Equation. (with Konstantin Turitsyn)

Starting equation:

$$-i\partial_z \vec{E} = \partial_t^2 \vec{E} + \frac{2}{3}\left(|E_1|^2 + |E_2|^2\right)\vec{E} + \frac{1}{3}\left(E_1^2 + E_2^2\right)\vec{E}^* + \hat{V}(z)\vec{E}. \quad (1)$$

$\hat{V}(z)$ is fast, i.e.

$$\hat{V}(z) = b\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}, \quad (2)$$

$$\frac{1}{b} \ll 1$$

„Slow variable" $\vec{\psi}$:

$$\vec{E} = T\exp\left(i\int_0^z dz'\, \hat{V}(z')\right)\vec{\psi} \quad (3)$$

If θ is smooth, and its correlation length $l \gg 1/b$, with exponential in bl accuracy:

$$\boxed{\begin{aligned} -i\partial_z \psi_1 &= \partial_t^2 \psi_1 + 2\left(|\psi_1|^2 + \tfrac{2}{3}|\psi_2|^2\right)\psi_1 \\ -i\partial_z \psi_2 &= \partial_t^2 \psi_2 + 2\left(|\psi_2|^2 + \tfrac{2}{3}|\psi_1|^2\right)\psi_2 . \end{aligned}} \quad (4)$$

anisotropic equations.

If $bl \sim 1$, $bz \gg 1$

$$-i\partial_z \vec{\psi} = \partial_t^2 \vec{\psi} + \frac{16}{9}\left(|\psi_1|^2 + |\psi_2|^2\right)\vec{\psi} \quad (5)$$

isotropic (Manakov) equation.

If $bz \sim 1$ averaged form of the equation has no meaning.

Fluctuations of the quantity $T\exp\left(i\int dz\,\hat{V}\right)$ and the mean value are of the same order.

Disorder (PMD) term:

$$i\hat{m}\,\partial_t \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix},$$

$$\hat{m} = \vec{h}\,\vec{\sigma},$$

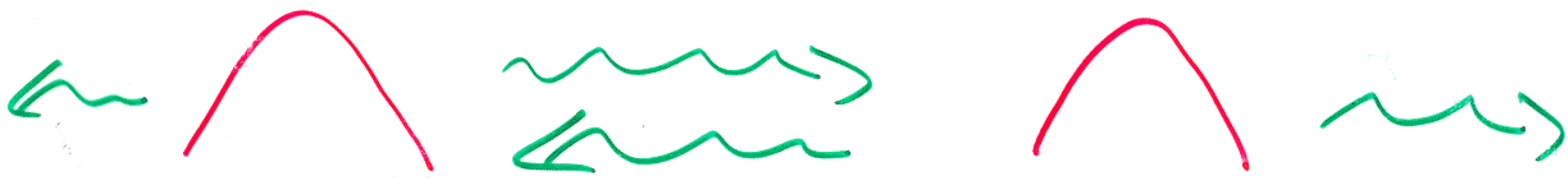
$$\langle h_\alpha(z)\, h_\beta(z')\rangle = D\,\delta_{\alpha\beta}\,\delta(z-z')$$

Soliton will loss energy; degradation law:

$$\eta = \frac{1}{(1+c\cdot Dz)^{1/2}}, \quad c \sim 1.$$

$z_{Lea} \sim 1/D$

Soliton - soliton long range forces due to radiation in disordered medium (M. Chertkov, I. Gabitov, I.K., V. Lebedev)



For anisotropic case there is reflection.

Two linearly polarized solitons + radiation:

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = e^{iz}\left\{\left(\frac{1}{\cosh(t-y_1)} + \frac{e^{i\alpha}}{\cosh(t-y_2)}\right)\cdot\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}v_1\\ v_2\end{pmatrix}\right\}$$

Due to PMD noise $v_2 \neq 0$.

Eigenmodes of linearized equation for v_1 are reflectionless.

But the evolution equation for v_2 near a given soliton:

$$i\partial_z v_2 + \left(\partial_t^2 - 1 + \frac{4}{3}\frac{1}{\cosh^2 t}\right) v_2 =$$

$$= (h_1 + i h_2)\frac{\sinh t}{\cosh^2 t}$$

correspond to the reflection-full potential. This leads to non-zero averaged force:

$$f = \int dt \frac{\sinh t}{\cosh^3 t}\left(4|v_1|^2 + \frac{4}{3}|v_2|^2 + v_1^2 + v_1^{*2}\right)$$

This force for large intersoliton distance y does not depend on y:

$$\langle f \rangle = c \cdot D, \quad c \sim 1$$

$$y(z) \sim \langle f \rangle z^2 \sim c \cdot D z^2,$$

$$z_c \sim \frac{1}{\sqrt{D}}$$

Due to interference effect for 3 solitons there is dependence on phase mismatches.

For Manakov's case:

$$\langle f \rangle = 0.$$