Forward message passing detector for probe storage.

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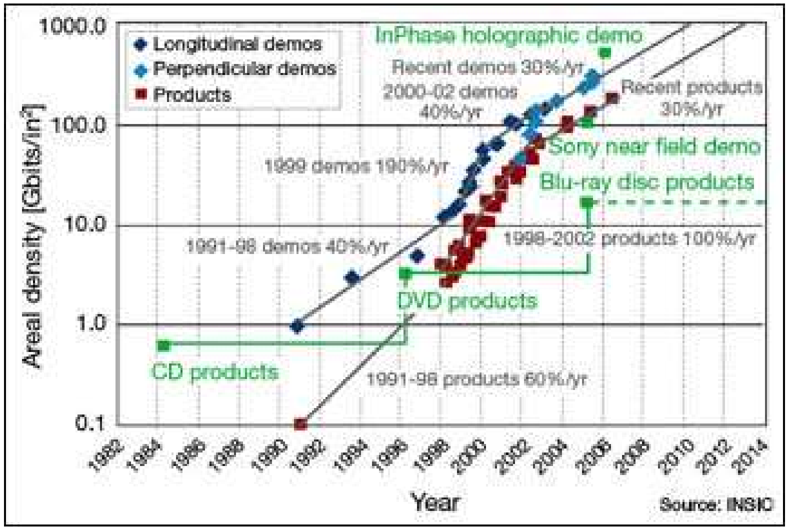
Plan.

- Thermomechanical probe storage.
- Detection/decoding schemes for probe storage devices.
- Channel model for probe storage.
- Soft output detection: forward message passing (FMP).
- FMP detector for probe storage.
- Performance analysis: mutual information and BER
- Performance analysis: SER
- Conclusions

Probe storage.

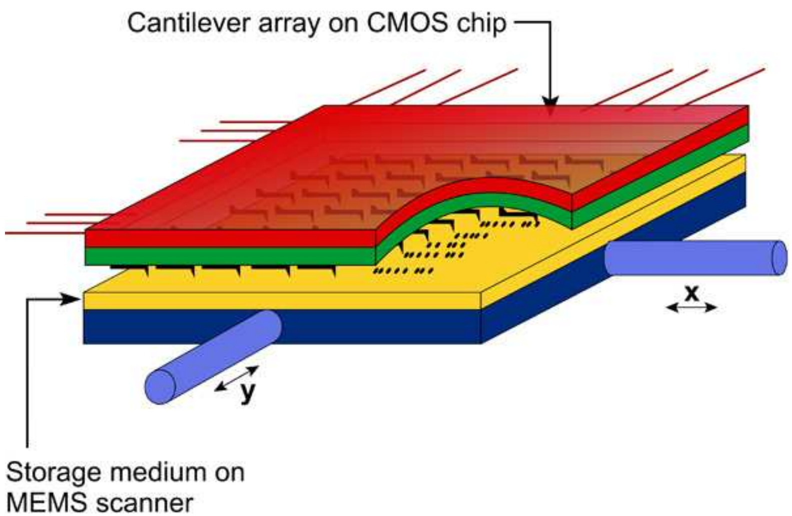
Storage density trends.

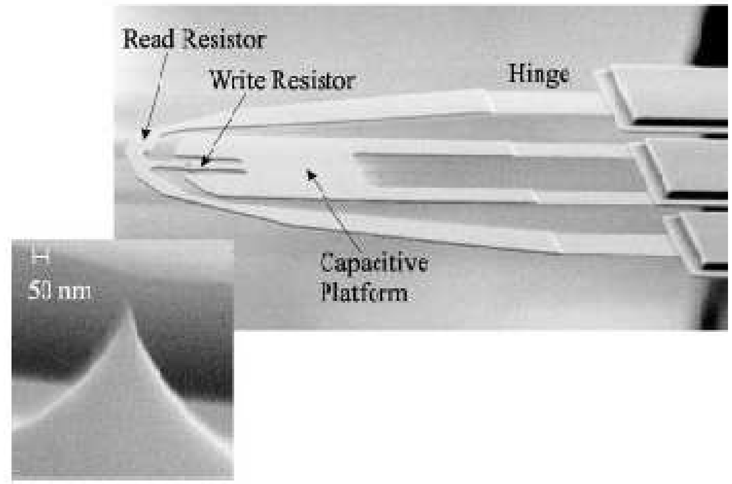


- Magnetic storage density is expected to reach $1\ Tb/in^2$
- Further growth is limited by the superparamagnetic effect.

- Probe storage has already demonstrated $1\ Tb/in^2$ with $4\ Tb/in^2$ demonstrators being developed.

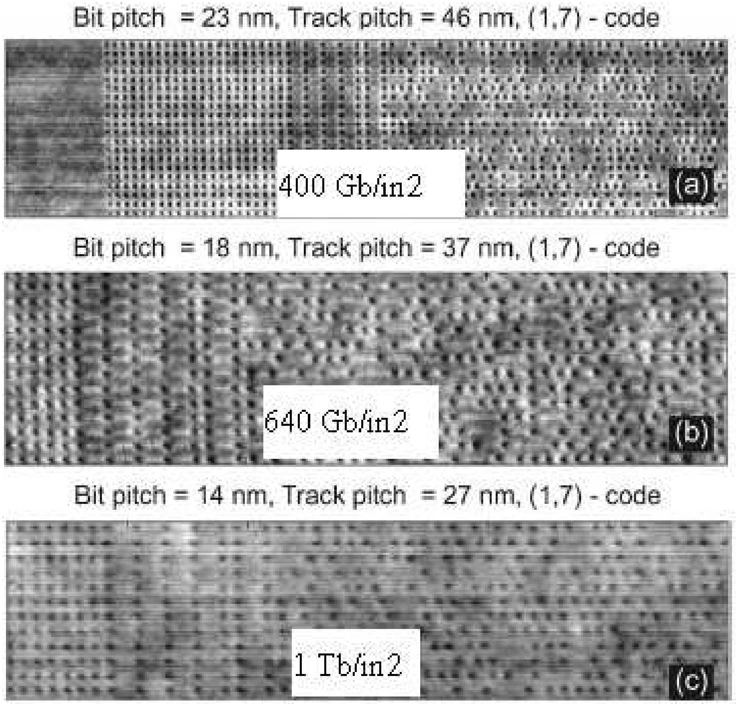
IBM's Thermomechanical Probe Storage Concept





- Thin polymer medium is positioned under the array of 64×64 atomic force probes.
- Each probe operates in its own field of size $100\mu m \times 100\mu m$. Tip radius $\sim 10\ nm$.
- Encoded data are stored as pits on the surface of the medium.

Read/Write



- Writing: the probe's tip is heated and pressed into the softened polymer film
- Reading: The probe heated to a smaller T follows the landscape of the polymer surface

- A probe inserted into a pit is cooler than the probe whose tip touches the surface. These variations are captured using a thermo-resistive sensor

Channel Model

Non-linear Inter-symbol Interference (ISI)

$$I_k(x_{k-1}, x_k, x_{k+1})$$

$$= x_k$$

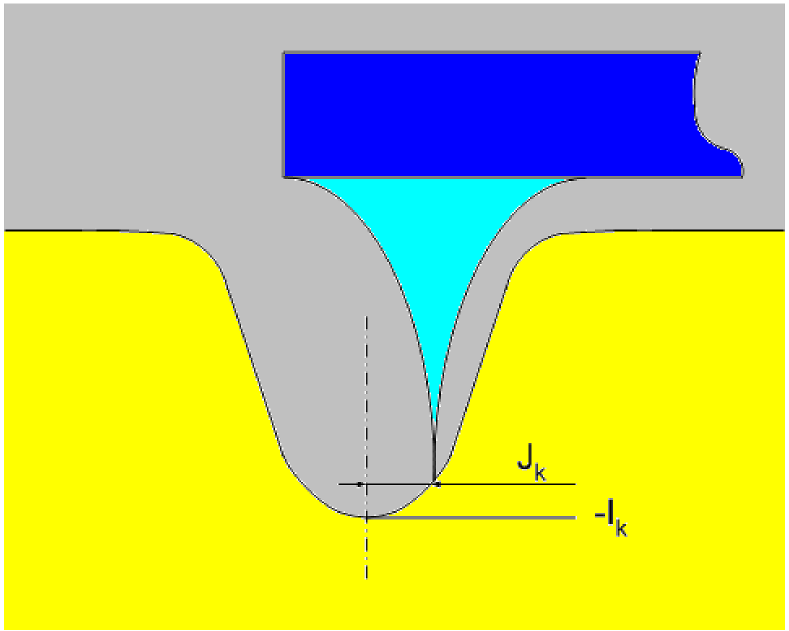
$$+(\alpha - 1)x_k x_{k+1}$$

$$+\beta x_k x_{k-1}$$

- Ideal readout at the k-th sampling point
- A signal due to an isolated pit at k
- Reduction in the signal strength due to plastic displaced from the $(k+1)$-st pit
- Signal enhancement due to plastic displaced into the $(k-1)$-st pit

- For experiments at $1\ Tb/in^2$, $\alpha \approx 0.8$, $\beta \approx 0.1$
- α, β depend on write parameters, tip shape and medium material properties

Position Jitter



- Jitter $=$ positioning error
- $J << $ pit width
- $\Delta I_k \sim J_k^2$
- Over 40% of total noise power is due to data-dependent position jitter

$$r_k \approx I_k \cdot \left(1 - \left(\frac{\sigma_j}{h} W_k\right)^2\right) + \sigma_e N_k,$$

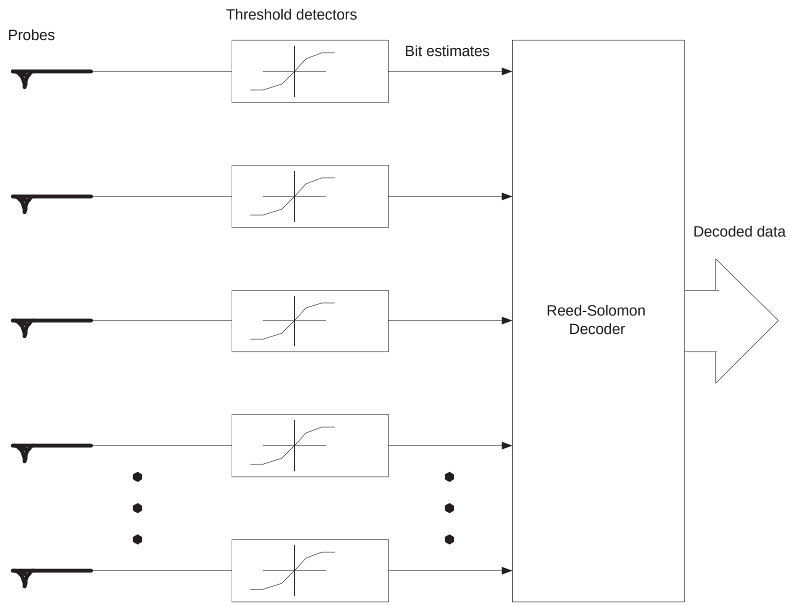
W_k, N_k are independent normal r. v.'s, h is the pit's radius of curvature; σ_j, σ_e are the strengths of jitter and electronics noise correspondingly

Statistics of Signal Distortion

- Let $\eta_k = \frac{r_k - I_k}{I_k}$
- Let $\epsilon = \frac{\sigma_j}{h}$, $\delta = \frac{\sigma_e}{I_k}$.
- $\rho(\eta) \sim \frac{Const_-}{|\eta|^{1/2}} e^{2\frac{\eta}{\epsilon^2}}, \;\; \eta << -\epsilon^2$
- $\rho(\eta) \sim \frac{Const_+}{\eta^{1/2}} e^{-\frac{\eta^2}{2\delta^2}}, \;\; \eta >> \delta$
- Signal distortion is **non-Gaussian**

Detection/decoding

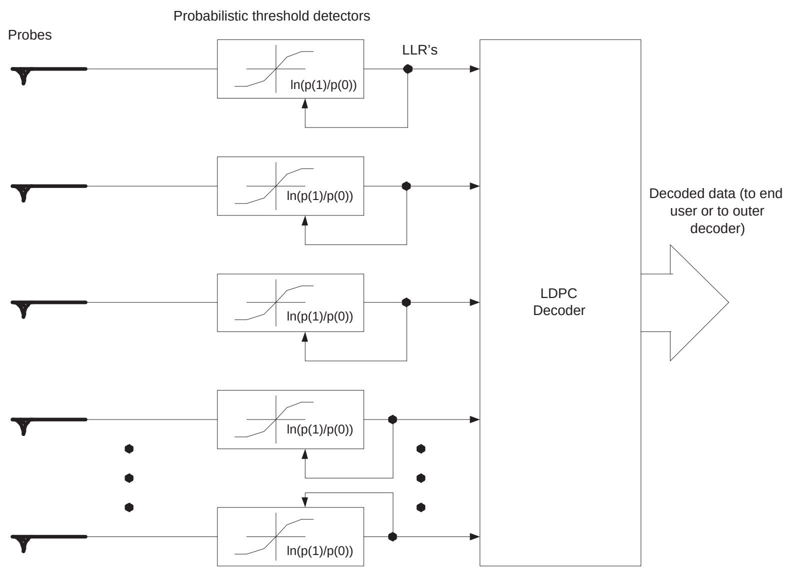
The currently employed scheme



- Read channel: hard output threshold detector
- ECC: Reed-Solomon code

- HDD read channel: a sector of data is detected as the *most likely* binary string given the digitised received string using Viterbi algorithm.
- A significant increase of recording density beyond $1\ Tb/in^2$ would require a significantly more advanced detection decoding scheme

Desired scheme



- Read channel: soft output data detector
- ECC: Soft input decoder for $LDPC$, $LDPC \circ RS$, $SPC \circ RS$, etc. code

- MAP detector per probe is too complex
- Any easy ways to generate soft outputs?

Soft detection via forward message passing

Soft threshold detector.

$$LLR_k \stackrel{def}{=} \ln \frac{Pr(x_k{=}1|r_k)}{Pr(x_k{=}0|r_k)} \stackrel{Bayes}{=\!=\!=} \ln \frac{Pr(r_k|x_k{=}1)}{Pr(r_k|x_k{=}0)}$$

- Threshold bit estimate: $\hat{x}_k = sign LLR_k$
- Information contained in $r_{k'}$: $k' \neq k$ is not used in the computation of LLR_k.

Forward message passing detector.

- Assume that r_k depends on $x_k, x_{k\pm1}$ only.
- Let $LLR_k = ln\left(\frac{Pr(x_k=1|\vec{r}_{k+1})}{\Pr(x_k=0|\vec{r}_{k+1})}\right)$,where $\vec{r}_k = \ldots r_{k-3}r_{k-2}r_{k-1}r_k$. Then

$$Pr(\vec{r}_k \mid x_{k+1}, x_k, x_{k-1}) = \frac{1}{2}Pr(r_k \mid x_{k+1}, x_k, x_{k-1}) \sum_{x_{k-2}=0}^{1} Pr(\vec{r}_{k-1} \mid x_k, x_{k-1}, x_{k-2})$$

- Message is an 8-dimensional vector of probabilities $Pr(\vec{r}_k \mid x_{k+1}, x_k, x_{k-1})$ propagated left-to-right using transfer matrix built out of conditional probabilities $Pr(r_k \mid x_{k+1}, x_k, x_{k-1})$.

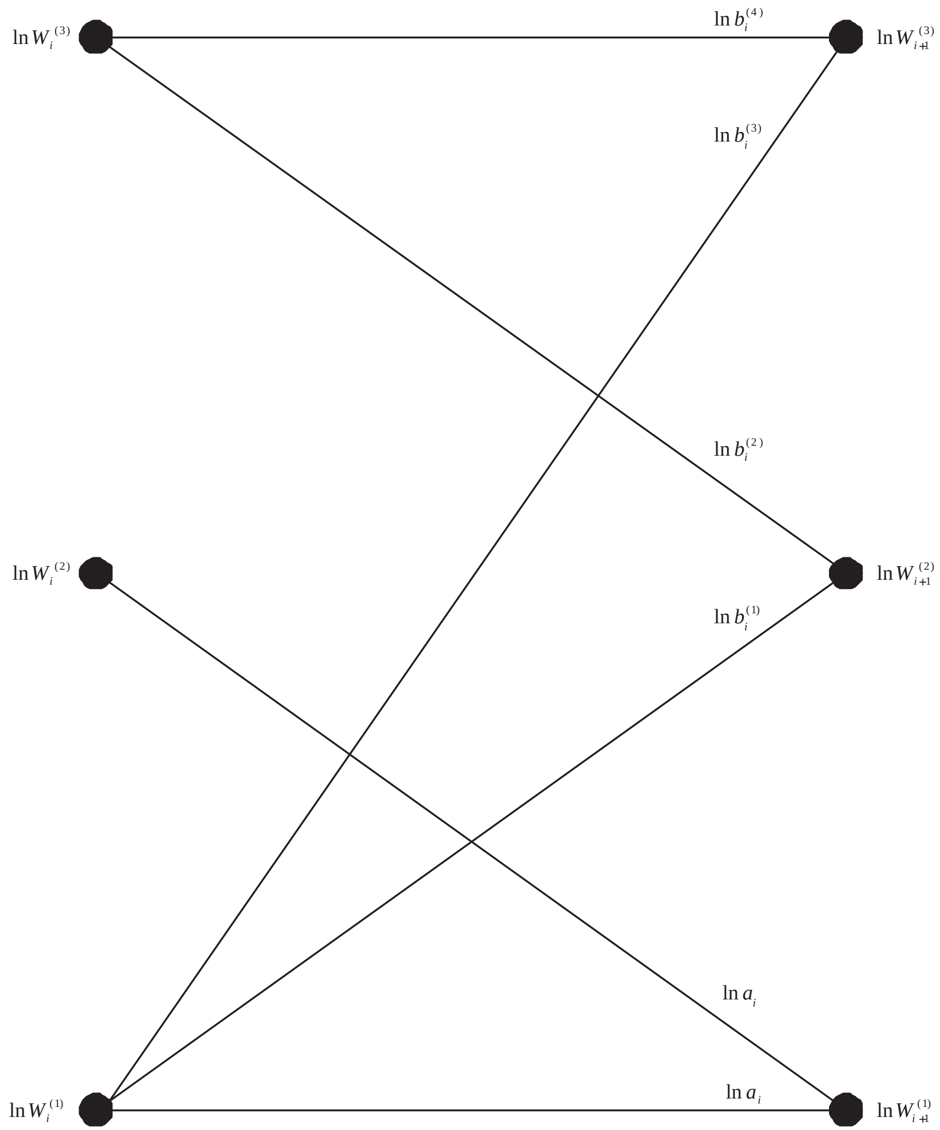
Transition matrix

$$
\mathbf{T} = \begin{pmatrix}
\alpha_i & \alpha_i & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \alpha_i & \alpha_i & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \beta_i^{(1)} & \beta_i^{(1)} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \beta_i^{(2)} & \beta_i^{(2)} \\
\alpha_i & \alpha_i & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \alpha_i & \alpha_i & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \beta_i^{(3)} & \beta_i^{(3)} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \beta_i^{(4)} & \beta_i^{(4)},
\end{pmatrix}
$$

where α's and β's are conditional probabilities.

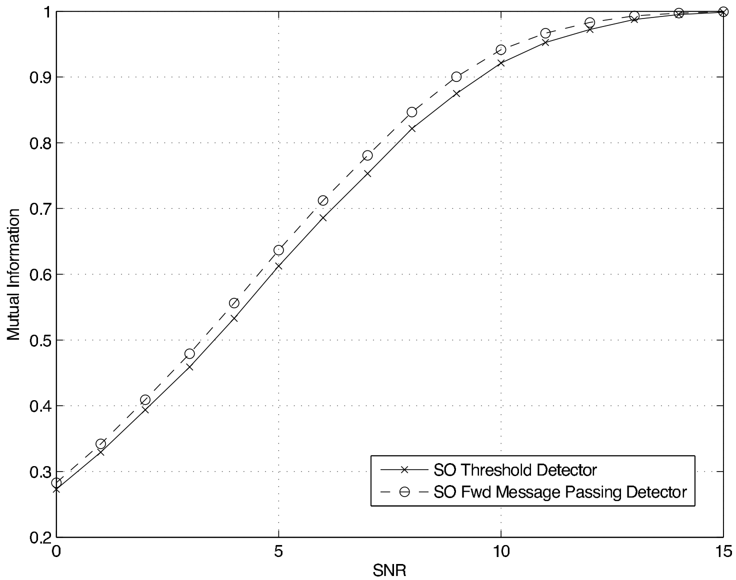
- T_k is time-dependent. But, there are 4 time independent right null vectors and 2 time independent left null vectors.
- Forward recursion can be reduced to a 3×3 recursion

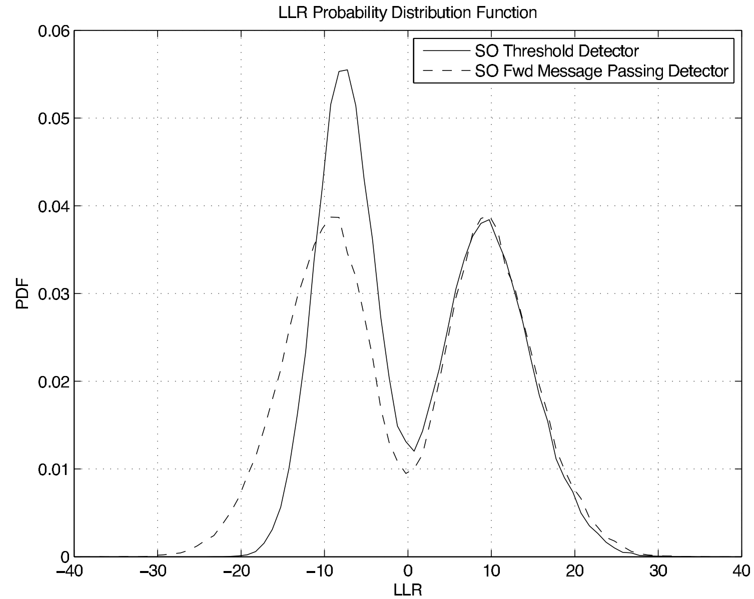
Reduced recursion.



Performance analysis

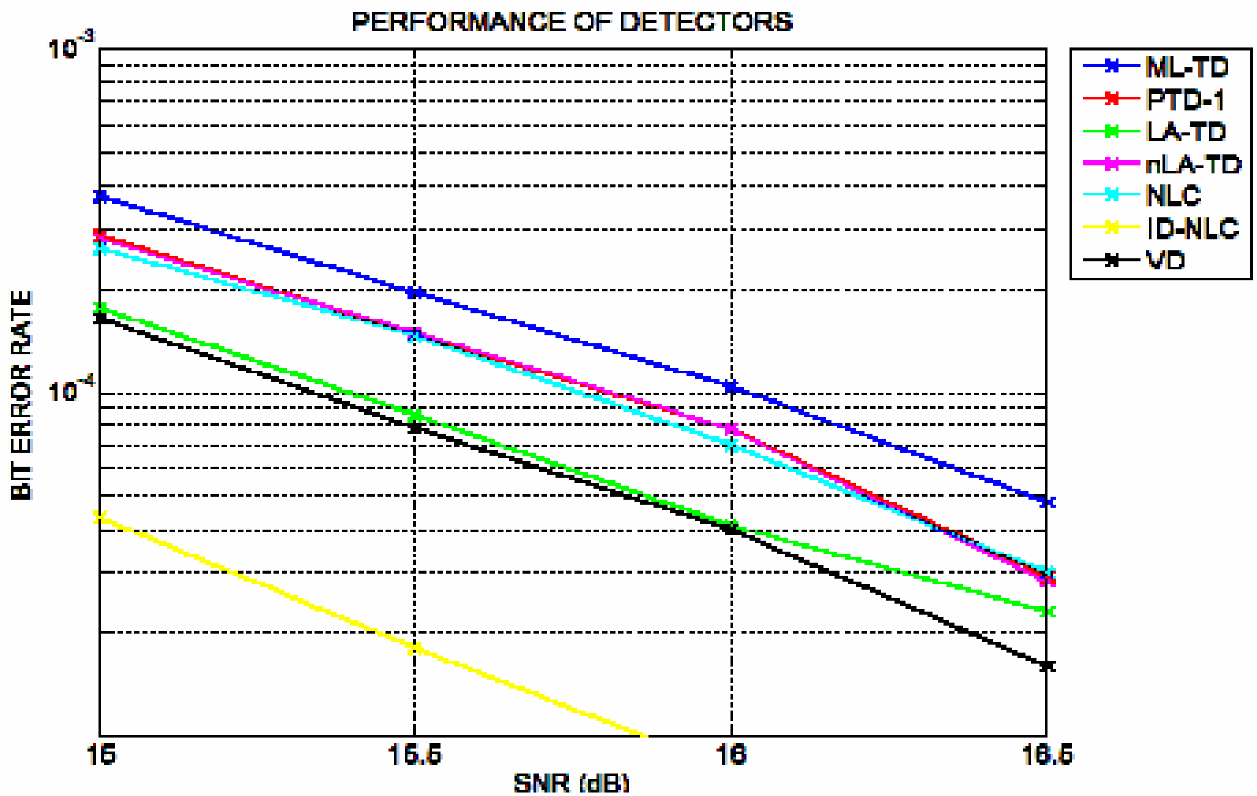
Mutual information





- Mutual info between data and output LLR's:
$I(X,L) = \mathbf{E}_X\left(\int_{-\infty}^{\infty} dl\rho(l \mid x) log_2\left(\frac{\rho(l|x)}{\rho(l)}\right)\right)$
- FMP detector resolves the asymmetry of LLR's. Channel capacity is increased by about 5% compared to THD channel

Bit error rate



- The performance of FMP (green curve) matches the performance of Viterbi detector (black curve)

Sector error rate: large deviations

Outer code: RS(w, τ, nN). **Inner code**: block size is $nw + t$ bits. Symbol error counts for different IC blocks are independent identically distributed random variables. Let $\vec{p} = \{p_0, p_1, \ldots, p_n\}$ be the probability distribution of symbol error count ξ in an IC block such that $\mathbf{E}(\xi) < \tau$. Then

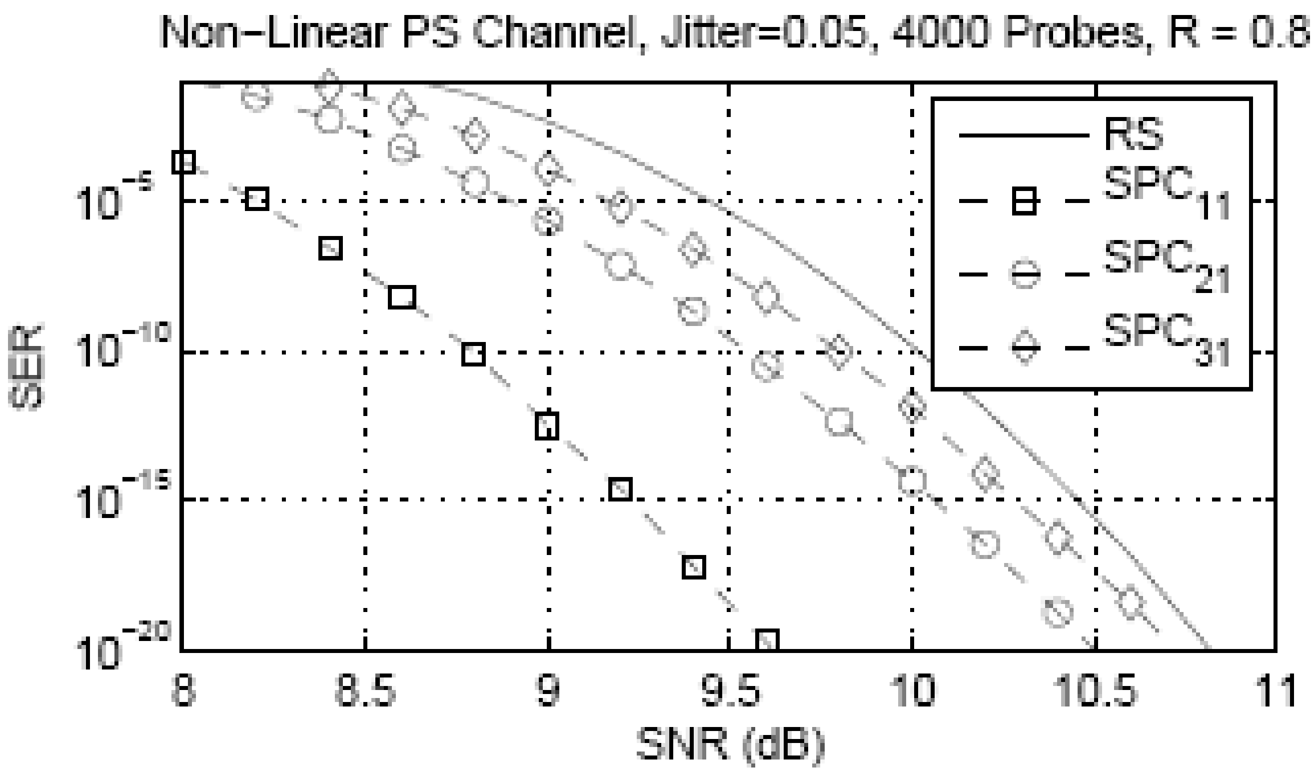
$$\frac{ln(P_{se})}{N} \nearrow -D_{KL}\left(\vec{q}||\vec{p}\right), \quad \text{as } N \to \infty.$$

where $\vec{q}$ is the effective probability distribution given by

$$q_k = \frac{p_k \mu^k}{\sum_{m=0}^{n} p_m \mu^m}, \;\; k = 0, 1, \ldots, n$$

and μ is the unique positive solution of the critical point equation, $\sum_{k=0}^{n} (k - n\tau)\, p_k \mu^k = 0$; D_{KL} is relative entropy

Sector error rate comparison



Conclusions-I

- Probe storage DSP is challenging to the extreme: on the one hand we have a very noisy channel, on the other - the allowed complexity of read channel per probe is severely restricted
- Forward message passing detector allows a generation of soft outputs at the complexity cost of a 3-state Viterbi detector without traceback unit with the performance matching that of the full 4-state Viterbi detector matched to the non-linear thermomechanical channel
- Large deviations analysis leads to an analytic expression for SER in probe storage, which is useful for sufficiently short inner codes
- Soft input $SPC + RS$ code outperforms hard input RS code of the same rate by about $1\ dB$ at $SER = 10^{-15}$.

Conclusions-II

- A twist in the tale: asymptotically, RS code is better!
- Research supported by PROTEM FP6 European network grant
- The reported results will be published in the proceedings of ICC2008.