

Gene Surfing on Population Waves

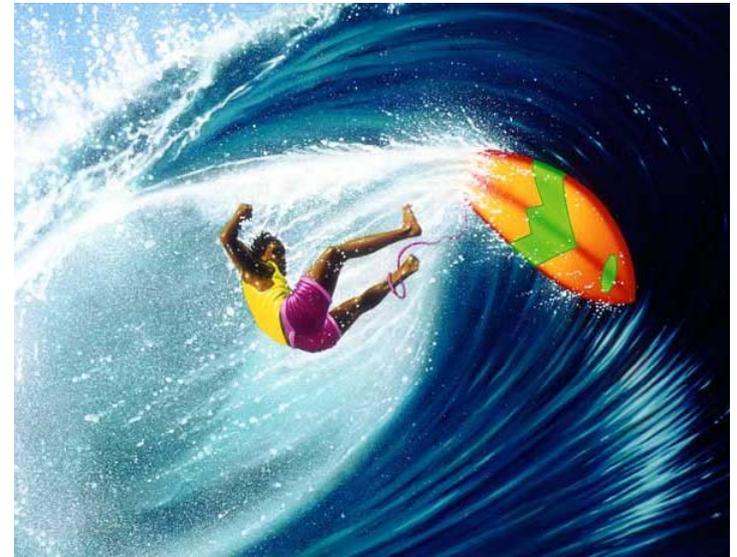
The fate of neutral mutations in a spreading population

O. Hallatschek and drn

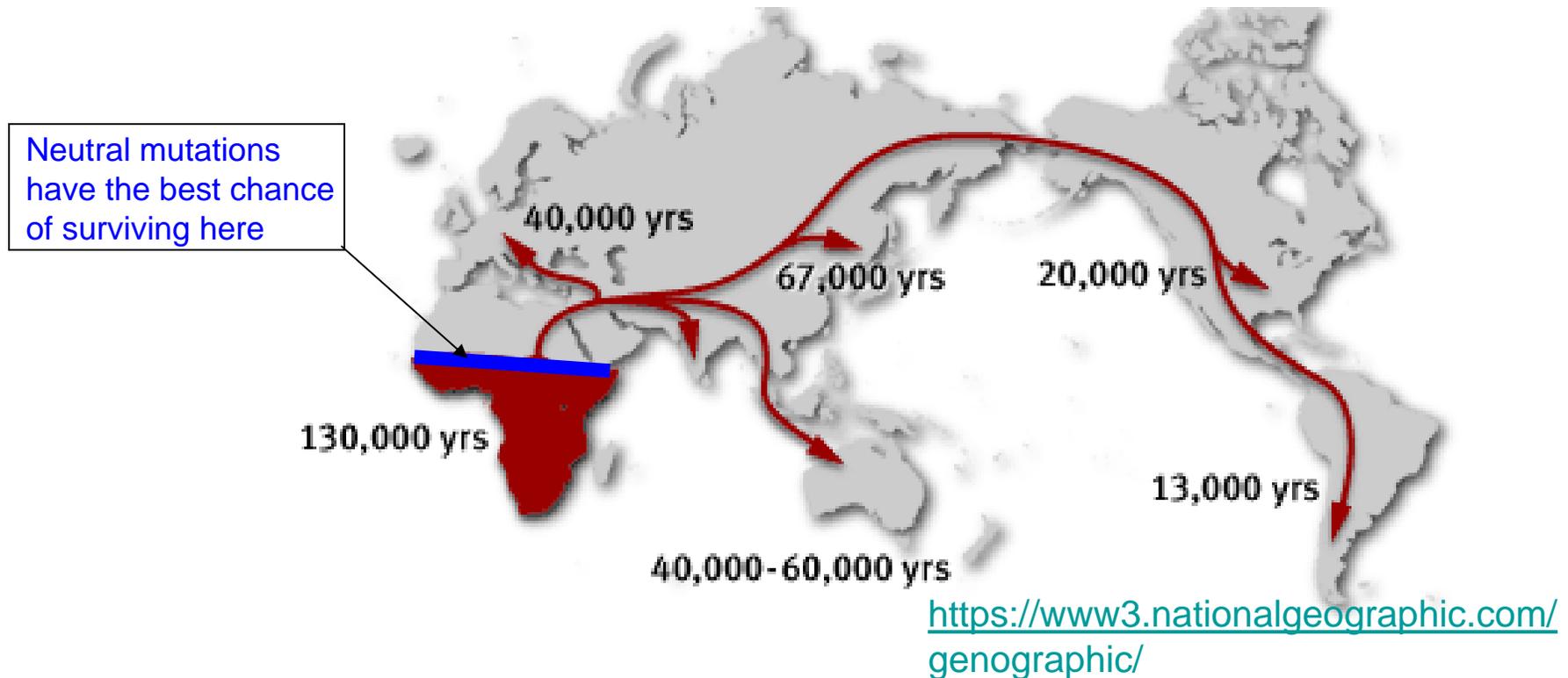
lots of help from the Ramanathan group.....



vs.

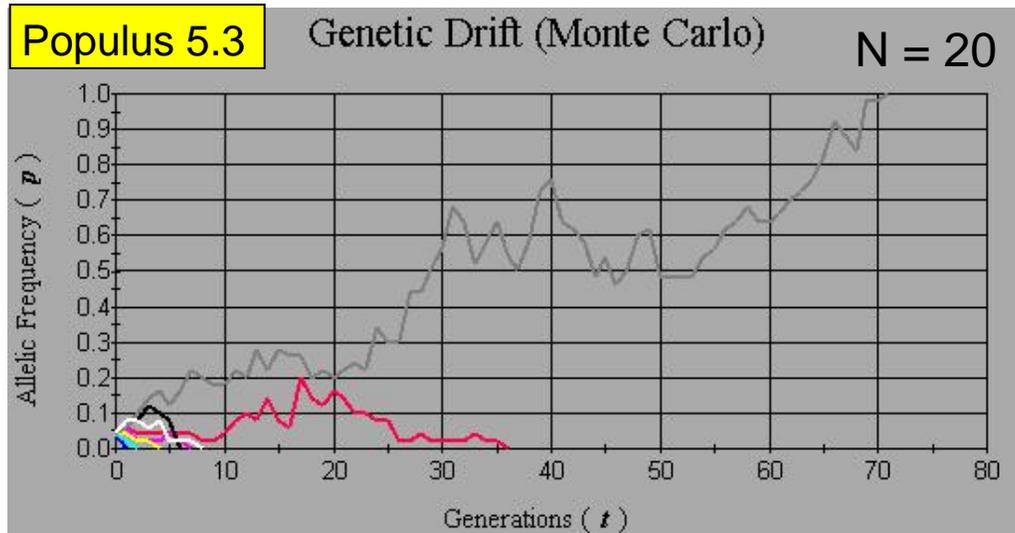


Neutral genetic markers can be used to infer direction, timing, and dispersal rates of colonization pathways



“Surfing genes” are the major footprints of humanity’s migrations

*Neutral mutations have an advantage
at an expanding population front*



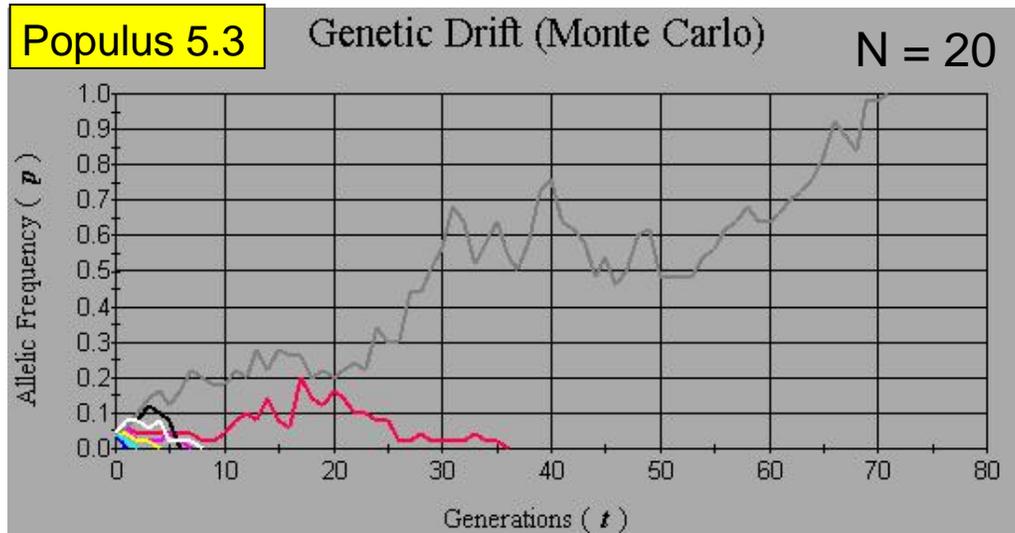
*Genetic drift for a
neutral mutation*

M. Kimura, Genetics 47, 713 (1962)

*$u(p,t)$ = probability allele A has
frequency p at time t .*

- Finite populations go to fixation for long times (using, e.g., Fisher-Wright population sampling)

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M. Kimura, Genetics 47, 713 (1962)

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frequency p at time t .*

•Finite populations go to fixation
for long times(using, e.g., Fisher-
Wright population sampling)

$$\frac{\partial u(p,t)}{\partial t} = \frac{\partial^2 [D_G \cdot u(p,t)]}{\partial p^2}$$

$$D_G = p(1-p)/(4N)$$

$$u(0,t) = 0; u(1,t) = 1$$

- Finite populations go to fixation for long times
- Probability of fixation of a single neutral mutation in a population of size N is just 1/N
- But N is small in the vicinity of an expanding population front!

Introduction to Population Dynamics

$c(t)$ = population of species at time t in region Ω

$$\frac{dc(t)}{dt} = \text{births} + \text{saturation} + \text{migration}$$

1798 T.R. Matthus $\frac{dc(t)}{dt} = ac(t), a > 0$

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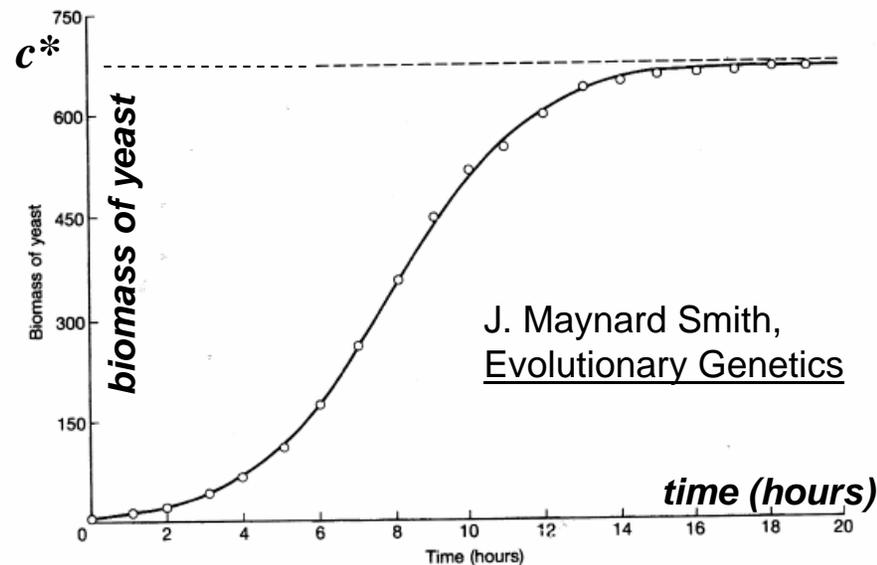
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stable
population
size: $c^* = a/b$



Fisher Waves and Population Dynamics

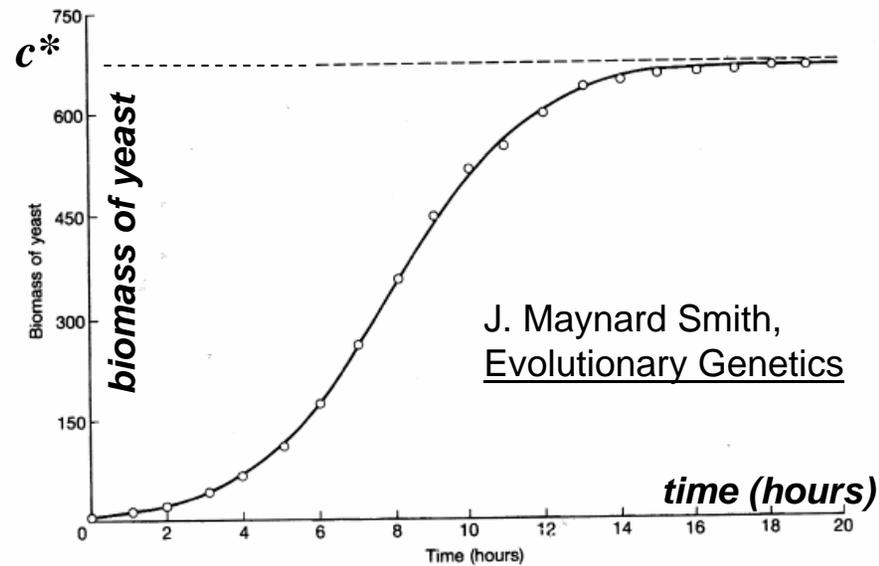
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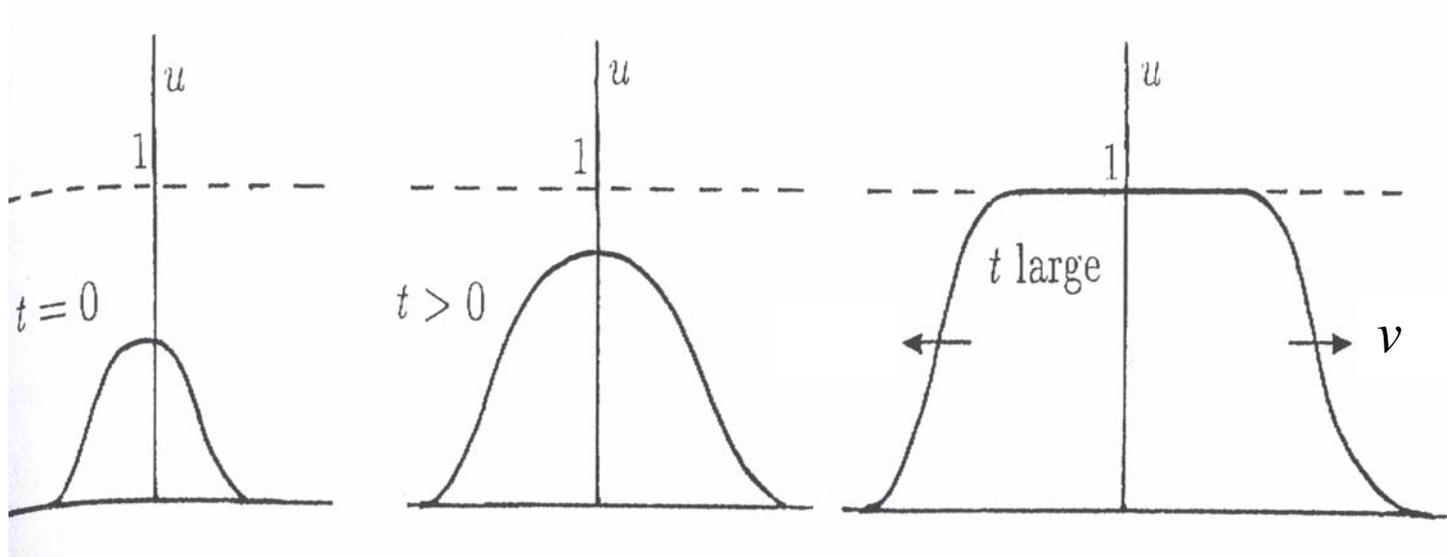


1937 R.A. Fisher

$$c = c(\vec{r}, t) \quad \frac{\partial c(\vec{r}, t)}{\partial t} = D\nabla^2 c(\vec{r}, t) + ac(\vec{r}, t) - bc^2(\vec{r}, t)$$

Fisher Waves In One Dimension

$$\frac{\partial}{\partial t} c(x,t) = D \frac{\partial^2}{\partial x^2} c(x,t) + ac(x,t) - bc^2(x,t); \quad \text{let } c(x,t) = f(x-vt)$$



Schematic time development of a wavefront solution of Fisher's equation on the infinite line. (J.D. Murray, Mathematical Biology)

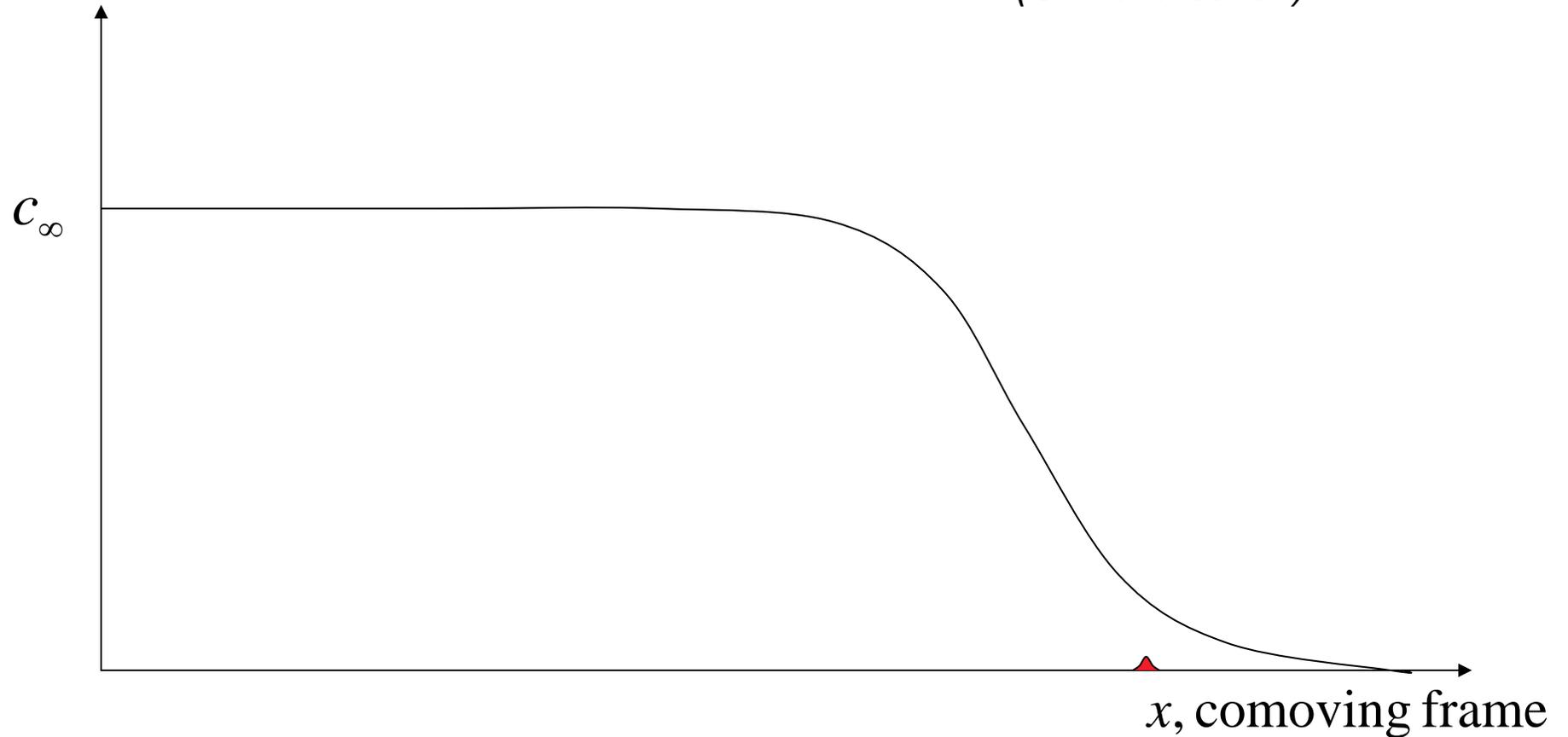
$$\text{Interface velocity} = 2\sqrt{Da}$$

$$\text{Interface width} = \sqrt{D/a}$$

Successful Surfing (1d)

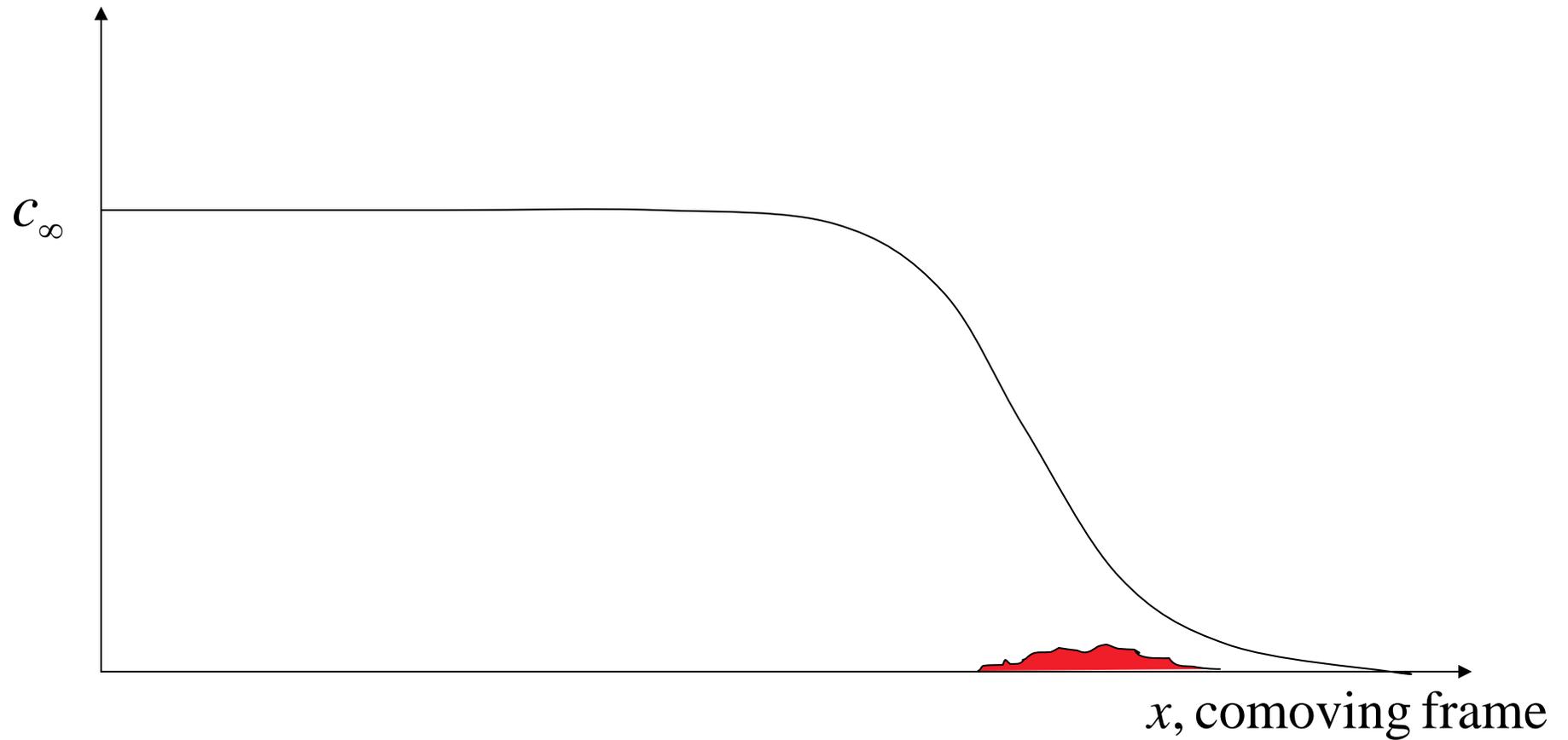
$c_s(x)$, steady state population density

(O. Hallatschek)



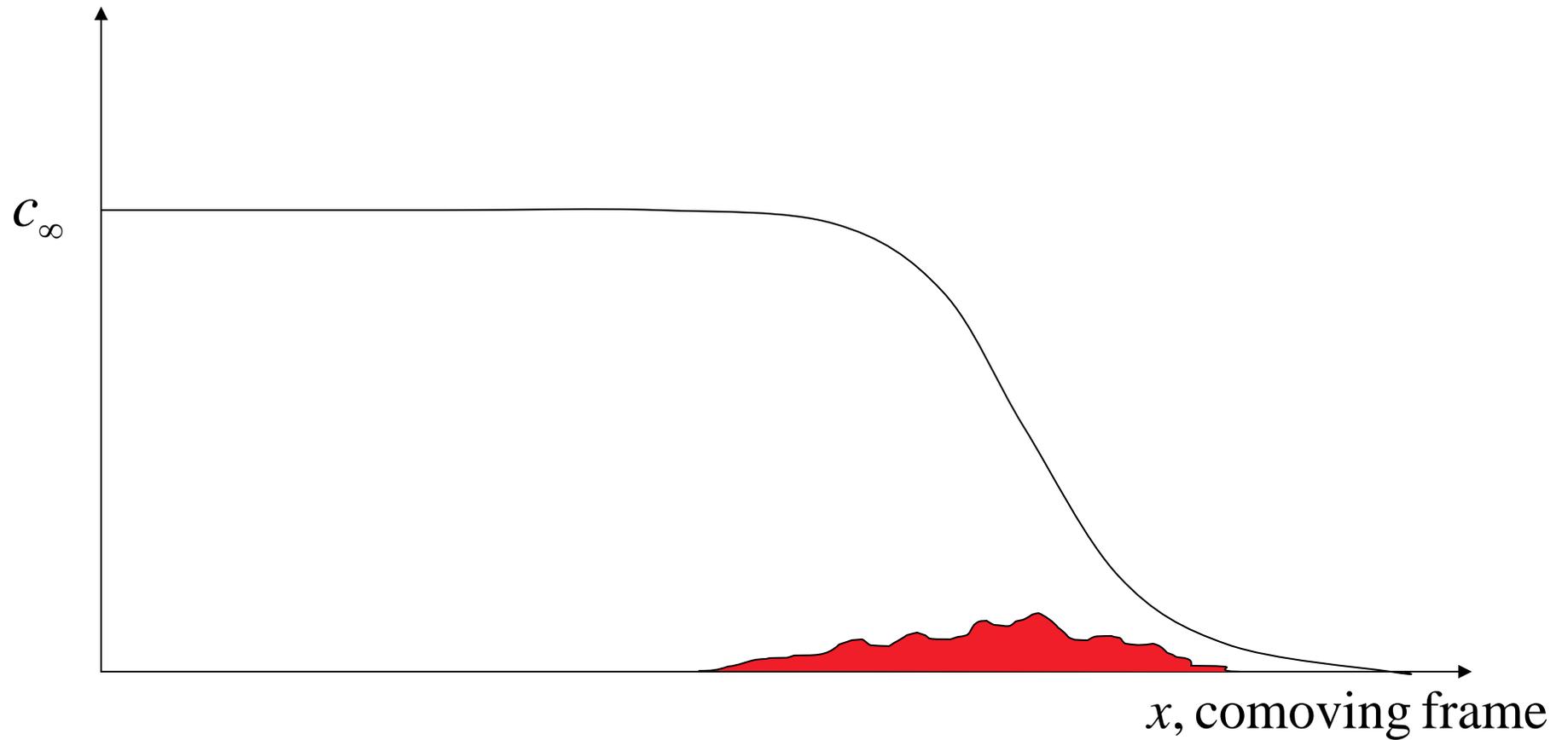
Successful Surfing (1d)

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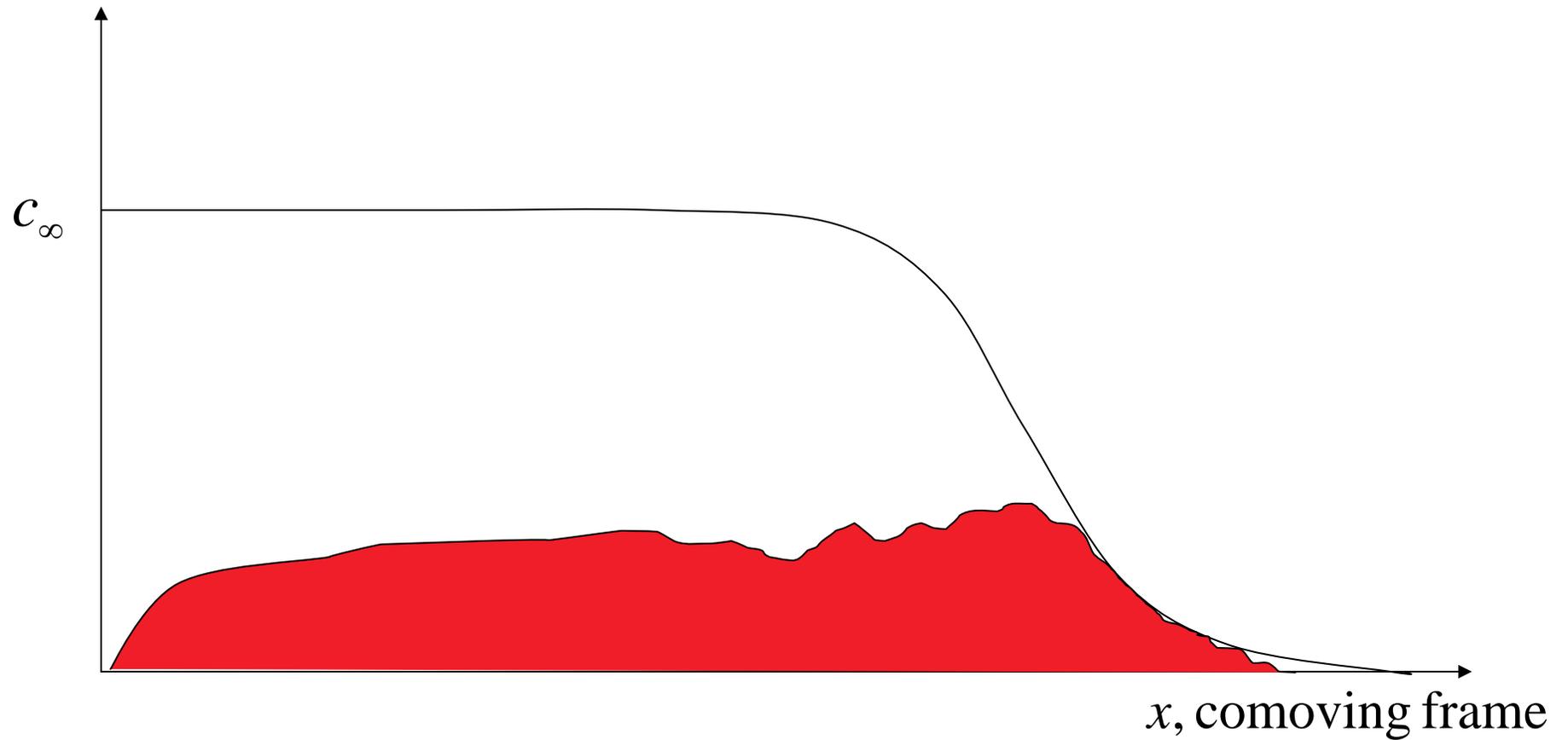
Successful Surfing (1d)

$c_s(x)$, steady state population density



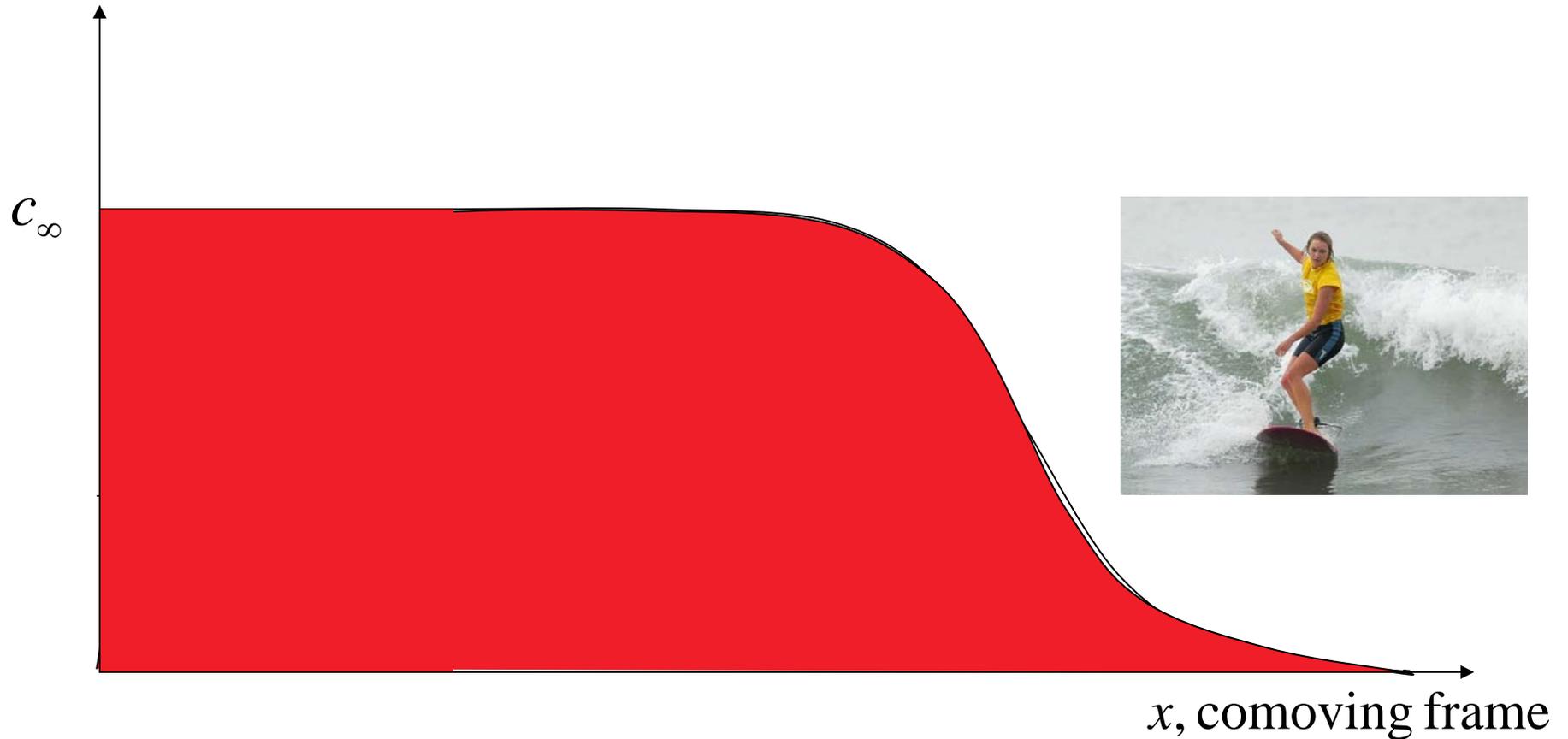
Successful Surfing (1d)

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Successful Surfing (1d)

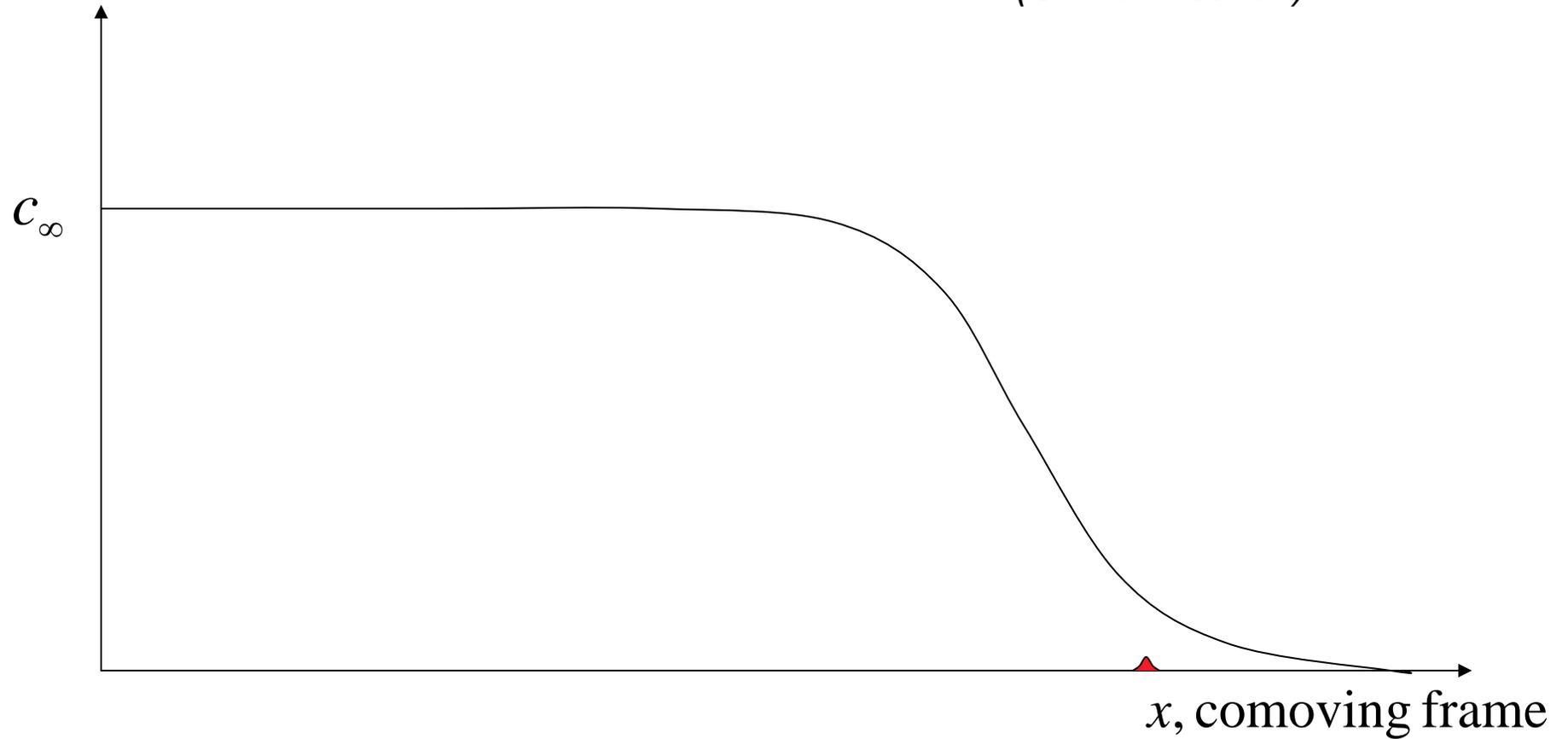
$c_s(x)$, steady state population density



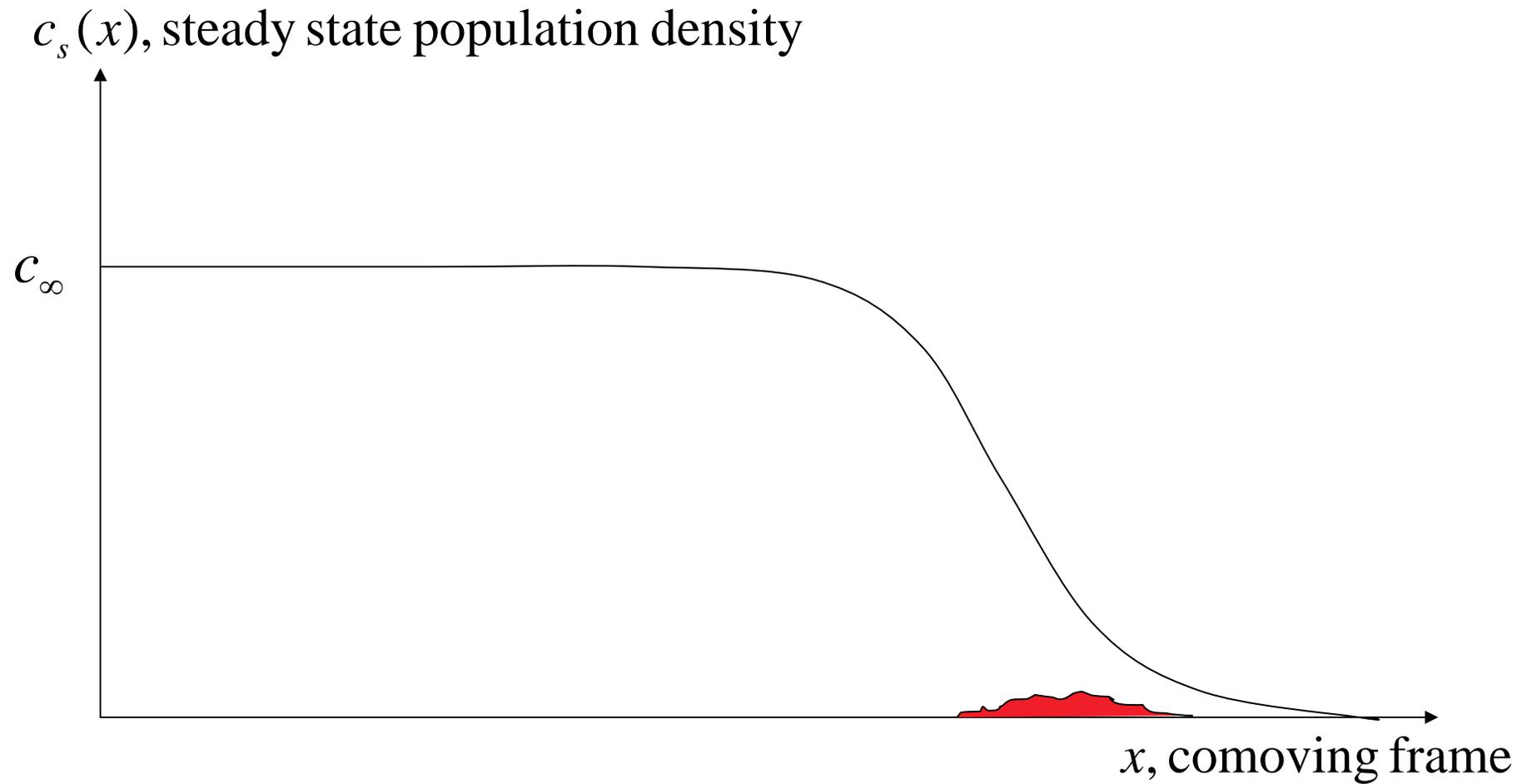
Often however ...

$c_s(x)$, steady state population density

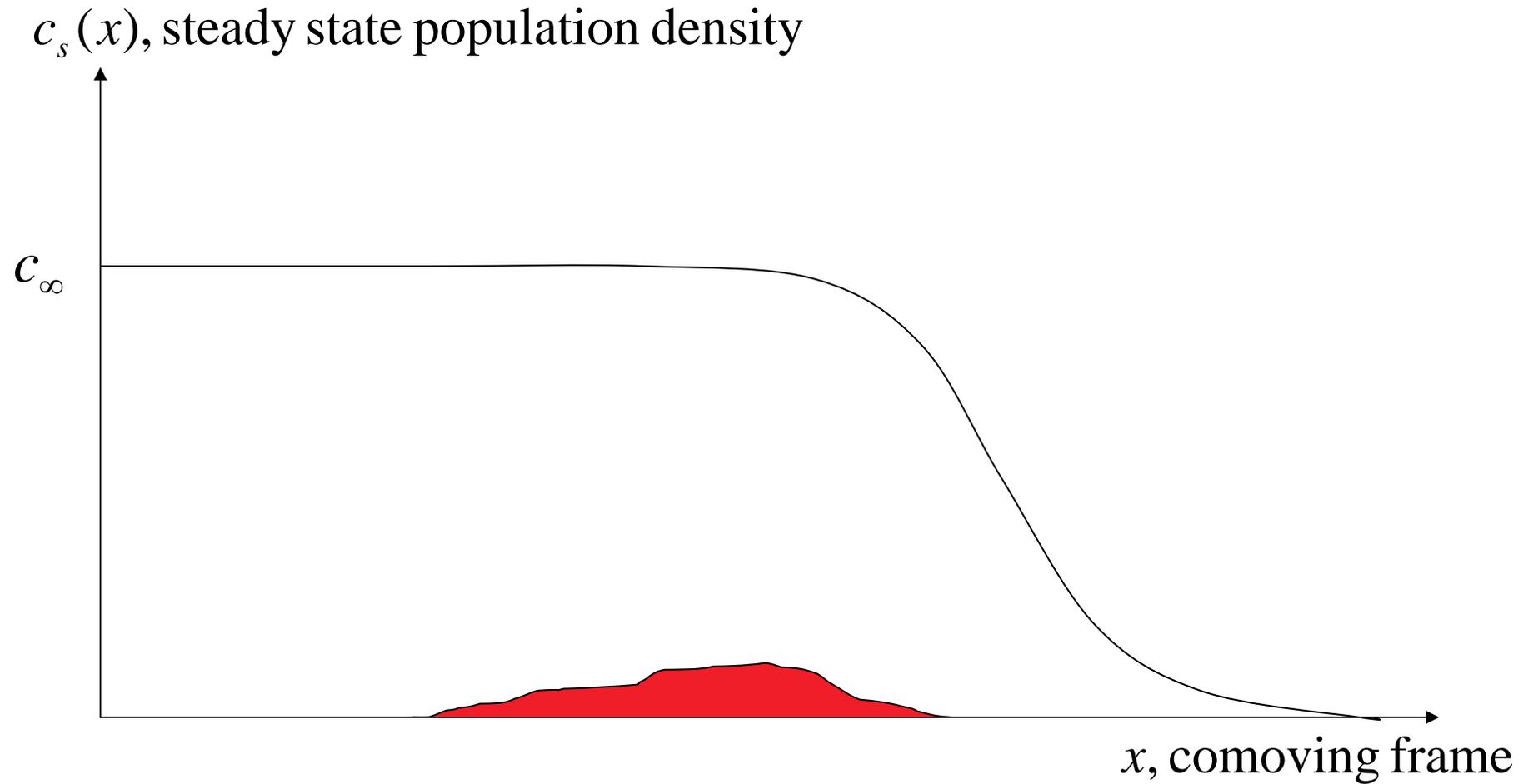
(O. Hallatschek)



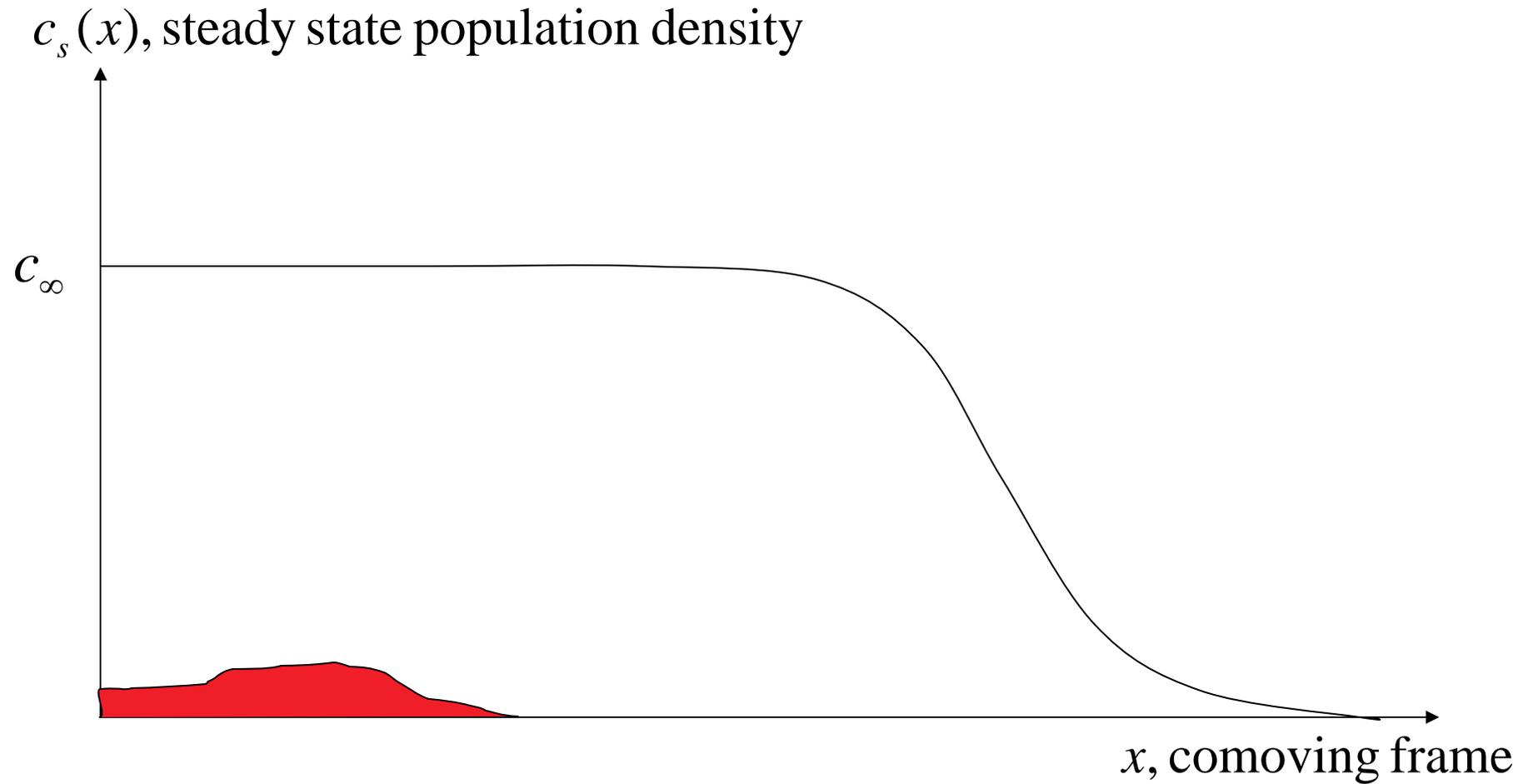
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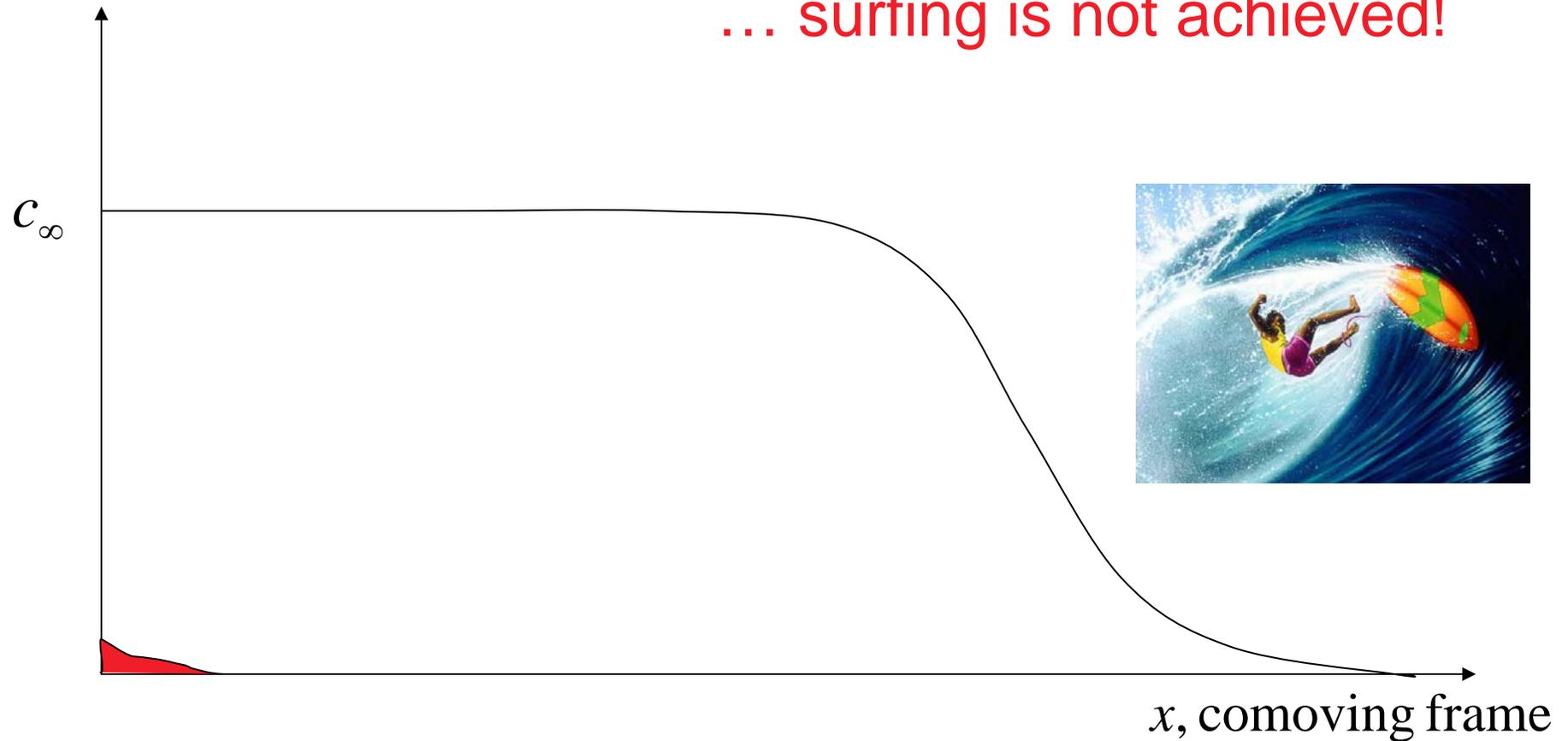
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$c_s(x)$, steady state population density

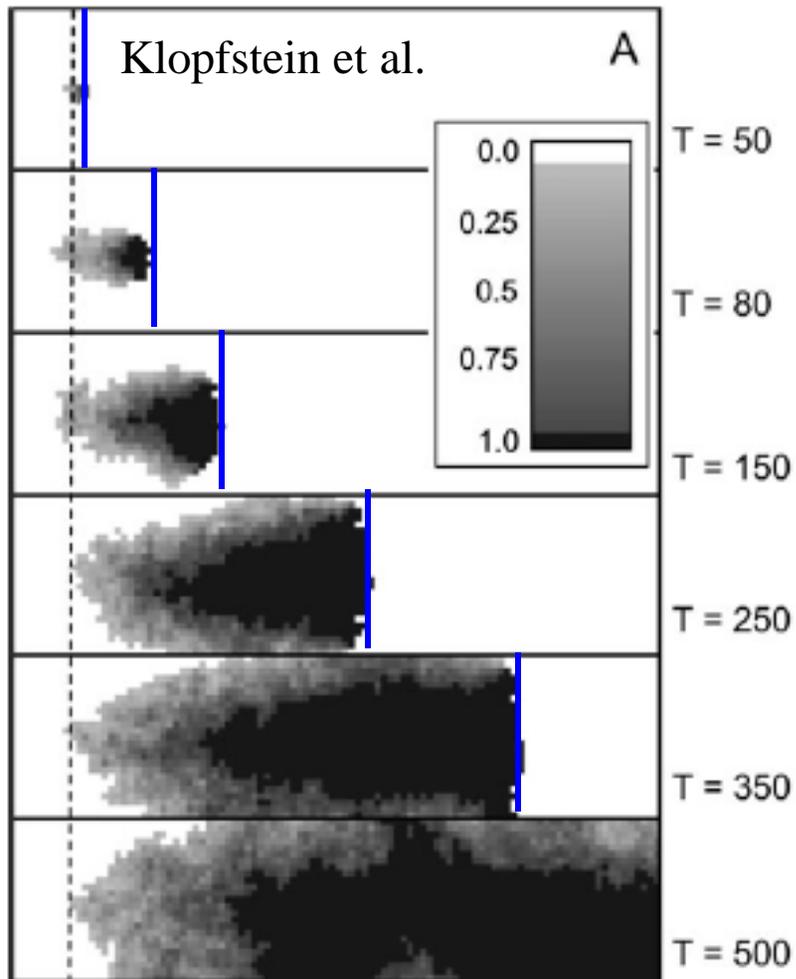
... surfing is not achieved!



Mutations Surfing on a Two-Dimensional Population Wave

S. Klopstein, M. Currat, and L. Excoffier, *Mol. Biol. Evol.* **23**, 482–490 (2006)

C. A. Edmonds, A. S. Lillie, and L. Cavalli-Sforza *PNAS* 975–979 (2004)



Effective population size at the front of a wave can be very small....

1. What is the probability of successful surfing?
2. How does this depend on the position of the mutation within the interface?
3. How does a successful mutation spread laterally downstream?
4. How do neutral mutations (initially well mixed) segregate in space when carried along a population wave?

Can Gene Surfing be Studied in Bacteria (or Yeast!)?



Twitching motility?

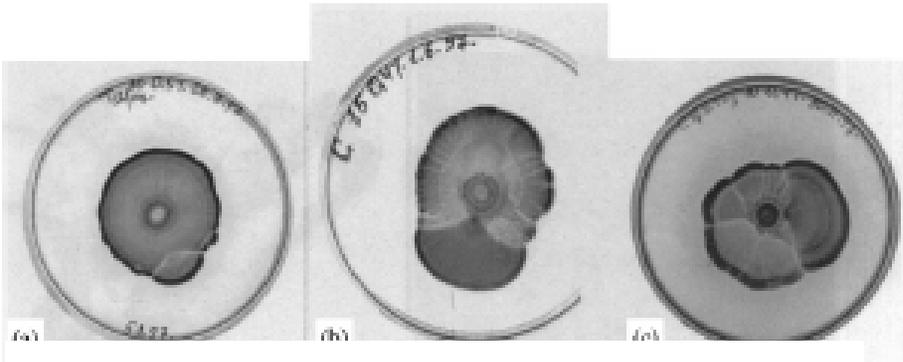
	Large mammals (with mitochondria)	E. coli with flagella	E. coli without flagella
t_2	9×10^8 sec. (30 years)	1500 sec.	1500 sec.
l_2	10^6 cm. (10 km.)	10^{-1} cm.	10^{-3} cm.
$D = l_2^2 / 2t_2$	10^3 cm ² /sec	2×10^{-6} cm ² /sec	7×10^{-10} cm ² /sec
velocity	1×10^{-3} cm/sec	1×10^{-4} cm/sec	1×10^{-6} cm/sec
$L(500)$	10^4 km	30 cm	0.5 cm

Range expansion in 500 generations

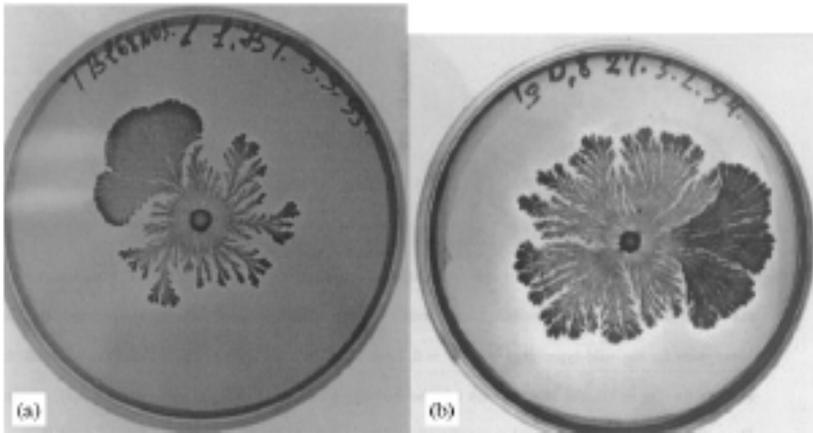
E. J. Stewart et al., PLoS, Biology, 3, 295 (2005)(Taddei Lab)

“Surfing” (= Sectoring) in *Paenibacillus dendritiformis*:

I. G. Ron et al. Physica A320, 485 (2003)



Emerging sectors in compact colonies of *P. dendritiformis*.



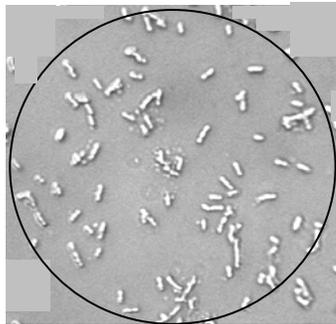
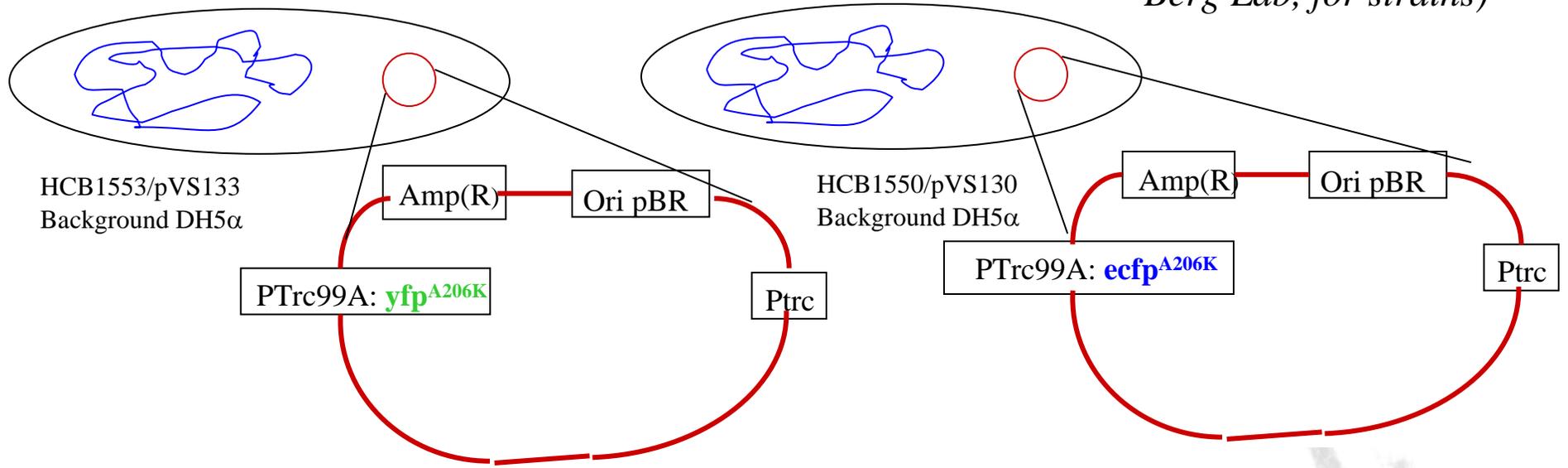
Emerging sectors in branching colonies of *P. dendritiformis*.

❁ Unless mutants have a different growth dynamics (and hence are not neutral!), they will go unnoticed...

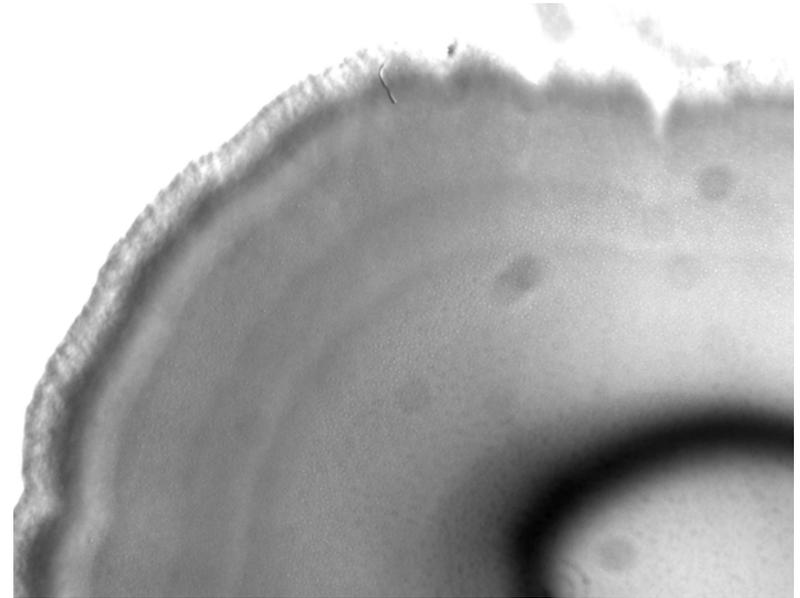
❁ Can we create strictly neutral mutations at the interface of a bacterial population front, and follow their subsequent dynamics?

Gene Surfing in nonmotile *E. coli*

(thanks to Tom Shimizu,
Berg Lab, for strains)

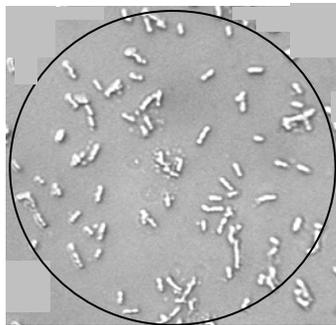
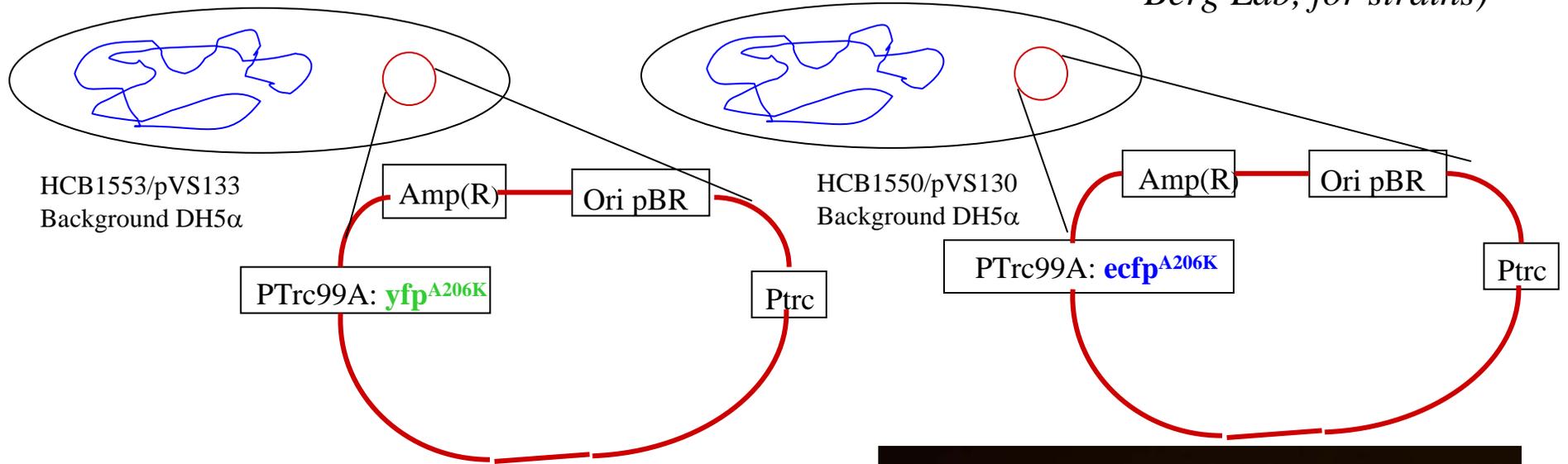


50-50 mixture,
1550/1553

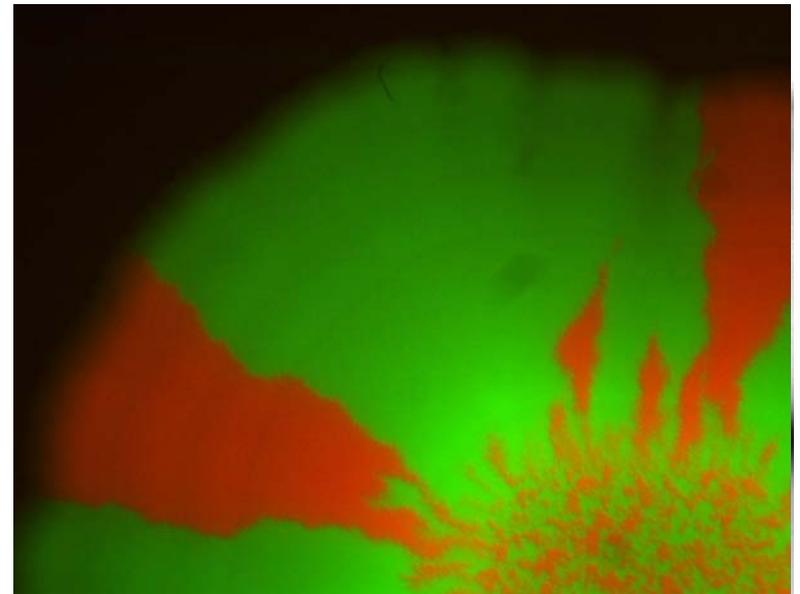


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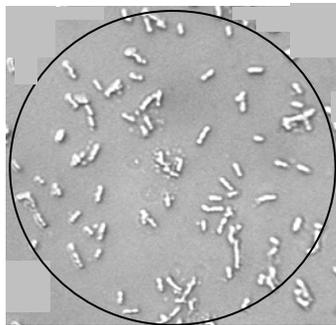
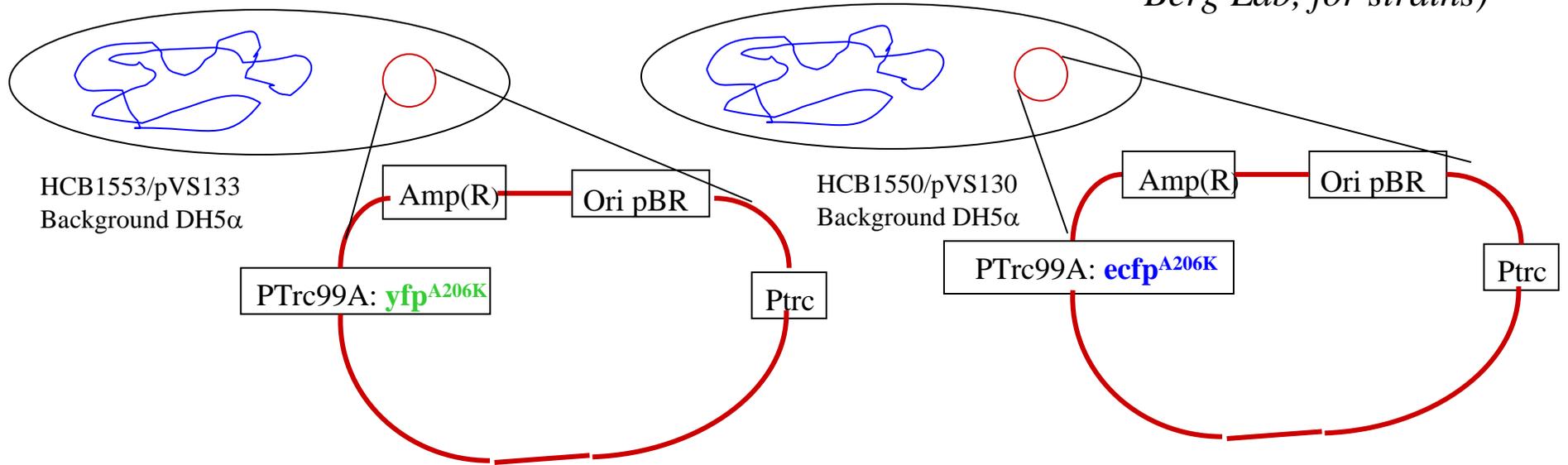
50-50 mixture,
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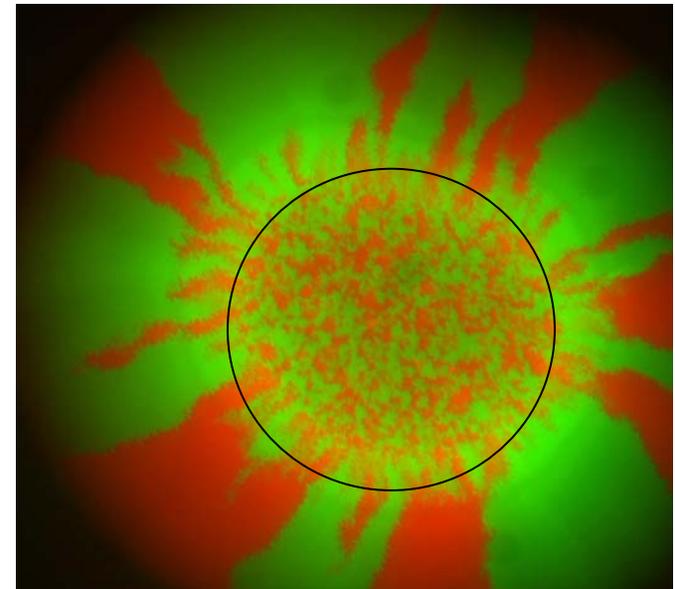
Cyan → Red

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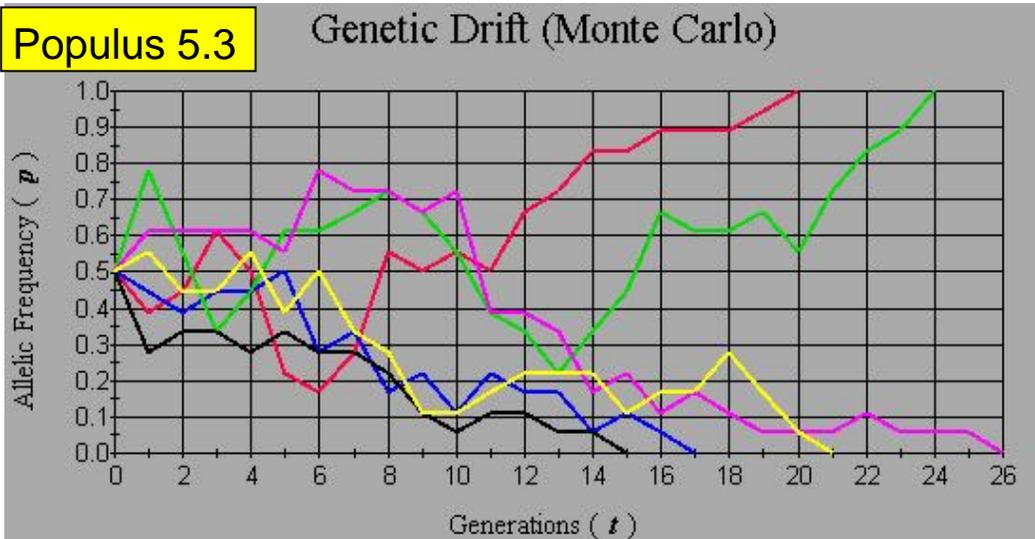
50-50 mixture,
1550/1553



Cyan \rightarrow Red

Genetic drift is enhanced at the frontier...

Consider a 50-50 “panmictic” solution of N_r red & N_b blue bacteria in a test tube



Let H_n = “heterozygosity”

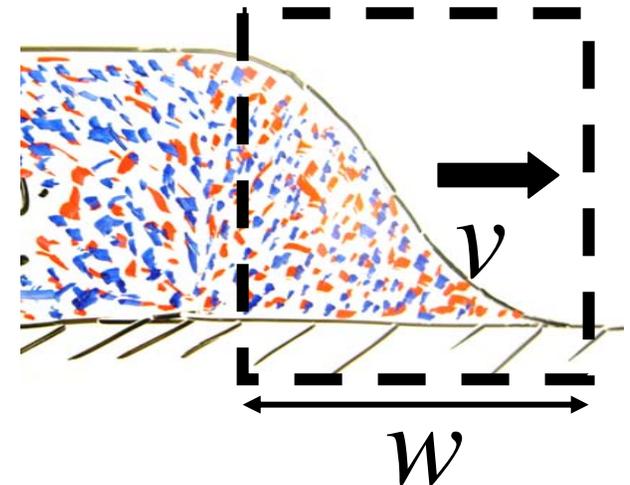
= probability that two randomly chosen bacteria differ in color

τ = generation time

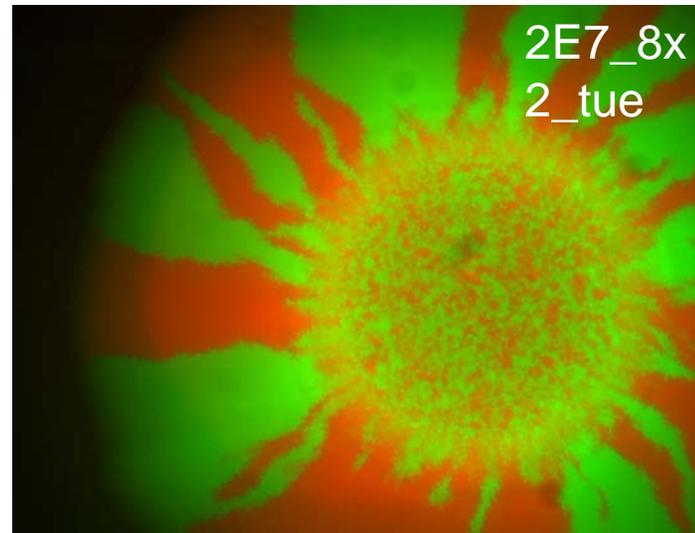
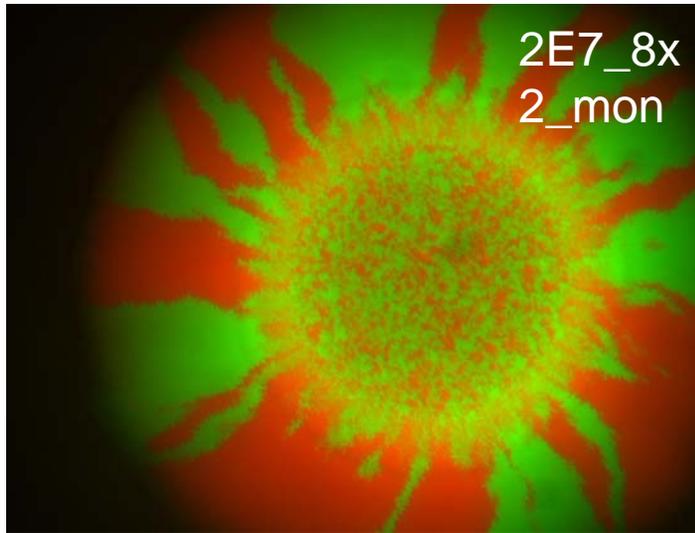
$$H(t) \approx H_0 e^{-t/\tau N_{eff}}$$

N_{eff} = effective population size

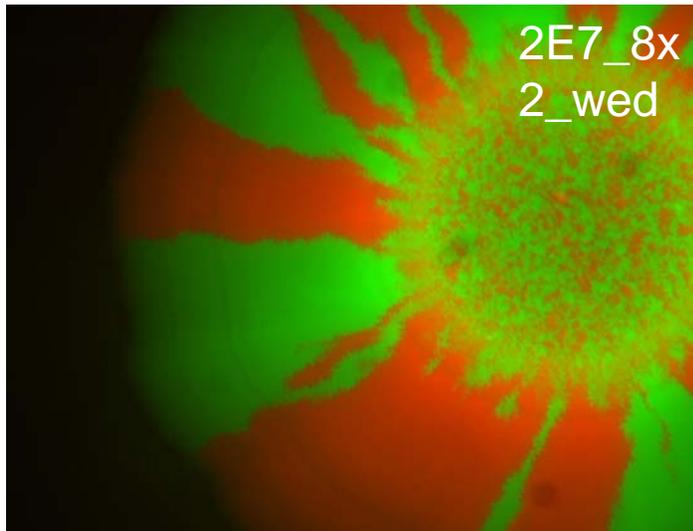
$N_{eff} \approx 10^9$, in an overnight culture, but
 $N_{eff} \approx 10$, for immotile bacteria
modelled by a Fisher wave



50-50 HCB1550/1553 circular inoculants



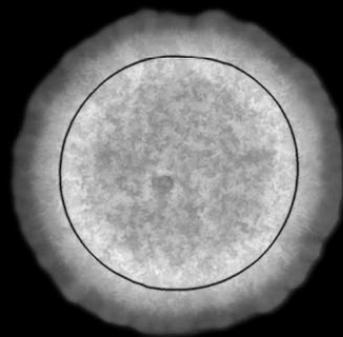
— = 2 mm.



Founder population \cong 5000

Segregation patterns both in the
“homeland” area and radial
“sectors” lock in behind wave

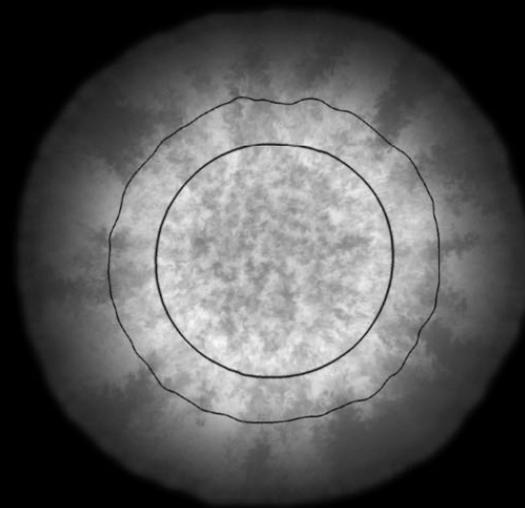
YFP-channel after 12 hours



1cm



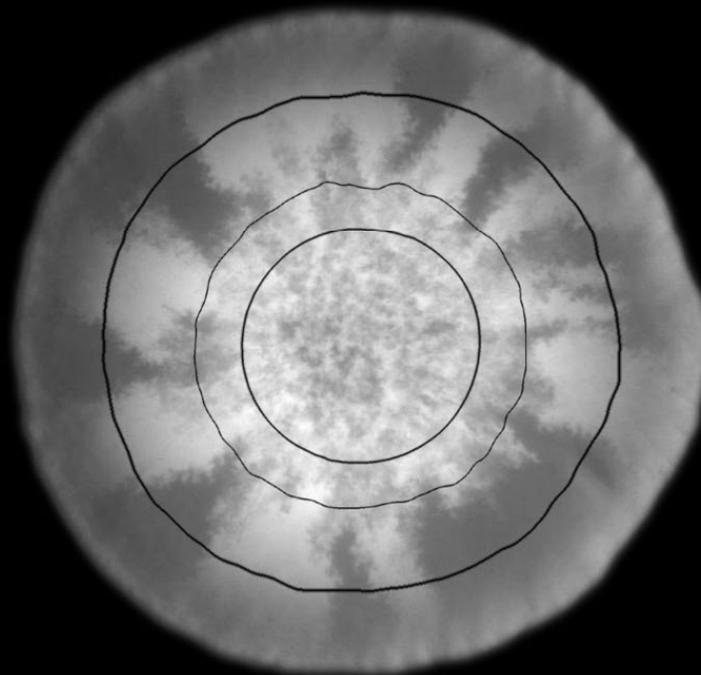
YFP-channel after 24 hours



1cm



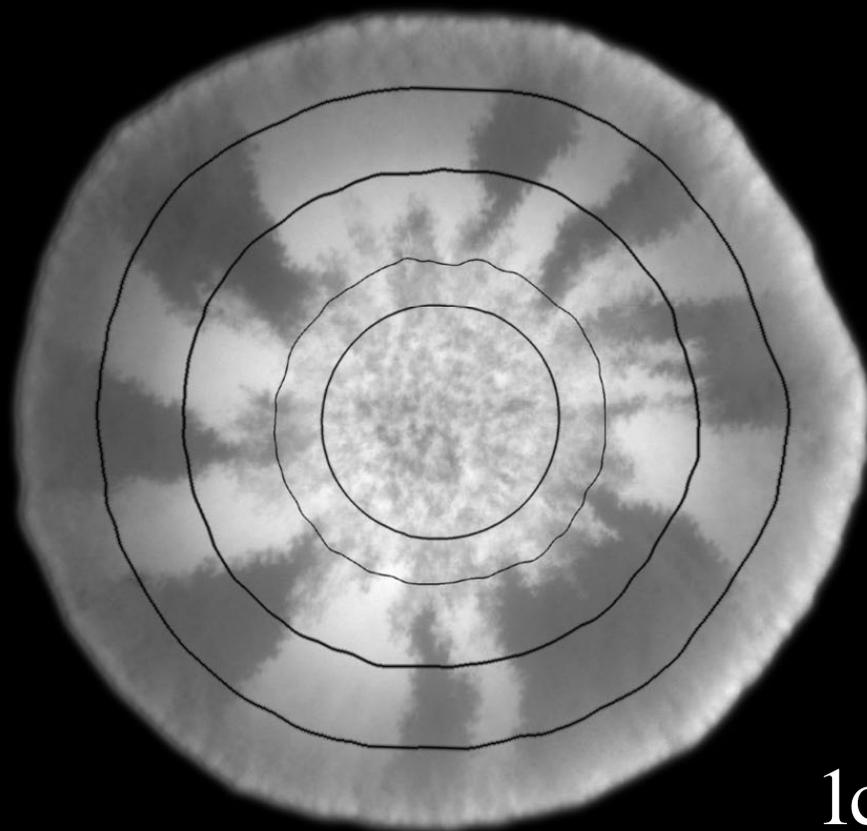
YFP-channel after 36 hours



1cm



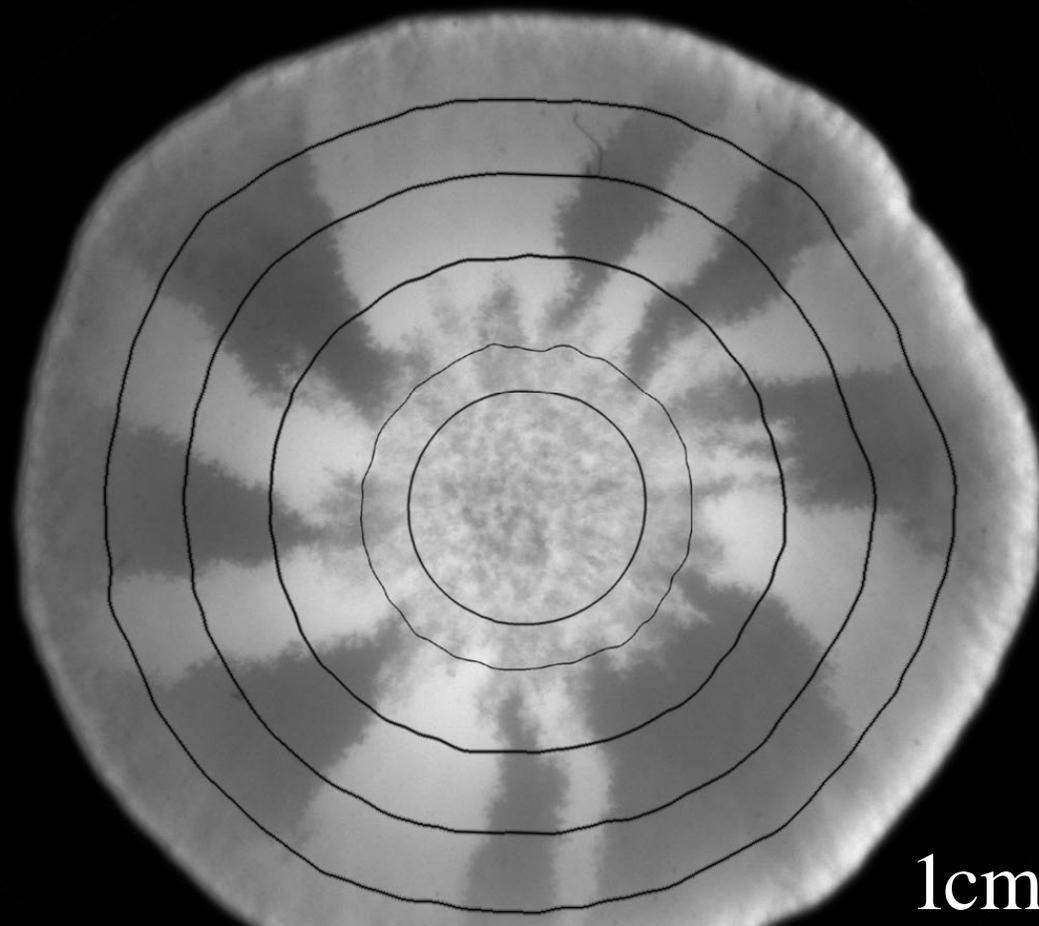
YFP-channel after 48 hours



1cm



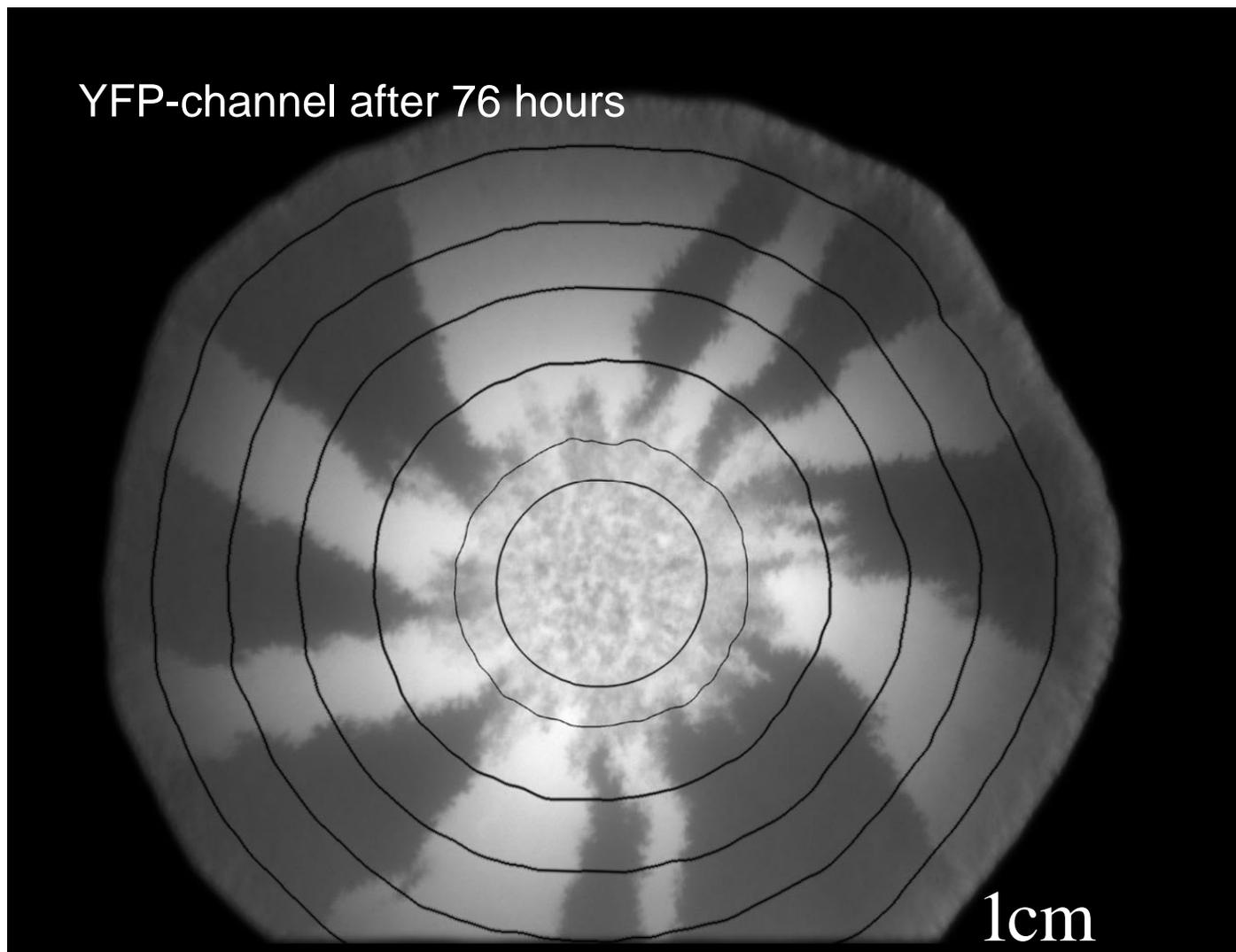
YFP-channel after 64 hours



1cm



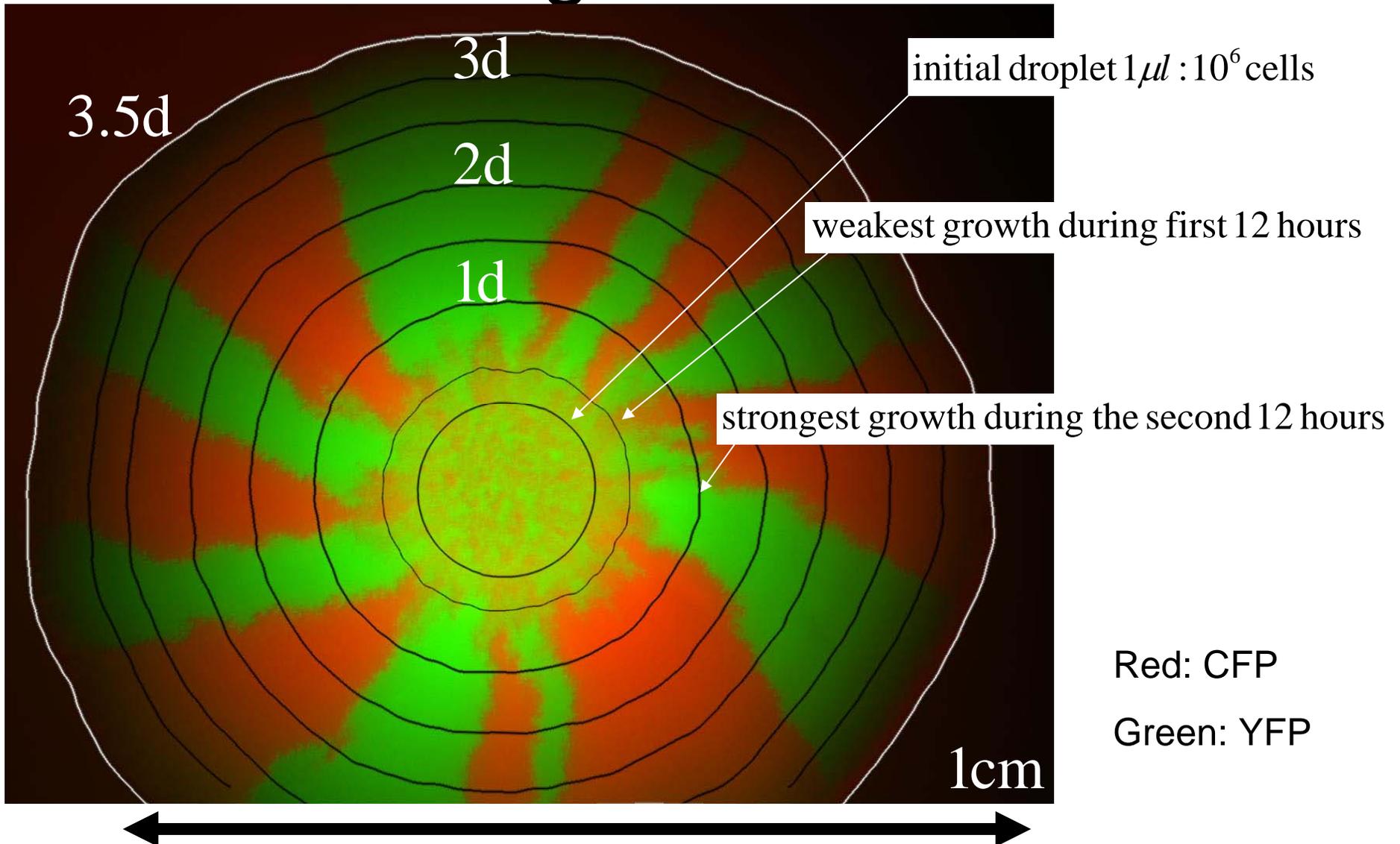
YFP-channel after 76 hours



1cm



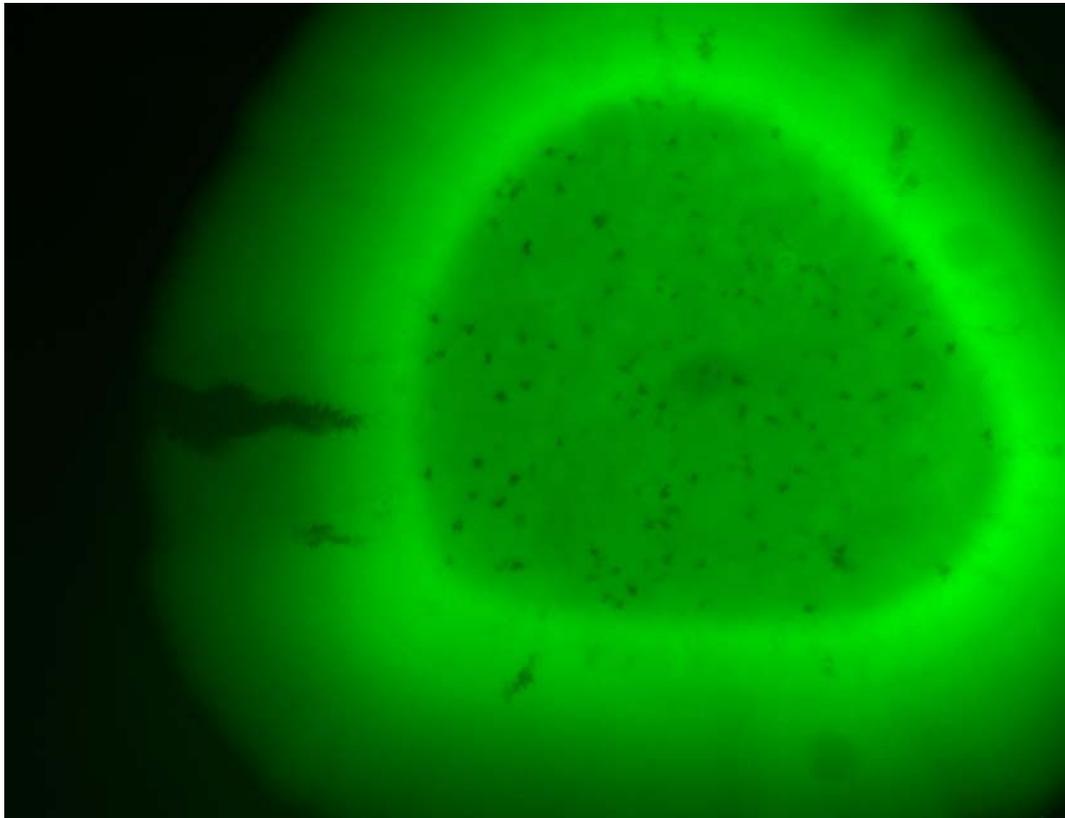
Gene surfing in *E. coli* colonies



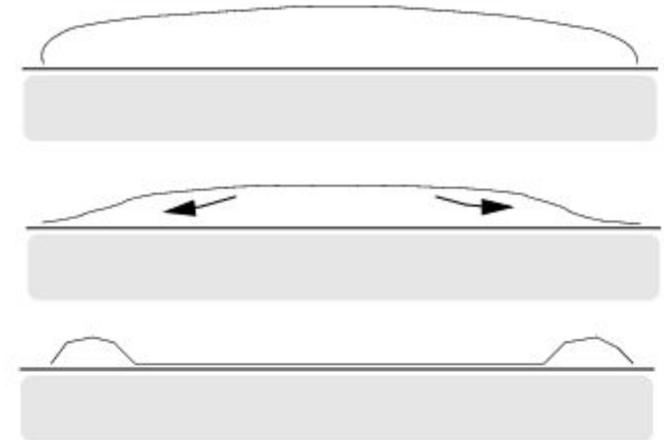
Sector creation in the dilute limit I

98%-2% mixture
green channel only
founder population~5000

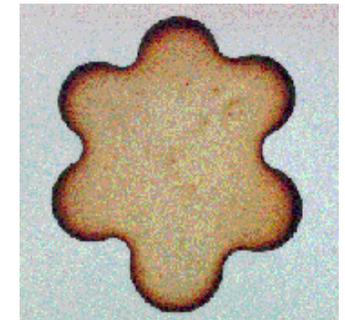
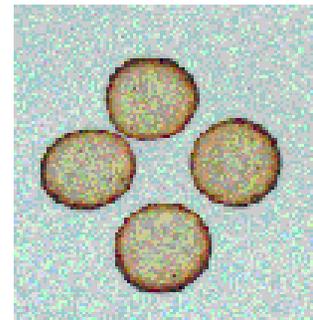
But why is there a bright rim???



Coffee stain effect

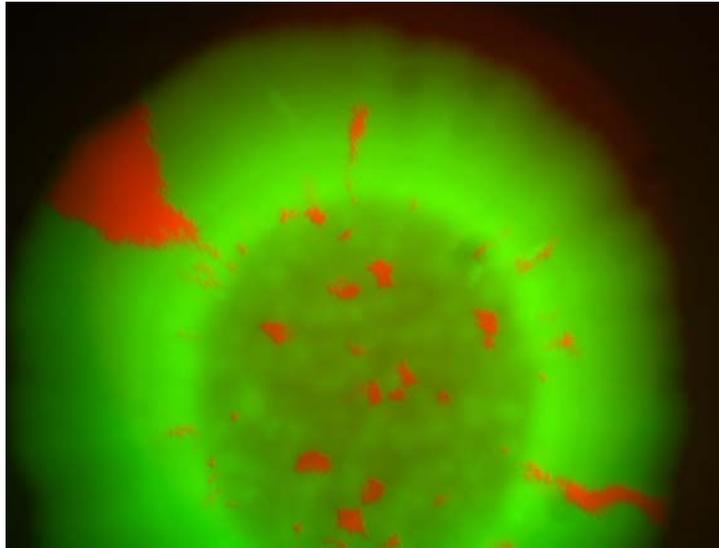


<http://visindavefur.hi.is/svar.asp?id=5513>

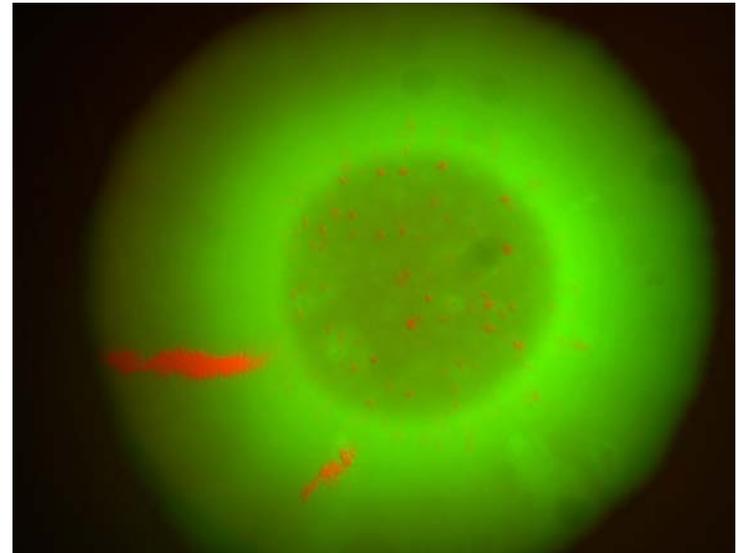


S. Nagel et al. Univ. of Chicago

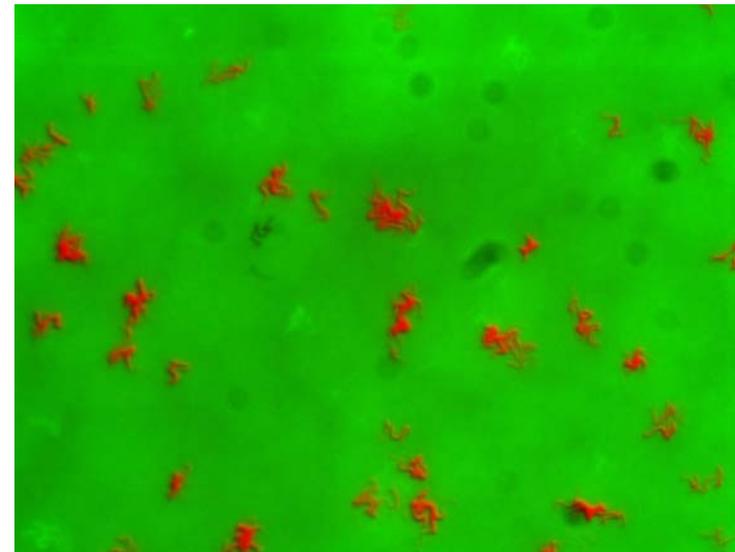
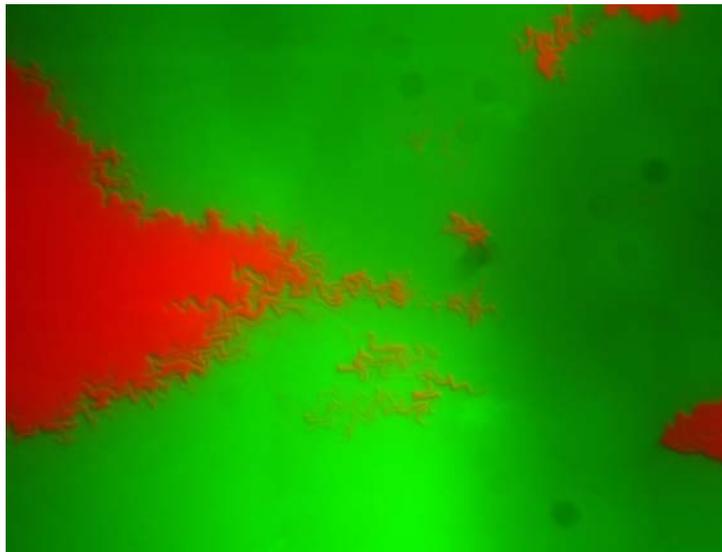
Sector creation in the dilute limit II



95%-5% mixture, founder population ~ 500

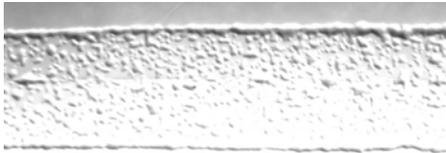


98%-2% mixture, founder population ~ 5000

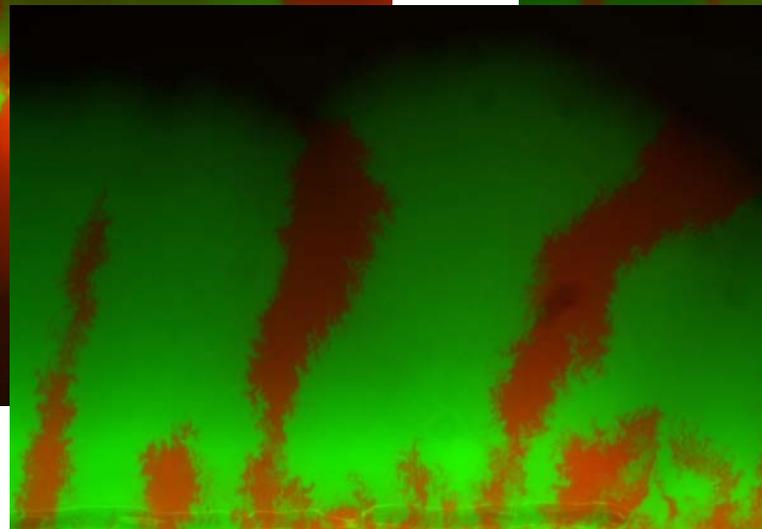
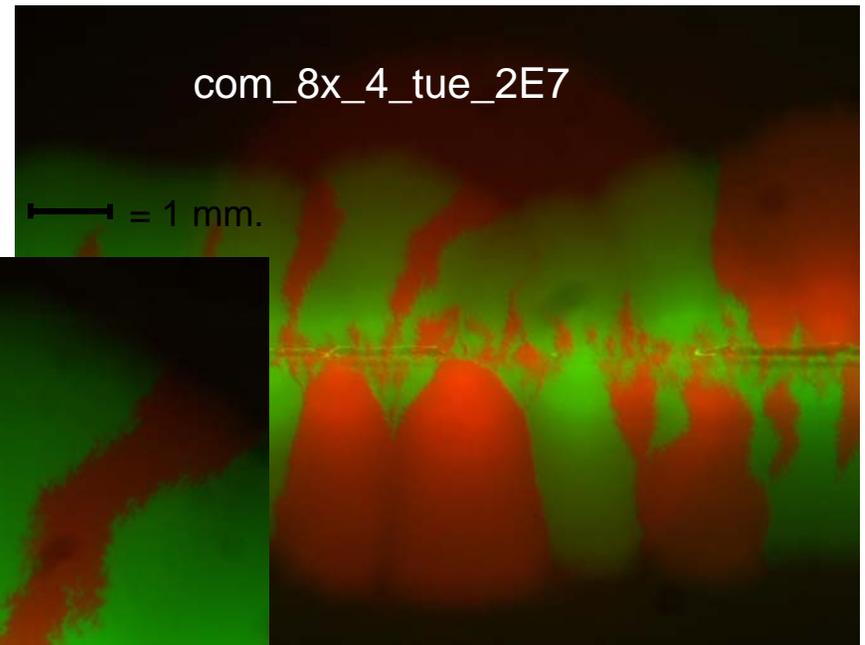
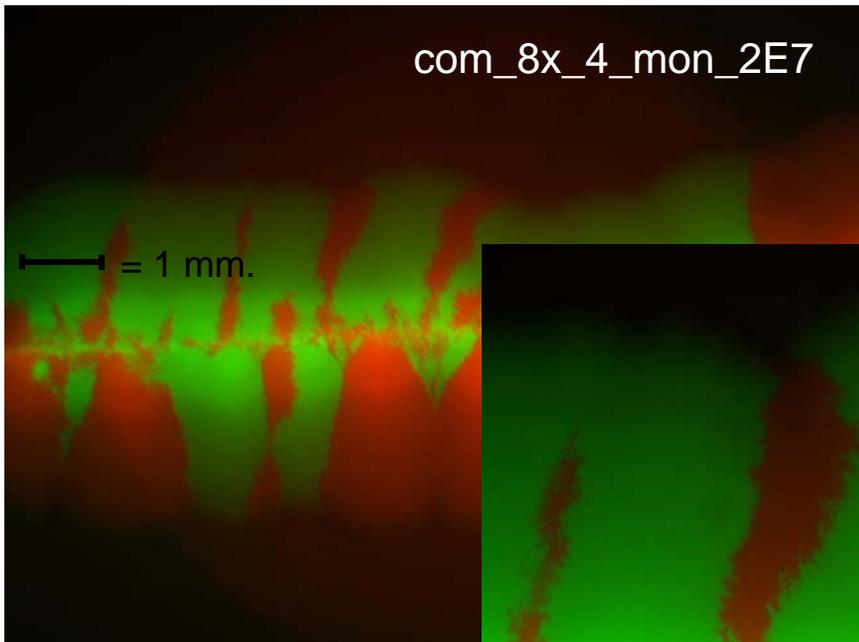
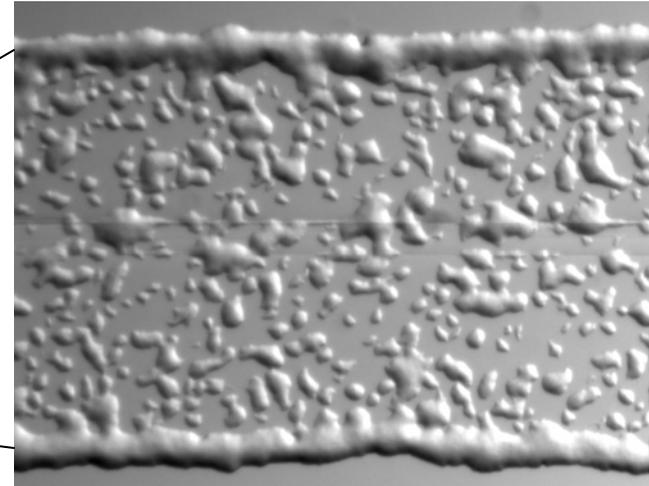


Linear inoculants (razor blade) 50%-50% mixtures

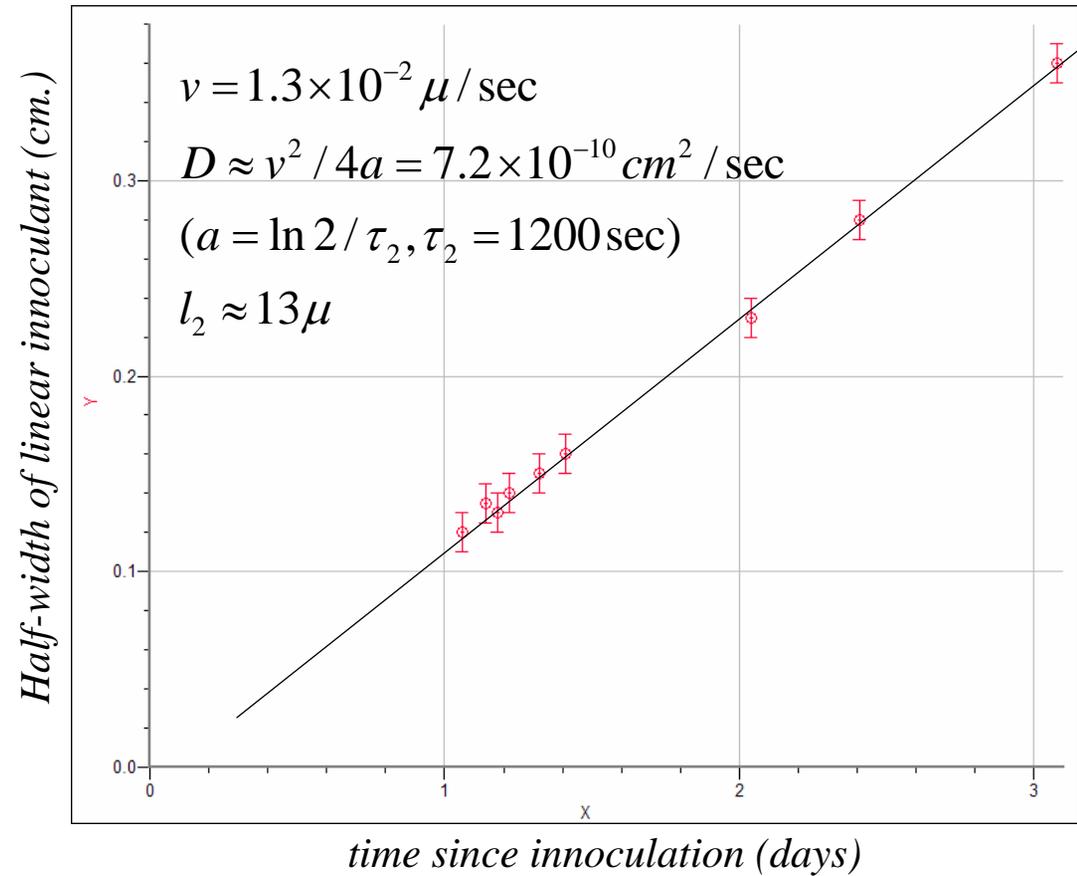
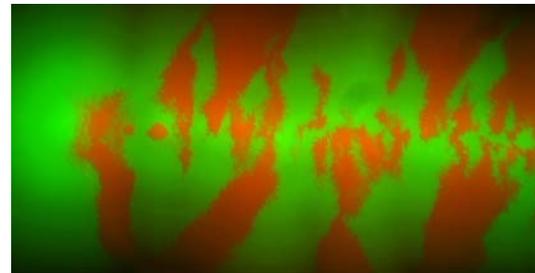
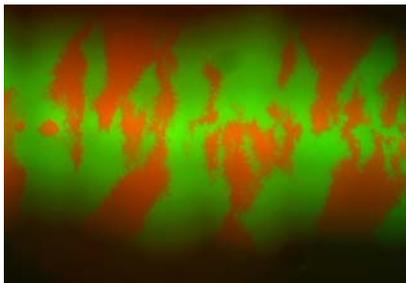
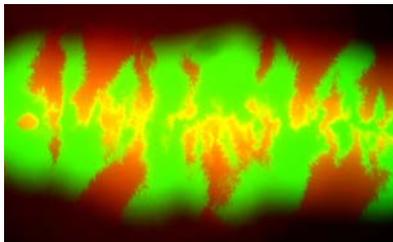
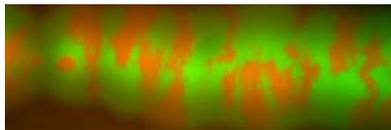
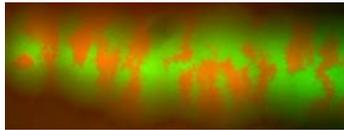
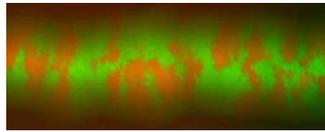
100X, 6h after inoculation



coffee stain effect!

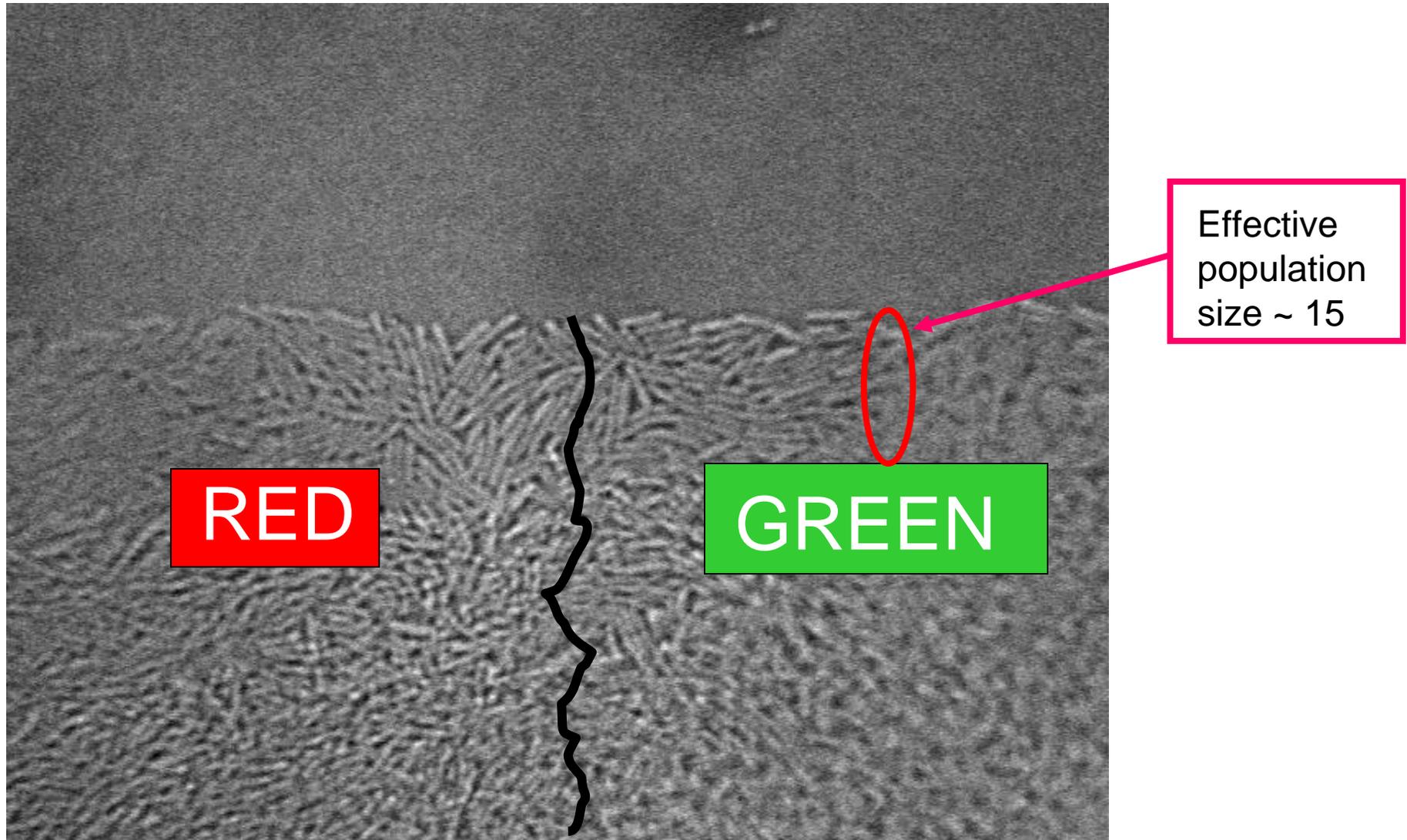


Growth velocity of linear inoculation

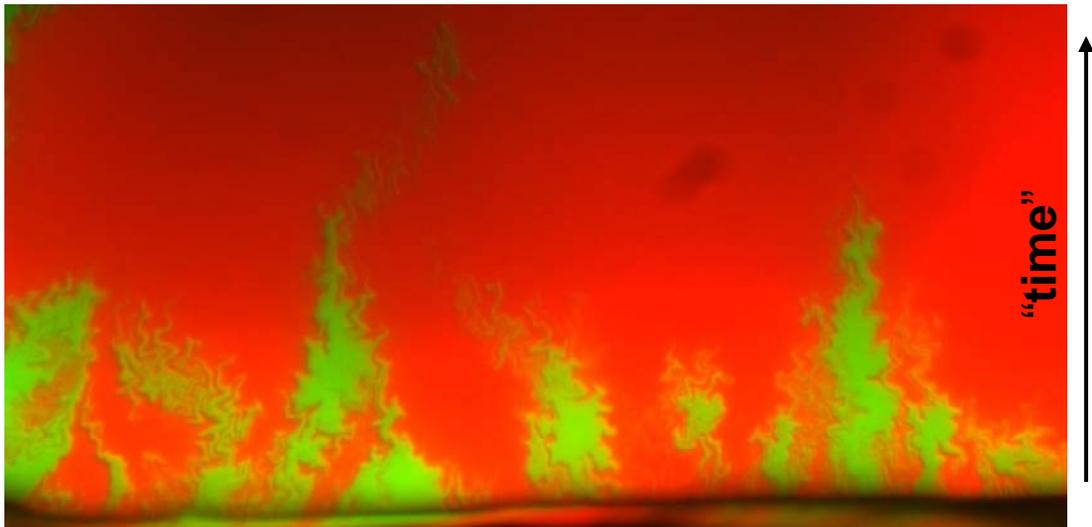
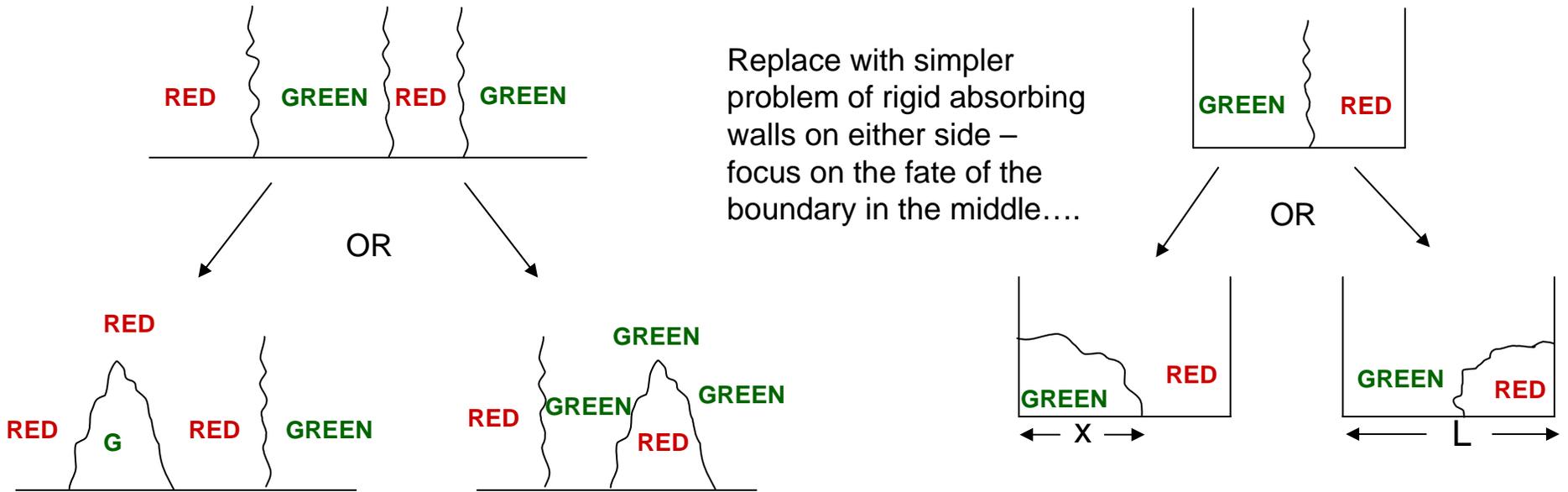


**Front moves ~15 cell widths
in a generation time...**

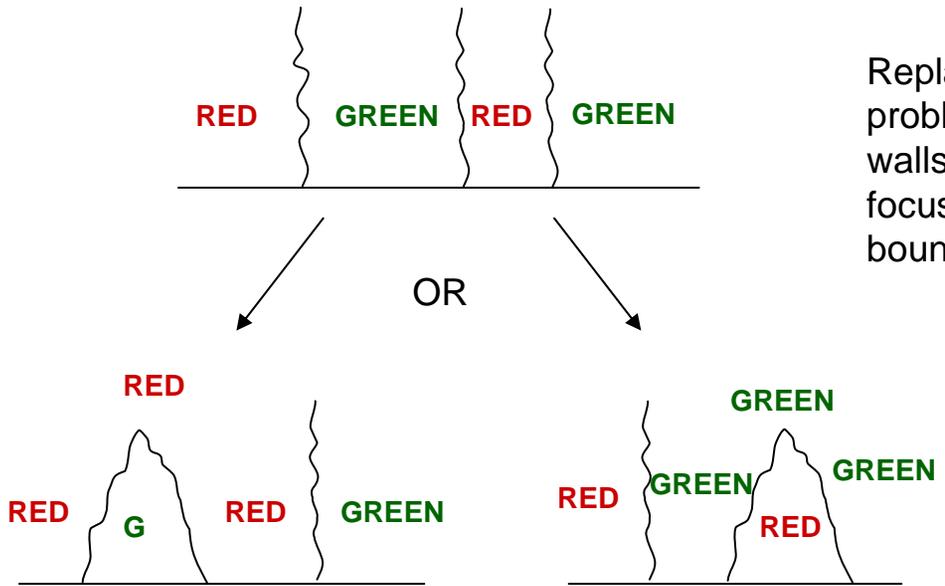
Are segregation boundaries between neutral mutations in bacteria unbiased random walks?



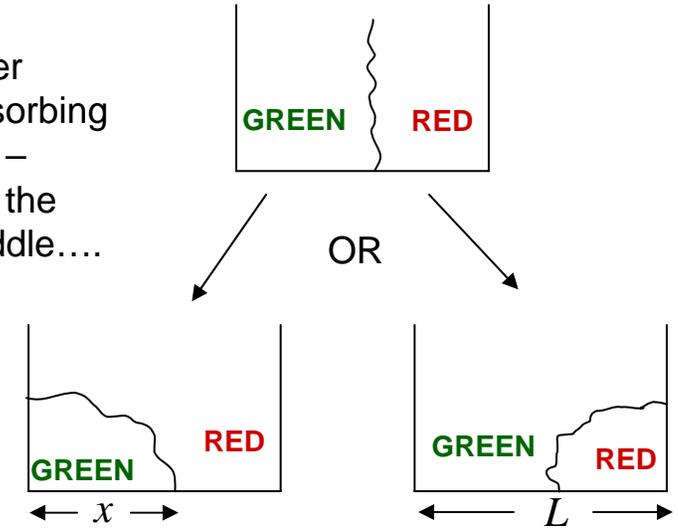
Linear Inoculations: Coarsening of Annihilating Random Walks



Linear Inoculations: Coarsening of Annihilating Random Walks



Replace with simpler problem of rigid absorbing walls on either side – focus on the fate of the boundary in the middle....



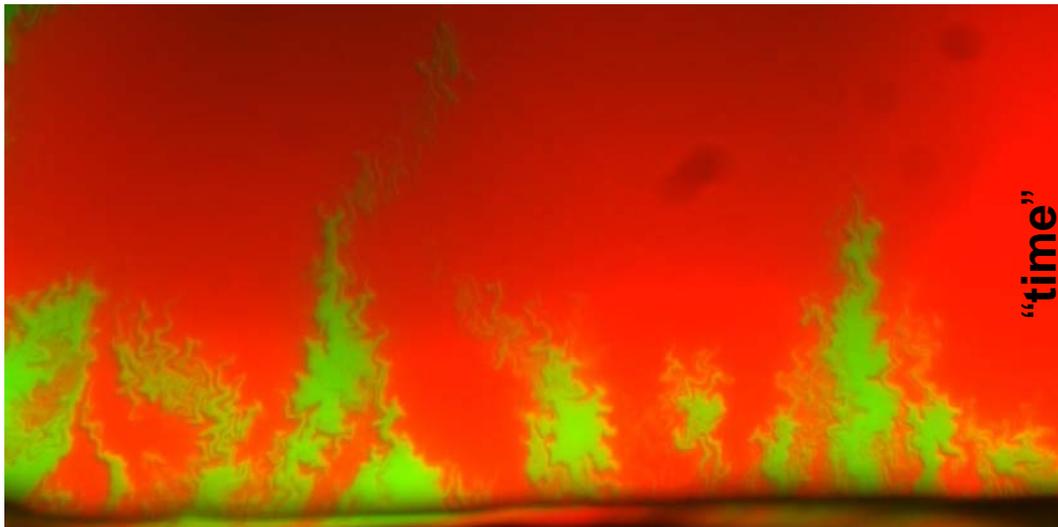
If $D_w =$ wall diffusion constant, length of flares given by....

$$W(x) = \text{mean distance to capture} \\ = x(L-x)/8D_w$$

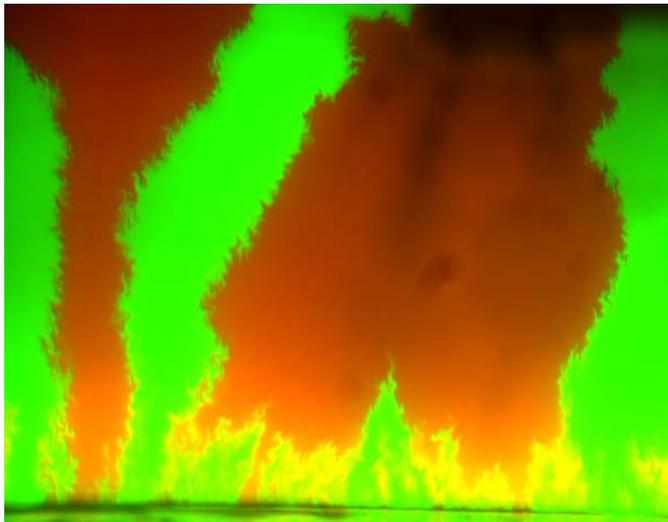
$$n_w(t) = \text{density of walls}$$

$$= 1/\sqrt{2D_w t}$$

$$\propto 1/t^{\zeta}, \text{ in general}$$



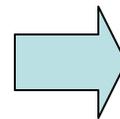
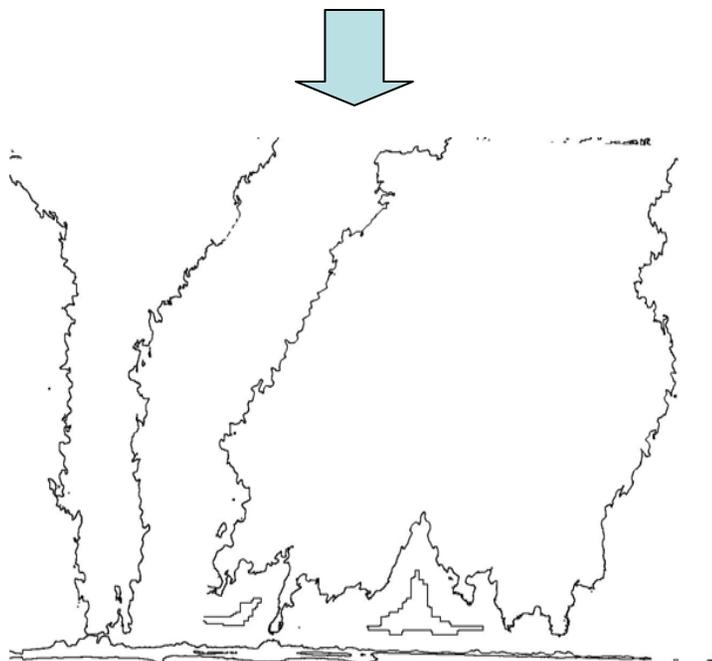
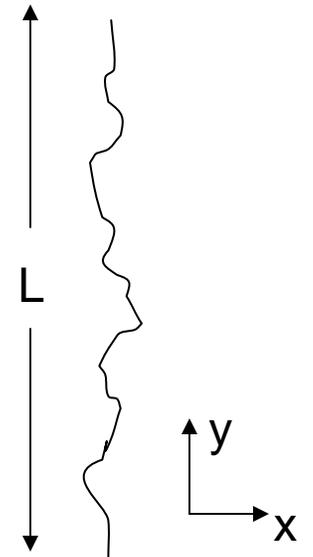
Boundary wandering, linear inoculation



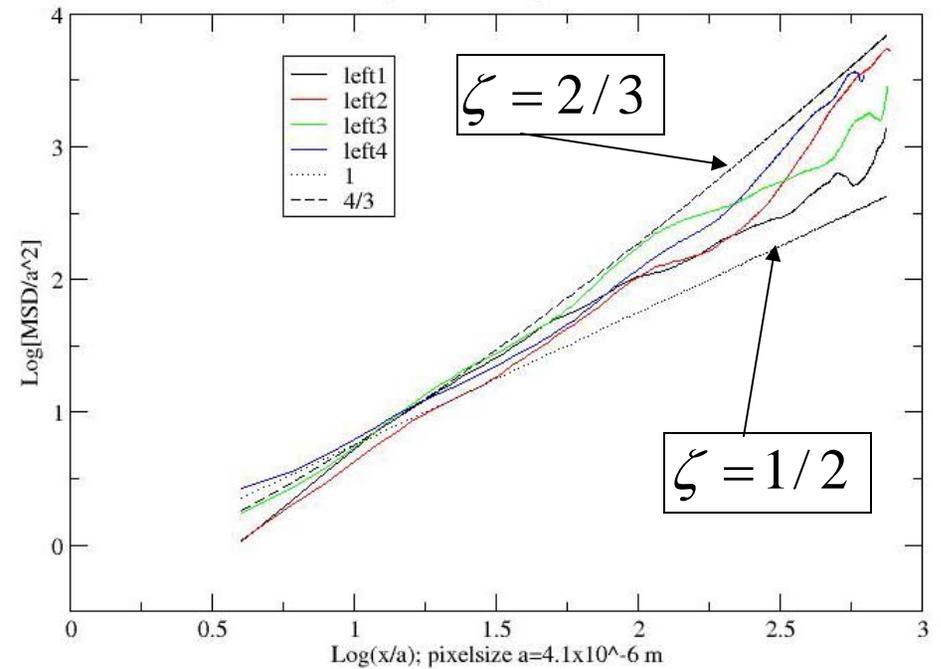
$$\langle [x(L) - x(0)]^2 \rangle \approx \text{const.} \times L^{2\zeta}$$

$\zeta = 1/2$, random walk

$\zeta = 2/3$, "Eden model"



Linear initial conditions
from picture "4-2E7-g8x-series-2"



Acknowledgements and Future Directions

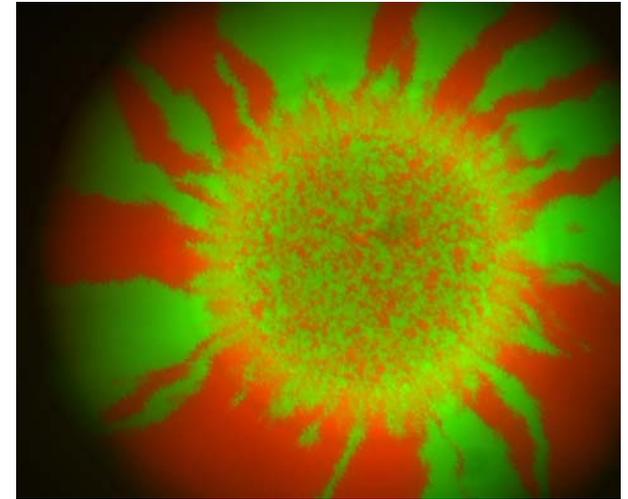


Sharad Ramanathan

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Thanks to Oskar Hallatschek

Thanks to members of the Ramanathan group

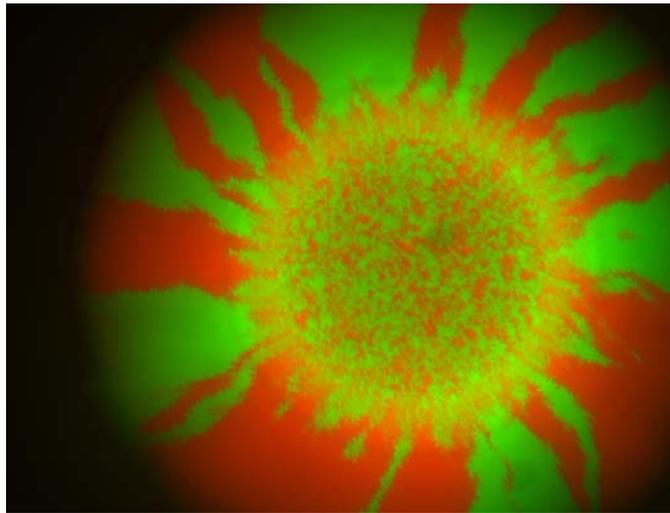


Effect of "Inflation": some sectors never die out!

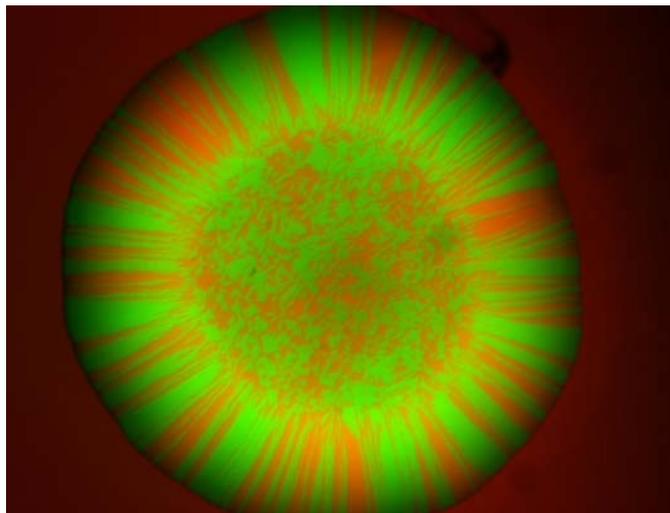
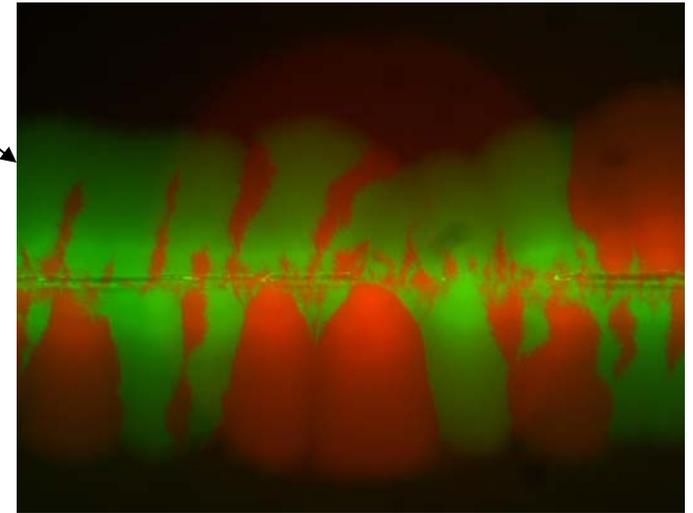
Future Directions

- Effect of 'inflation' and selective pressure on radial inoculations
- Gene surfing in yeast; how do *round* micro-organisms behave?
- Population segregation in *flagellated* bacteria (diffusion constant is larger by four orders of magnitude!)
- Why the crinkled patterns in the early stages? Effect of boundary conditions?

Segregation in *E. coli* vs. segregation in *Mat α* haploid yeast

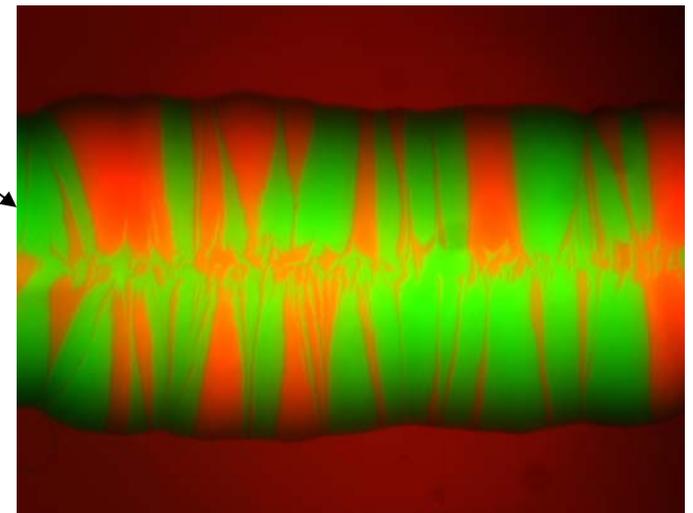


bacteria

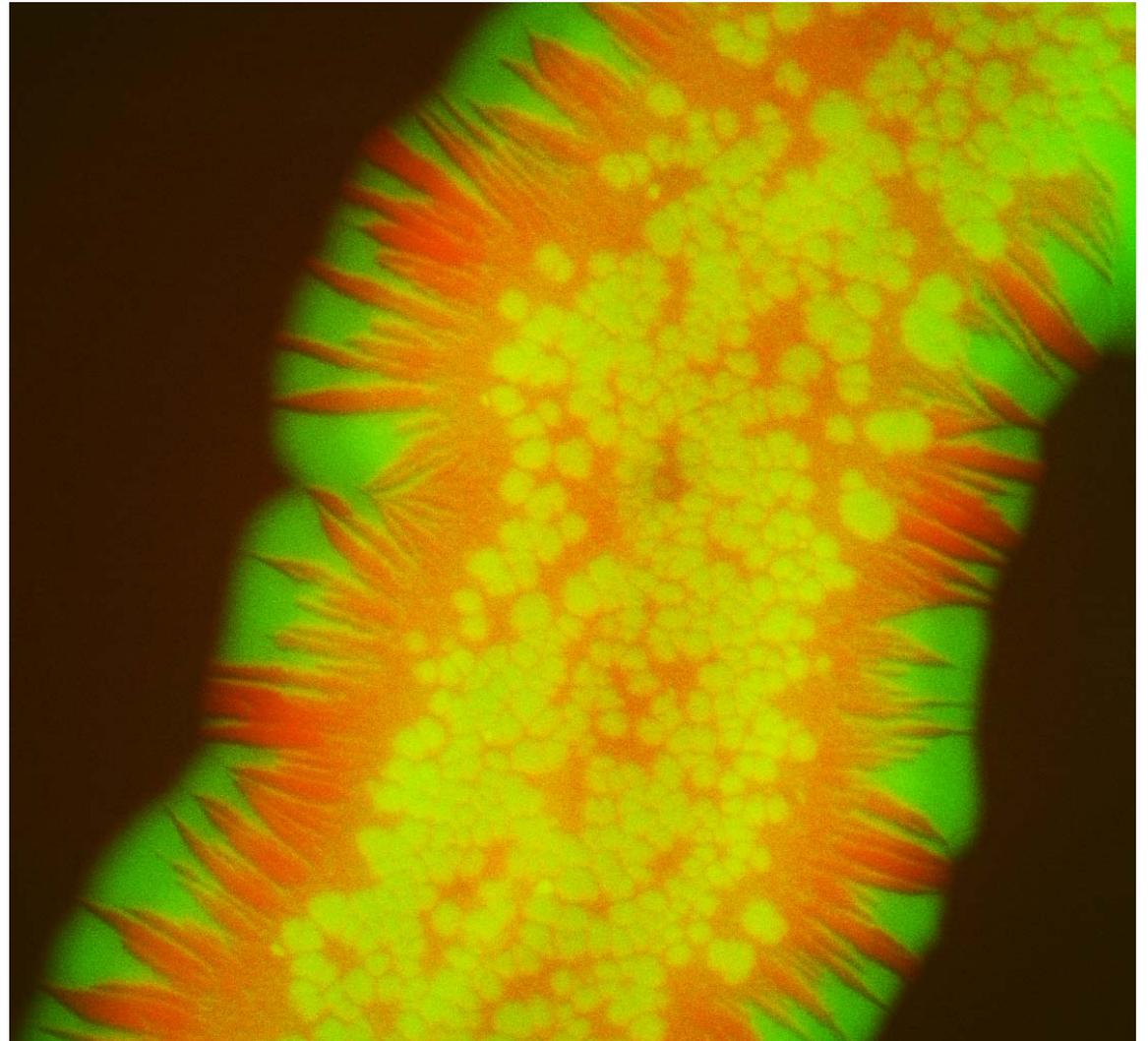
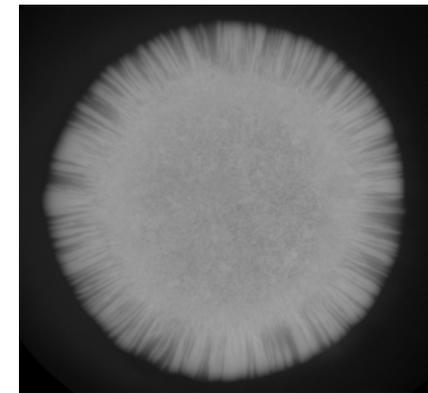
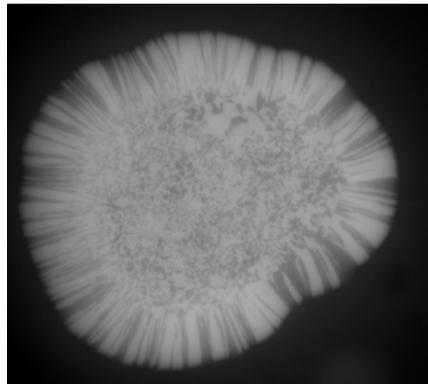
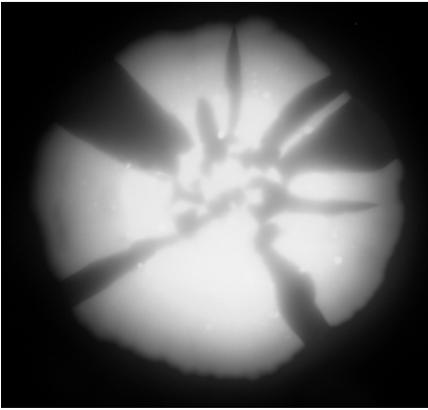


yeast

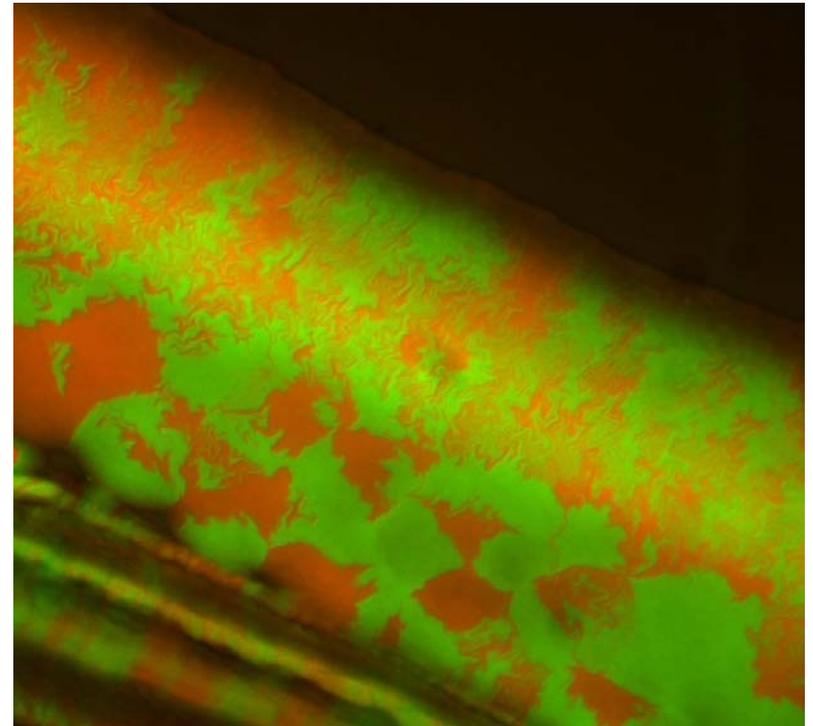
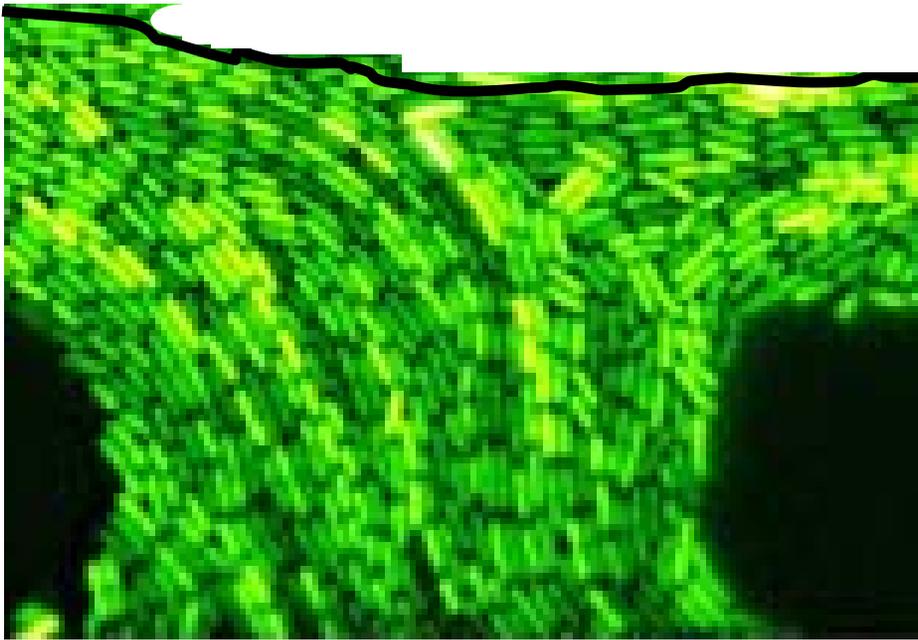
Effective population size probably much larger at the frontier...



Colored yeast with a
selective advantage !



Why are there buckles and folds in the segregation textures?



HOMELAND

adapted from, N. Guido et al. Nature V439, 856 (2006). (J. J. Collins Lab, Boston University)