Potsdam.
Contact: haakh@uni-potsdam.de

German-Israeli Foundation for Scientific Research and Development

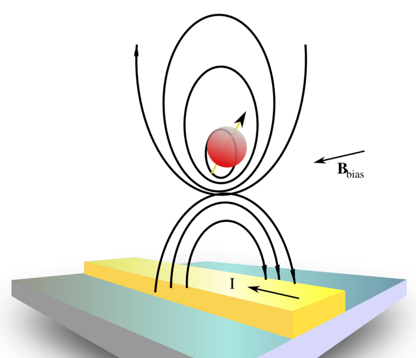
## Introduction

### Modern microtrap experiments [1,2]

- require exact knowledge of the Casimir-Polder (CP) interaction between atoms and conductors.
- **magnetic fluctuations** play an important role in the **trap stability** [3,4].

### This work:

Magnetic dipole contribution to the atom-surface interaction.

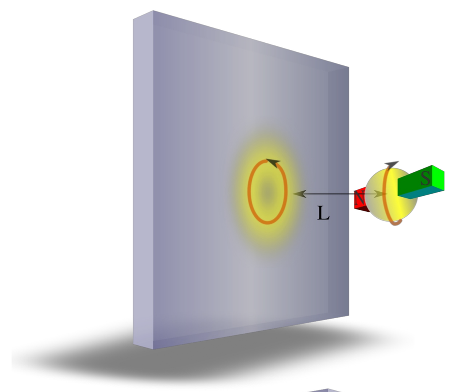



### Setups:

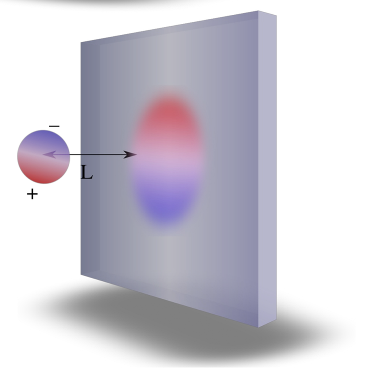
- Metal or superconducting surfaces
- Thermally excited atoms
- Ground state atoms
- Atoms prepared in a trappable hyperfine state.

### Electric and magnetic dipole coupling

**Magnetic dipole**
- interaction is dominated by **surface currents**,
- subject to **ohmic losses**.




**Electric dipole**
- coupling is determined by **surface charges**.
- **Similar in all conductors**.




**Separation** between the contributions:
- **differential measurements**, using isotopic or Zeeman shift.
- **Rydberg atoms.**

## Main results

- **Electric and magnetic surface forces differ strongly**.
Magnetic coupling shows important features not present in the electric case.

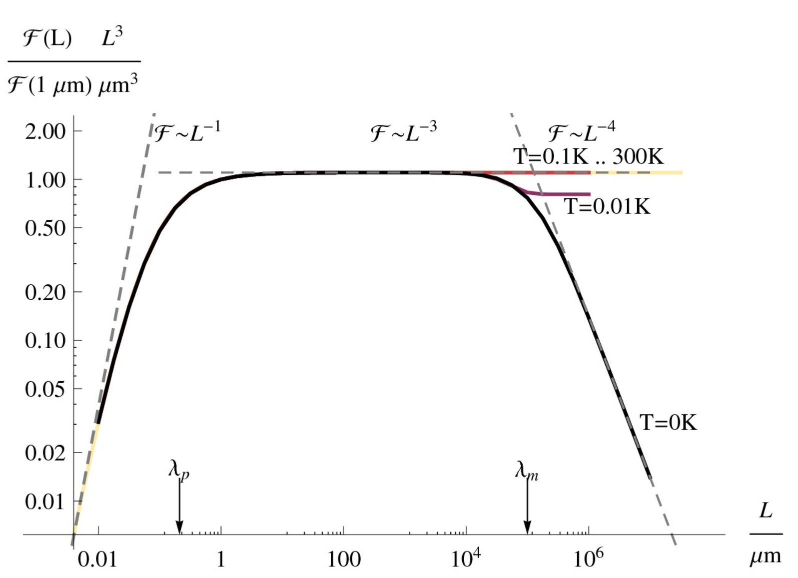
- **Magnetic coupling is highly sensitive to dissipation**.
Thermal decoupling allows for precise tests of cavity QED.

- **Strong resemblance to two-plate Casimir interaction**
New ways to decide open questions in the thermal Casimir effect experimentally.

## Interaction potential at thermal equilibrium

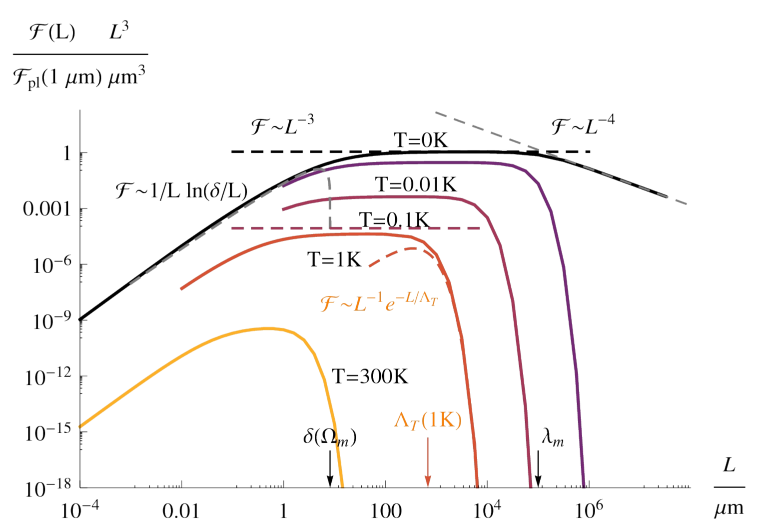
### Plasma model

- Completely **repulsive** force.
- **Thermal enhancement** at large L>>$\Lambda_T$.
- Quick **convergence to the high** T limit.



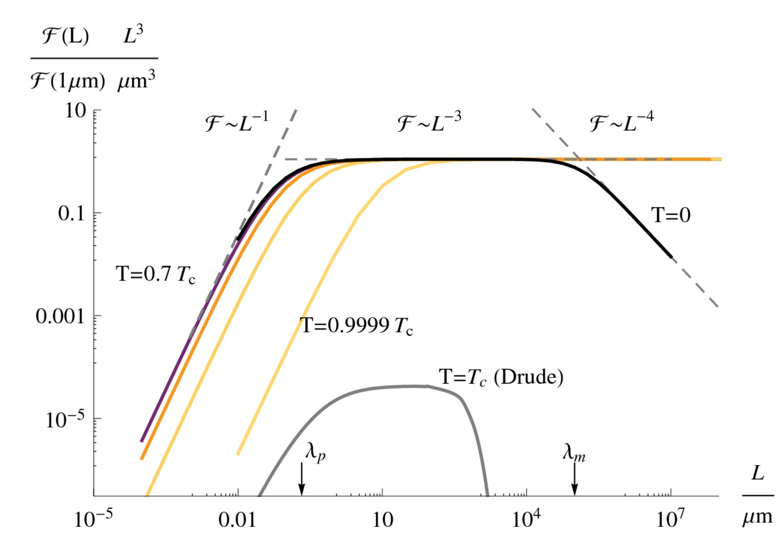

### Drude model

- **Thermal decoupling** at L>>$\Lambda_T$, not anticipated from T=0 [7].
- **Logarithmic correction** at small L w.r.t. the plasma.




### Superconductors

- Growth of the **effective plasma wavelength** as T / $T_c$: Small L curves move towards Drude limit.
- Rapid change from plasma to Drude behavior.
- Sudden suppression of large distance CP interaction at the **onset of thermal decoupling**.

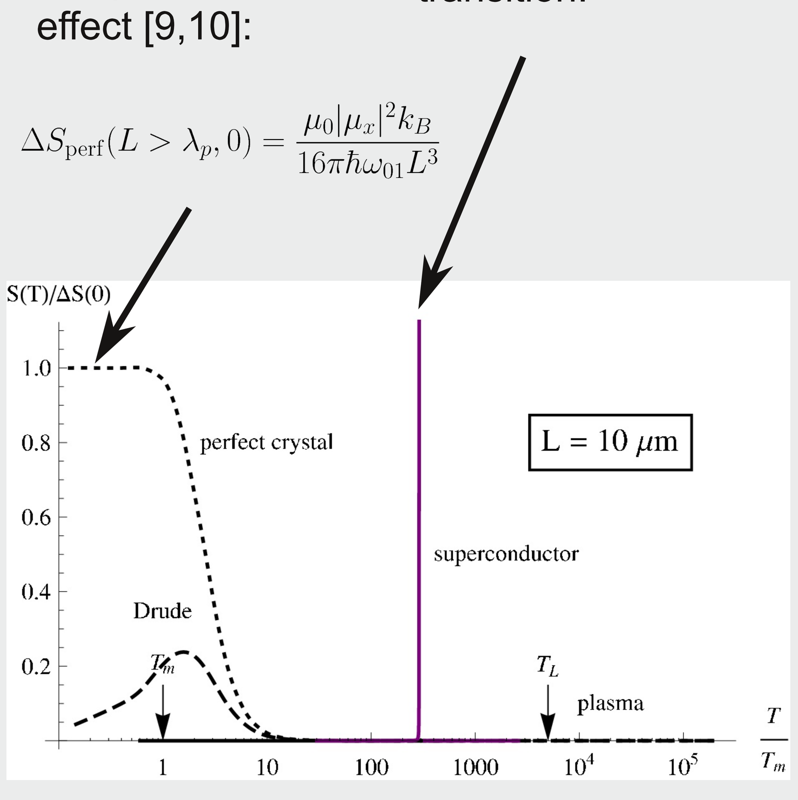



### Entropy

**Residual entropy at T=0** in the perfect crystal as in the two-plate Casimir effect [9,10]:

**Entropy cusp** at $T_c$: Participation of the atom in the phase transition.

$$\Delta S_{\rm perf}(L > \lambda_p, 0) = \frac{\mu_0 |\mu_x|^2 k_B}{16\pi\hbar\omega_{01} L^3}$$

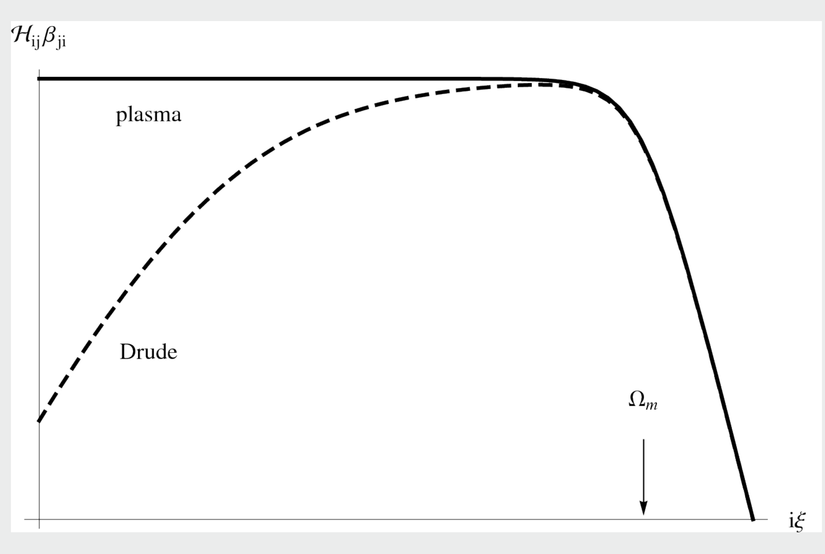



### Thermal decoupling

Strong impact of dissipation

→ **Asymptotic transparency** for low frequency magnetic fields in dissipative media (Bohr-van Leeuwen theorem [11]).

→ **Thermal decoupling**: Strong decay of the magnetic CP interaction at large T, L.




## Calculation of the Casimir-Polder interaction

### Surface response

- Encoded in dielectric functions $\epsilon(\omega)$.
- Reflection amplitudes: local (Fresnel) approximation.

**Plasma model**

$$\epsilon_{\rm Pl}(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

**No ohmic dissipation**, concides with a superconductor at T=0. Response lacks causality.

**Drude model**

$$\epsilon_{\rm Dr}(\omega) = 1 - \frac{\omega_p}{\omega(\omega + i\gamma)}$$
$$\gamma = const.$$

Dissipation dominated by **impurity scattering**, independent of T.

**Perfect crystal**

$$\gamma = \gamma(T)$$

Dissipation through **electron-electron** or **electron-phonon scattering** (Bloch-Grüneisen law).

**Superconductors**

Two-fluid model [5]

$$\epsilon(\omega, T) = \eta(T)\epsilon_{\rm Pl}(\omega) + [1 - \eta(T)]\epsilon_{\rm Dr}(\omega)$$
$$\eta(T) = \left[1 - (T/T_c)^4\right]\theta(T_c - T)$$

**Good agreement with BCS calculations [6]** for realistic values of γ, $T_C$ and the BCS-Gap.

Parameters used in numerics:

$$\omega_p = 8.95 \cdot 10^{16}{\rm s}^{-1}$$
$$\gamma = 1 \cdot 10^{-2}\omega_p$$
$$\Omega_m = 3 \cdot 10^{9}{\rm s}^{-1}$$
$$T_c = 13{\rm K}$$
$$\mathcal{F}_{pl}(1\mu{\rm m}, 0{\rm K}) = 9.8 \cdot 10^{-37}{\rm J}$$

### Interaction free energy

The interaction free energy is obtained from thermal linear response theory [7]:

$$\mathcal{F}(L,T) = -k_B T \sum_{n=0}^{\infty}{}' \beta^a_{ij}(i\xi_n, T)\mathcal{H}_{ji}(L, i\xi_n) + \sum_b n(\omega_{ba})\mu_i^{ab}\mu_j^{ba}{\rm Re}\mathcal{H}_{ji}(L,\omega_{ba}) \,,$$

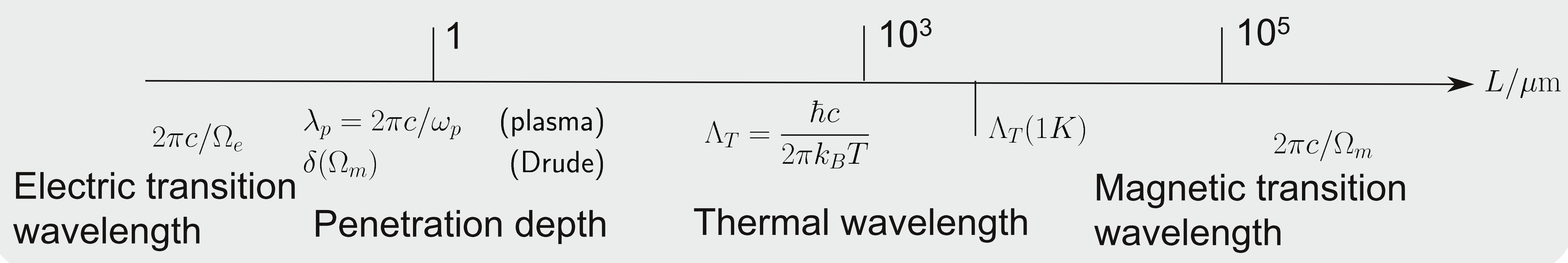
Green tensor
$$\boldsymbol{\mathcal{H}}(L,\omega) = \frac{1}{4\epsilon_0 c^2}\int\frac{d^2k}{(2\pi)^2}\kappa\left[\left(r^{TE}(\omega) + \frac{\omega^2}{c^2\kappa^2}r^{TM}(\omega)\right)[\hat{\boldsymbol{x}}\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}}\hat{\boldsymbol{y}}] + 2\frac{k^2}{\kappa^2}r^{TE}(\omega)\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}\right]e^{-2L\kappa} \,,$$

Equilibrium polarizability
$$\beta_{ij}(\omega,T) = \sum_{a,b}\frac{\mu_i^{ab}\mu_j^{ba}}{\hbar Z}e^{-\frac{\hbar\omega_{a0}}{k_B T}}\frac{2\omega_{ba}}{\omega_{ab}^2 - (\omega + i0^+)^2},$$

$\mu_i^{ab}$ dipole matrix element, $\kappa = \sqrt{k^2 + \frac{\xi^2}{\omega^2}}$.

### Length scales

Separation of scales → asymptotic expansions [8,12]



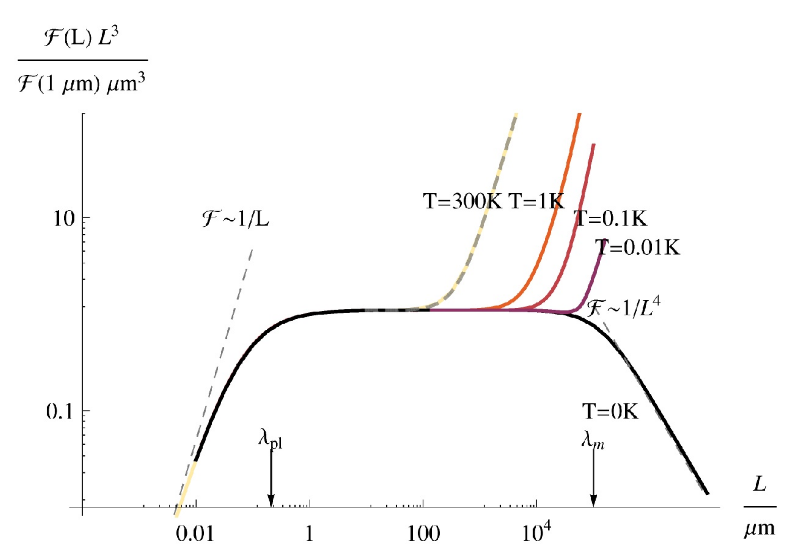

## Interaction of non-thermal atoms

The atom is prepared in a well-defined state; surfaces and fields are in thermal equilibrium at T.

### Ground state two-level atom

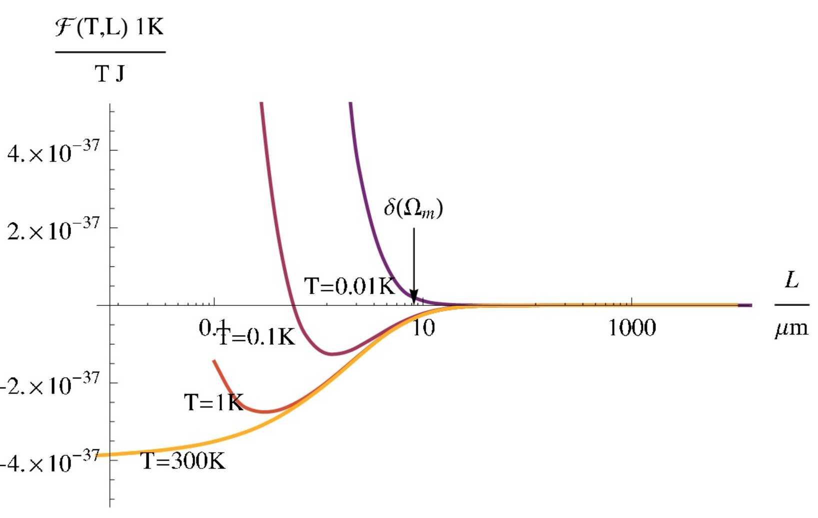
**Plasma model**

- Completely **repulsive** force.
- **Thermal enhancement** at large L >> $\Lambda_T$.
- Small distance limit **independent of T**.



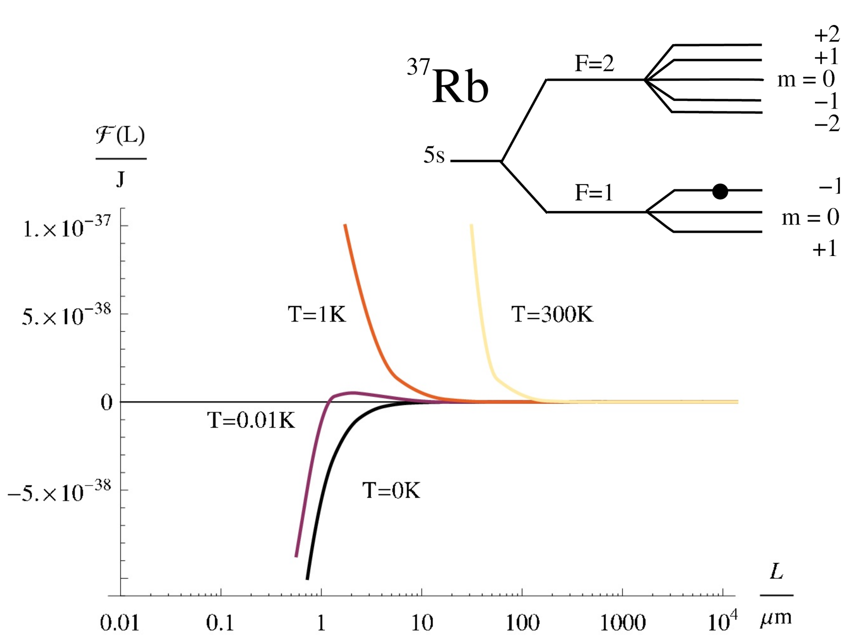

**Drude model**

- **Potential minimum:** transition from repulsive to attractive regime.
- Repulsive vacuum interaction at low T.
- Dominating attractive (resonant) thermal contribution, asymptotically linear in T.




### Rubidium atom in trappable hyperfine state

- **Population inversion** yields **global minus** w.r.t. ground state atom.
- **Potential barrier**: transition from attraction to repulsion.
- Effects occur in **experimentally accessible** ranges of temperature and distance.





## References

[1] D. M. Harber, J. M. Obrecht, J. M. McGuirk and E. A. Cornell, Phys. Rev. A 72 (2005) 033610
[2] T. Nirrengarten et al., Phys. Rev. Lett. 97 (2006) 200405
[3] C. Henkel, S. Pötting and M. Wilkens, Appl. Phys. B 69 (1999) 379
[4] K. Joulain, R. Carminati, J.-P. Mulet and J.-J. Greffet, Phys. Rev. B 68 (2003) 245405
[5] J. R. Schrieffer, "Theory of superconductivity" (Perseus Books, Reading MA 1999)
[6] G. Bimonte, H. Haakh, C. Henkel and F. Intravaia, arXiv:0907.3775 (2009)
[7] J. M. Wylie and J. E. Sipe, Phys. Rev. A 32 (1985) 2030
[8] C. Henkel, B. J. Power and F. Sols, J. Phys.: Conf. Ser. 19 (2005) 34
[9] K. A. Milton, J. Phys. A: Math. Gen. 37 (2004) R209;
G. L. Klimchitskaya and B. Geyer, J. Phys. A: Math. Gen. 41 (2008) 164032
[10] F. Intravaia and C. Henkel, J. Phys. A: Math. Gen. 41 (2008) 164018
[11] G. Bimonte, Phys. Rev. A 79 (2009), 042107
**[12] More Details: H. Haakh, F. Intravaia, C. Henkel, S. Spagnolo, R. Passante, B. Power, F. Sols, in preparation.**