Multi-channel soliton collisions

Solitons and solitary waves have been the subjects of intensive research efforts in recent years since they appear in a wide variety of nonlinear phenomena in fields ranging from plasma physics and nonlinear optics to solid state physics and particle physics. In many of these nonlinear systems, random processes play an important role in the dynamics of solitons. Of particular interest are optical solitons, due to their potential application as information carriers in long distance communications systems and in other nonlinear optics systems. In optical fiber communication systems, a sequence of solitons launched into the fiber forms a bit pattern that codes the transmitted message. Each soliton in the sequence is positioned at the center of time slot that is allocated for the bit.

In an ideal fiber each soliton in the bit pattern propagates without any changes in its parameters and without emitting any radiation owing to the exact balance between second order dispersion and Kerr nonlinearity. In practice, however, certain physical processes break this ideal picture and lead to the deterioration of the bit patterns and eventually, to loss of information. Of these, physical processes that involve noise and disorder appear to be the most prominent. In this case, the soliton parameters become random variables and one needs to calculate their statistics. The performance of optical fiber communication systems is characterized by Bit-Error-Rate (BER), which represents the probability for an incorrect identification of a bit at the system output. Estimation of the BER are usually based on the assumption that the statistics of the soliton parameters can be approximated by Gaussian distributions. However, since the standards of telecommunication systems require the BER to be as small as 10^{-9}, estimations of the BER are very sensitive to the statics of the soliton parameters and deviations from Gaussian statistics can lead to significant errors in the BER evaluation. Therefore, an accurate calculation of the statistics of soliton parameters in optical fiber transmission systems is of utmost importance.

There exist many types of disorder effects in optical fibers, which includes amplifier noise, chromatic dispersion disorder, birefringent disorder, and electrostriction. In all these cases, the temporal separation between two solitons in the pulse sequence becomes a Gaussian random variable. The variance of the corresponding Gaussian distribution grows with the third power of propagation distance in the cases of birefringence disorder [1],[2] and disorder in the second order dispersion coefficient [3].

A different type of randomness appears in optical fiber transmission systems with multiple frequency channels (see e.g., Fig. 1).

Transmission of information through such systems is advantageous since it allows for sending many pulse sequences (as many as 100 pulses) through the same optical fiber. The solitons in each frequency channel in these systems propagate with the same group velocity, but the group velocity is different for solitons from different channels. As a result, collisions between solitons from different channels are very frequent. In an ideal case, these interchannel collisions are elastic, that is, the soliton amplitude, frequency and shape do not change by the collisions and no radiation is emitted. In real optical fibers, however, this ideal elastic nature of the collisions breaks down due to the presence of additional physical processes that can be considered as high order corrections to the ideal Nonlinear Schrödinger equation (Fig. 2). In this case interchannel soliton collisions can lead to changes in the soliton amplitude and frequency, radiation emission, corruption of the shape of other undesirable effects. Once soliton propagation under multiple interchannel collisions is considered, randomness appears quite naturally due to the quasi-random nature of soliton sequences in different frequency channels. In this study, we focus our attention

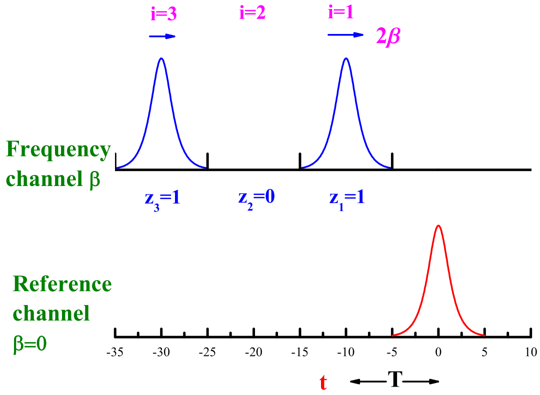

Figure 1: Soliton propagation in multiple frequency channel.

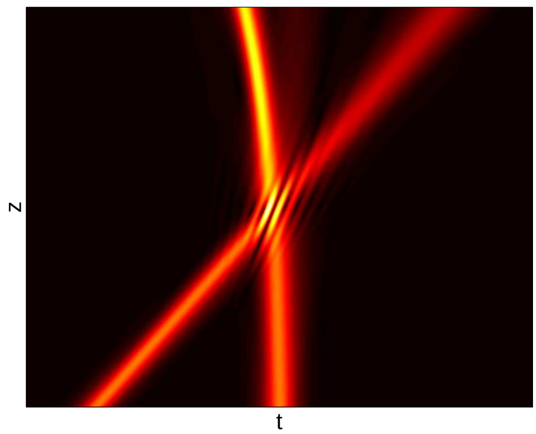

Figure 2: Energy exchange in single collision in the presence of Raman response.

on the influence of delayed Raman response on soliton dynamics under multiple interchannel collisions, since it is dominant compared to other inelastic effects in long distance multi-channel transmission systems. The main effect of delayed Raman response on single soliton propagation is the self-frequency shift, which is caused by energy transfer from higher frequency components of the pulse to its lower frequency components. In multi-channel transmission systems, we show that as a result of the random nature of the pulse sequences and the Raman induced cross talk, the soliton amplitude becomes a random variable with a lognormal distribution [4, 5] (Fig. 3).

$$F(\eta_0) = (8\pi\mathcal{D}\epsilon_R^2 z)^{-1/2}\eta_0^{-1}\exp\left[-\frac{\ln^2(\eta_0)}{8\mathcal{D}\epsilon_R^2 z}\right] \quad (1)$$

This distribution function is *different* from the Gaussian distribution function, which is usually encountered in situations in fiber optics communication systems where noise or disorder exist. As a result of this difference, a naive Gaussian approximation for $F(\eta_0)$ will simply not work. The most important property of the distribution function $F(\eta_0)$ is that its moments grow exponentially with both n and z: $\langle\eta_0^n\rangle = \exp(2n^2\mathcal{D}\epsilon_R^2 z)$.

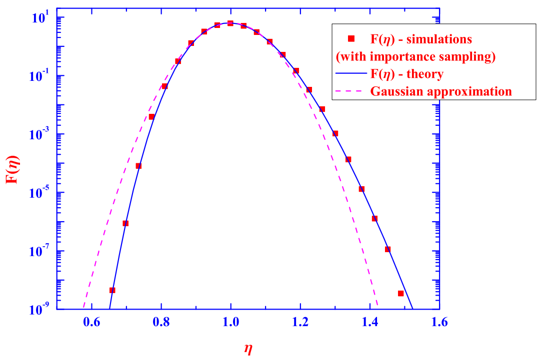

Figure 3: Probability distribution function of soliton amplitude in reference channel.

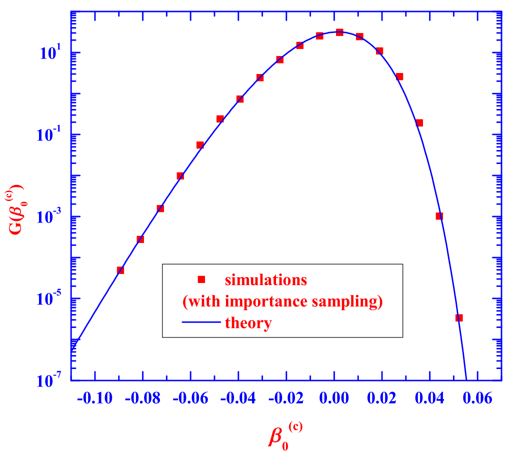

Figure 4: Probability distribution function of soliton frequency in reference channel.

In particular, the variance $\sigma_{\eta 0}^2$ is given by

$$\sigma_{\eta 0}^2 = \exp(4\mathcal{D}\epsilon_R^2 z)\left[\exp(4\mathcal{D}\epsilon_R^2 z) - 1\right] , \quad (2)$$

from which it follows that η_0 is not self-averaging. Moreover, we showed that the soliton frequency is a random variable that is not self-averaging whose distribution functions is also lognormal [4, 5] (Fig. 4),

$$G(\beta_0^{(c)}) = \frac{\exp\left\{-\ln^2\left[1 - 3|\beta|\beta_0^{(c)}/2\right]/(32\mathcal{D}\epsilon_R^2 z)\right\}}{(32\pi\mathcal{D}\epsilon_R^2 z)^{1/2}|\beta_0^{(c)} - 2/(3|\beta|)|}. \quad (3)$$

Eq. (3) suggests that fluctuations in soliton amplitude and frequency play a very important role in multi-channel optical fiber transmission systems.

It is interesting to compare the effects of the multi-channel disorder induced by solition collisions in the presence of delayed Raman response with the effects of other types of disorder that already exist in the single channel optical fiber telecommunication systems. For the multi-channel disorder the statistics of the soliton amplitude and frequency is strongly non-Gaussian, whereas, for the single-channel types of disorder the statistics of the affected soliton parameters is Gaussian. In addition, the variances of the distributions of the parameters grow algebraically with propagation distance for single-channel disorder, and exponentially for the multi-channel disorder. In view of this, we conclude that multi-channel disorder induced by soliton collisions in the presence of delayed Raman response is potentially much more harmful that other major types of single-channel disorder in massive multi-channel soliton transmission.

References

[1] Y. Chung, V. V. Lebedev, and S. S. Vergles, *Phys. Rev. E*, **69**, 046612, (2004).

[2] Y. Chung, V. V. Lebedev, and S. S. Vergles, *Opt. Lett.*, **29**, 11, 1245, (2004).

[3] M. Chertkov, Y. Chung, I. Gabitov, A. Dyachenko, I. Kolokolov, and V. Lebedev, *Phys. Rev. E*, **67**, 036615, (2003).

[4] A. Peleg, Opt. Lett. **29** 1980, (2004).

[5] Y. Chung and A. Peleg, *Nonlinearity*, in press (2005).

Contact Information: Yeo-Jin Chung – Center for Nonlinear Studies and Complex Systems Group, Theoretical Division, Los Alamos National Laboratory, MS-B258, Los Alamos, NM, 87545, Phone: (505) 667-6896, email: ychung@cnls.lanl.gov.