Hamiltonian Cellular Automata in 1D

Pawel M. Wocjan

School of Electrical Engineering and Computer Science

University of Central Florida

Orlando

wocjan@eecs.ucf.edu

Joint work with

- Daniel Nagaj (MIT)

  `quant-ph/0802.0886`

Quantum Circuit Model

- abstracts from the details of concrete physical systems and states that the required elementary operations are
  - initialization in basis states
  - implementation of one and two-qubit gates
  - measurement of single qubits in basis states
- many other models such as measurement-based qc, adiabatic qc, topological qc
- however, the common principle underlying all these models is that the computation process is driven by applying a sequence of control operations

Hamiltonian Quantum Computer

- prepare an initial state in the computational basis that encodes both the data and program
- let the Hamiltonian time evolution act undisturbed for a sufficiently long time
- measure a small subsystem in the computational basis to obtain the result of the computation with high probability

Hamiltonian Quantum Cellular Automato

- more specifically, we call it a Hamiltonian quantum cellular automaton provided that
  - the Hamiltonians acts on qudits that are arranged on some lattice
  - is invariant with respect to transitions along the symmetry axis of the lattice
  - contains only finite range interactions
- most natural Hamiltonians have these properties, so it is important to construct HQCA that are as close as possible to natural interactions

Continuous-time versus Discrete-Time QC

- the evolution of discrete-time QCA proceeds in discrete update steps (tensor products of local unitary operations)
- the execution of updates on overlapping cells is synchronized by external control
- in contrast, the states of the HQCA change in a continuous way according to the Schrödinger equation (time-independent Hamiltonian)
- all the couplings (interactions) are present all the time $\Rightarrow$ they have to include a mechanism that ensures the logical operations are executed in the correct order

Motivation

- fundamental question in thermodynamics of computation how to realize computational processes within a closed physical system
- HQCA could trigger new ideas for reducing the set of necessary control operations in current proposals by using the inherent computational power of the interactions
- this model can show the limitations of current and future methods in condensed for simulating the time evolution of translationally invariant systems

Related works

- Benioff/Feynman/Margolus
- Janzing/W.
  initial state is a canonical basis state; finite range interactions in 2D

  univ. nn H. in 2D acting on qutrits; is trans. inv. only if translated over several lattice sites
- Vollbrecht/Cirac
  univ. trans. inv. nn H in 1D acting on 30-dimensional qudits
- Chase/Landahl
  univ. nn H in 1D acting on 8-dimensional qudits; but not trans. inv.

Our Hamiltonian

- acts on 10-dimensional qudits arranged on a 1D chain
- contains only nearest-neighbor couplings (pair interactions)
- is translationally invariant (translation by one cell)

Universal Quantum Gates

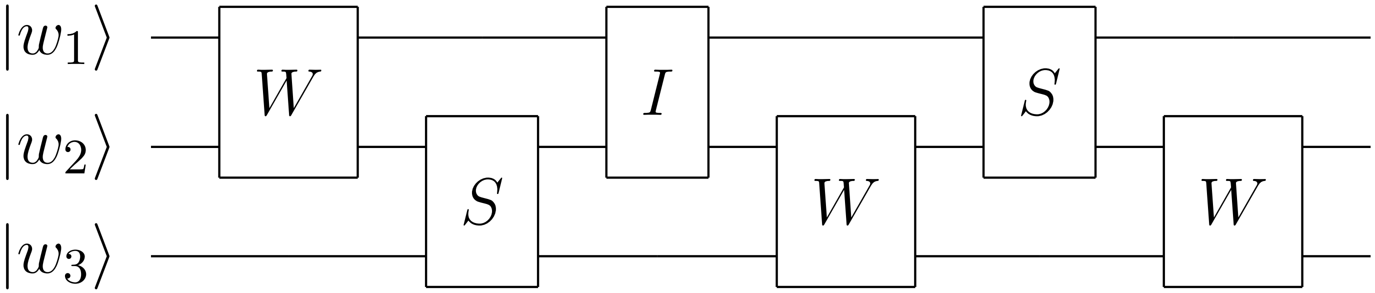
- we can realize universal QC with the controlled gate

$$W = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

  provided that we can apply it to arbitrary pairs of qubits

- however, we want to operate only on adjacent qubits $\Rightarrow$ we have to include the swap gate S

Universal "Staircase" Quantum Circuits



Hilbert space of the HQCA

- a cell $\mathcal{H}_c$ consists of a program subcell and a data subcell

$$\mathcal{H}_c = \mathcal{H}_p \otimes \mathcal{H}_d = \mathbb{C}^5 \otimes \mathbb{C}^2$$

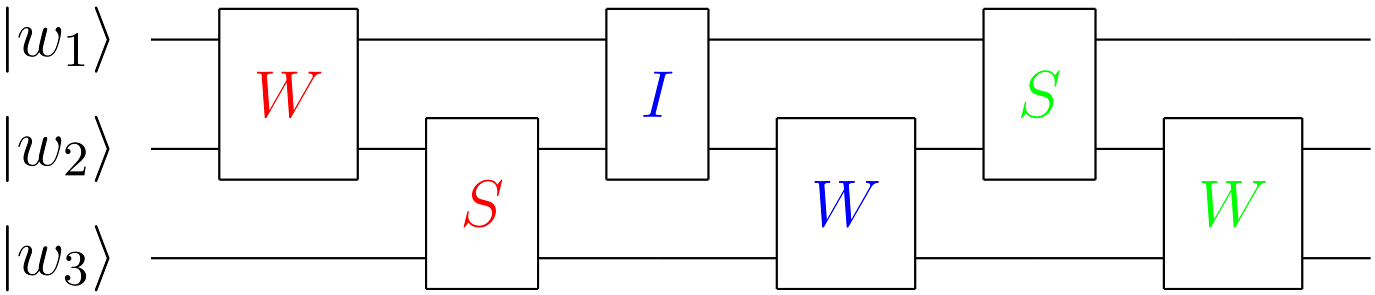
- the canonical basis states of $\mathcal{H}_p$ are
  - $| \blacktriangleright \rangle$ pointer symbol
  - $| \, . \, \rangle$ empty spot symbol
  - $|W\rangle$, $|S\rangle$, $|I\rangle$ gates symbols

- the canonical basis states of the $\mathcal{H}_d$ are $|0\rangle$ and $|1\rangle$

- the Hilbert space of the HQCA is

$$\mathcal{H}_c \otimes \mathcal{H}_c \otimes \cdots \otimes \mathcal{H}_c$$

Initialization of the HQC



the initial state $|\varphi\rangle$ is a canonical basis state

■	■	▶	■	■	▶	■	■	▶	I	W	S	I	I	W	I	S	W	I
0	0	0	0	0	0	0	0	0	w_1	w_2	w_3	0	0	0	0	0	0	0

Transition Rules of the HQCA

■	■	▶	■	■	▶	■	■	▶	I	W	S	I	I	W	I	S	W	I
0	0	0	0	0	0	0	0	0	w_1	w_2	w_3	0	0	0	0	0	0	0

Transition Rules of the HQCA

▪	▪	▶	▪	▪	▶	▪	▪	I	▶	W	S	I	I	W	I	S	W	I
0	0	0	0	0	0	0	0	0	w_1	w_2	w_3	0	0	0	0	0	0	0

I has been applied to 0 and w_1

Transition Rules of the HQCA

▪	▪	▶	▪	▪	▶	▪	I	▪	▶	W	S	I	I	W	I	S	W	I
0	0	0	0	0	0	0	0	0	w_1	w_2	w_3	0	0	0	0	0	0	0

I has moved one spot to the left

Transition Rules of the HQCA

▪	▪	▶	▪	▪	▶	▪	I	▪	W	▶	S	I	I	W	I	S	W	I
0	0	0	0	0	0	0	0	0	w_1	w_2	w_3	0	0	0	0	0	0	0

W has been applied to w_1 and w_2

Transition Rules of the HCA

■	■	▶	■	■	▶	■	I	■	W	S	▶	I	I	W	I	S	W	I
0	0	0	0	0	0	0	0	0	w_1	w_2	w_3	0	0	0	0	0	0	0

S has been applied to w_2 and w_3

Transition Rules of the HCA

- a gate particle $A \in \{W, S, I\}$ can move one spot to the left if that spot is empty

$$\textbf{.} \; \Big| \; A \quad \rightarrow \quad A \; \Big| \; \textbf{.}$$

- if a gate particle meets the pointer ▶, then A and ▶ swap positions and the gate A is applied to the two qubits below

$$\begin{array}{c|c} \blacktriangleright & A \\ \hline x & y \end{array} \quad \rightarrow \quad \begin{array}{c|c} \blacktriangleright & A \\ \hline \multicolumn{2}{c}{A(x,y)} \end{array}$$

Hamiltonian of the HCA

- the Hamiltonian is

$$H = \sum_j (F + F^\dagger)_{(j,j+1)}$$

- the forward-time operator

$$F = \sum_{A \in \{W,S,I} \left[|A \,\blacksquare\rangle\langle \blacksquare\, A|_{p,p'} \otimes I_{d,d'} + |A \blacktriangleright\rangle\langle \blacktriangleright A|_{p,p'} \otimes A_{d,d'} \right]$$

realizes the transition rules

Analysis of the Run-Time

- the time evolved state $|\varphi(t)\rangle = \exp(-iHt)|\varphi\rangle$ can be written as

$$\sum_C \alpha_C(t)|C\rangle \otimes |\psi_C\rangle \otimes |00\cdots 0\rangle$$

  - C is a configuration of the program band
  - $|\psi_C\rangle$ is the state of the data register corresponding to C
  - $|00\cdots 0\rangle$ is the (invariant) state of the data band outside the data register

Analysis of the Run-Time

- choose t so that all gate particles of the program code have moved to the left of the data register

Dynamics of hard-core bosons in 1D

- $W\ S\ I \quad \mapsto \quad 1$ $\qquad$. ▶ $\quad \mapsto \quad 0$
- the initial state is mapped onto

  $$\cdots 000000000000000000|{\color{blue}1111111111111}1111111111111111\cdots$$

- how long does it take the L blue particles to move to the left of $|$ with high probability under the dynamics

  $$H = \sum_j \Big(|01\rangle\langle 10| + |10\rangle\langle 01| \Big)_{j,j+1}$$

  if we let the system evolve for t chosen uniformly at random from $[0, \Theta(L \log L)]$, the probability is at least $5/6 - O(1/\log L)$

Conclusion/Future Research

- Hamiltonian Quantum Cellular Automaton
  - does not require any control operations during computational process
  - requires only the preparation of product states in the canonical basis and measurement of a small subsystem in the canonical basis
- reduce the dimension of the cells even further
- incorporate fault-tolerance in HQCA