Quantum Shock Waves and Emergence of Quantized Edge States

P. Wiegmann

Together with **Alexander Abanov** and **Eldad Bettelheim**

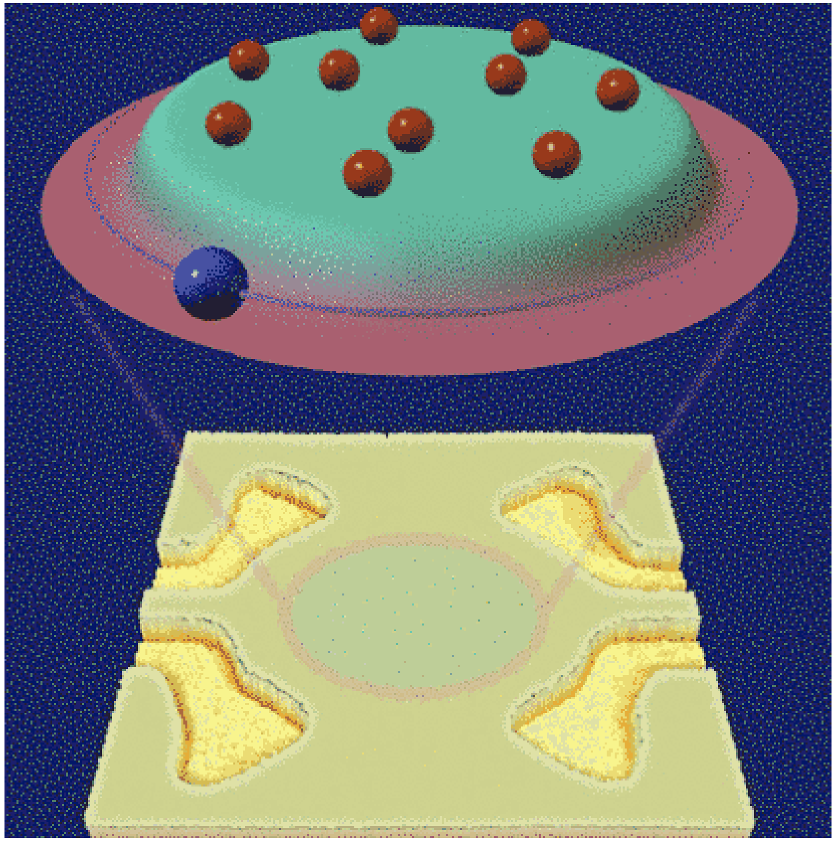
FQHE - topological quantum liquid:

a gapped state on a closed manifold with a monodromy.

The ground state is degenerate on a multiply-connected closed maniold

Probing is possible only through a boundary

Point like, or extended



Extended boundary - an Edge

Topological liquid is not gapped any longer -

Edge states propagate as a sound - subject to decoherence

Edge state carries the information of the topology:

Edge tunneling, noise measurements.

Topological aspects:

fractional charge and braiding

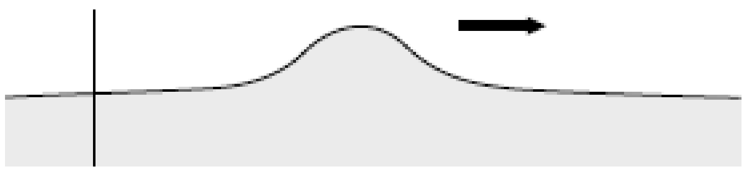
can be measured in edge states out-of- equlibrium

Quantum Shock Wave

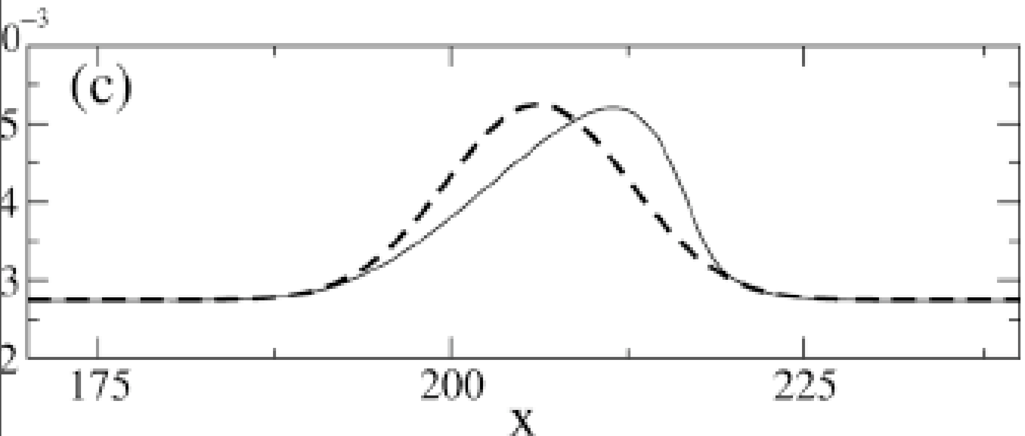
A smooth bump in density propagating:

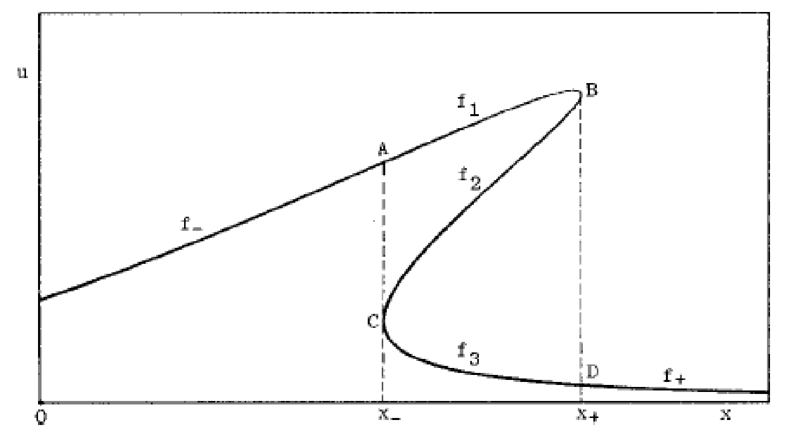
all gradients << Fermi scale

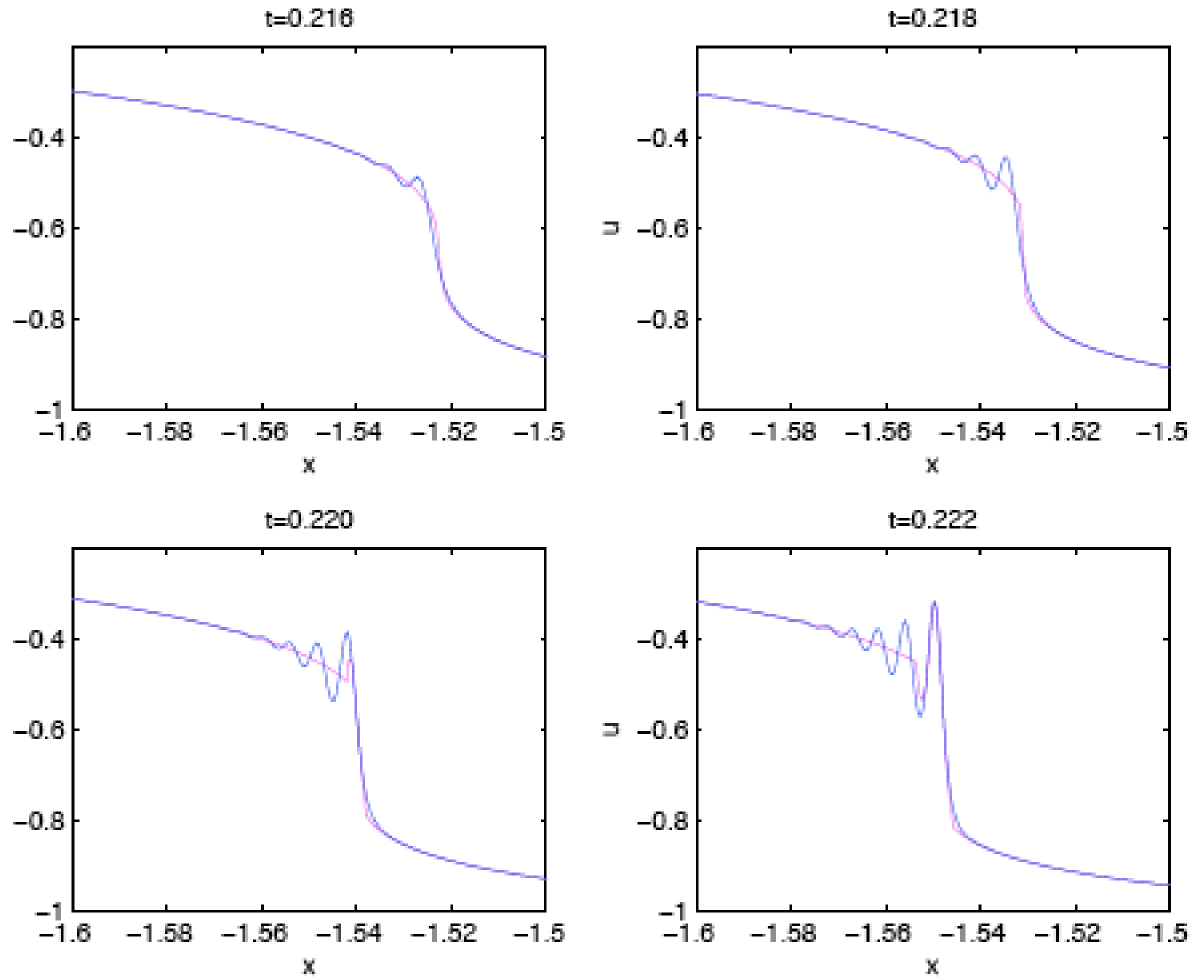
$\rho(x)$



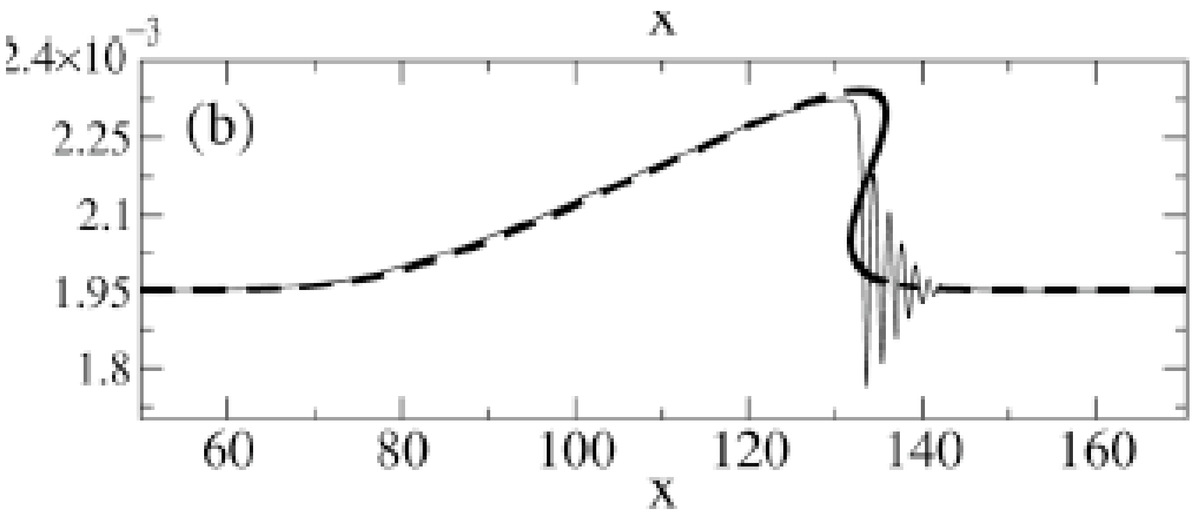
(i) transport on the edge is essentially non-linear,

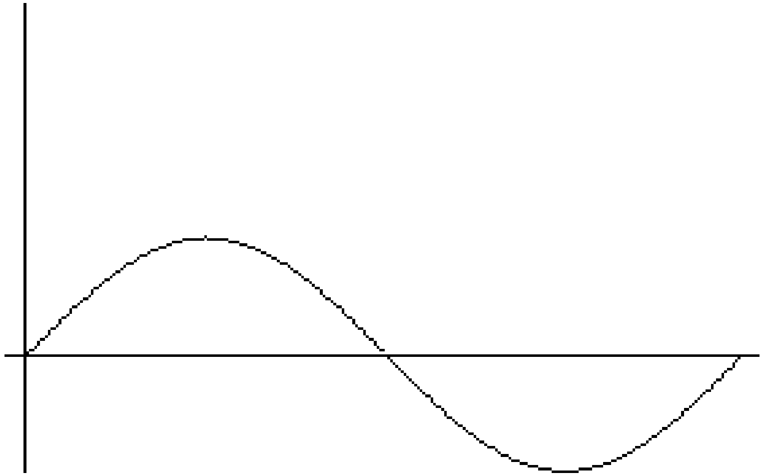





t=0.216
t=0.218
t=0.220
t=0.222
u
x
−0.4
−0.6
−0.8
−1
−1.6
−1.58
−1.56
−1.54
−1.52
−1.5

Disipativeless shock waves





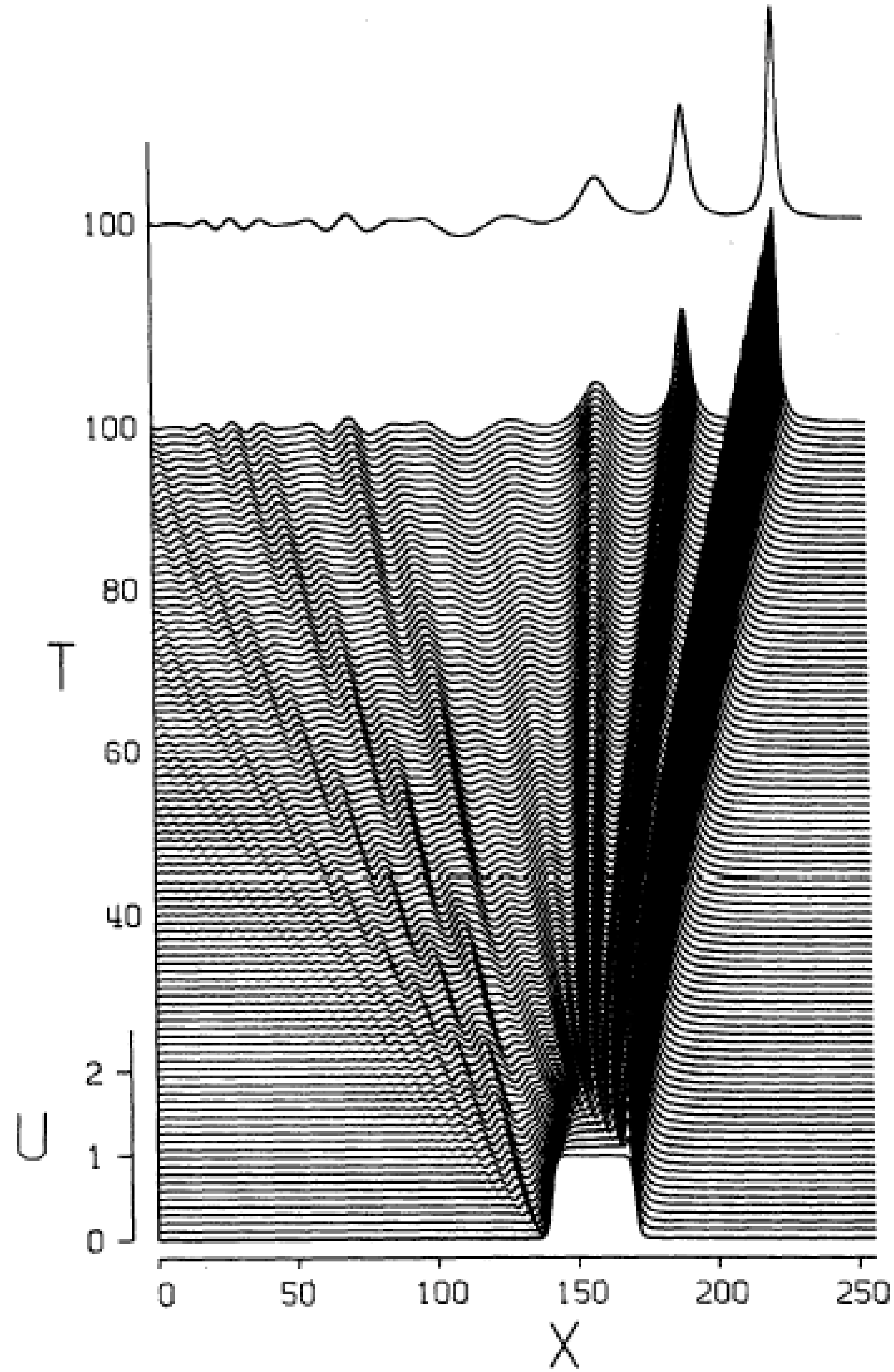
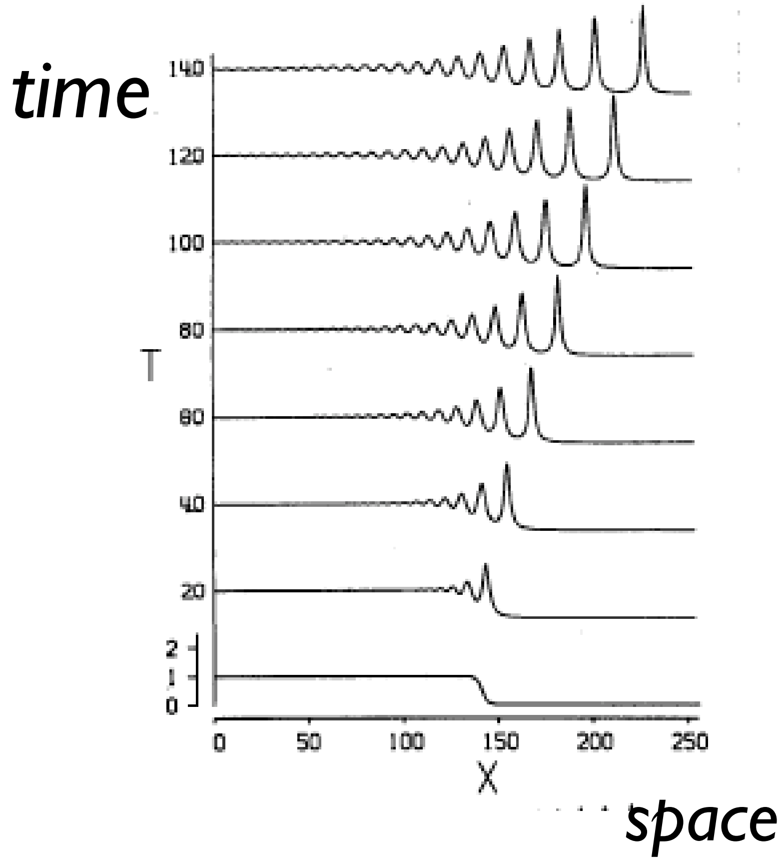


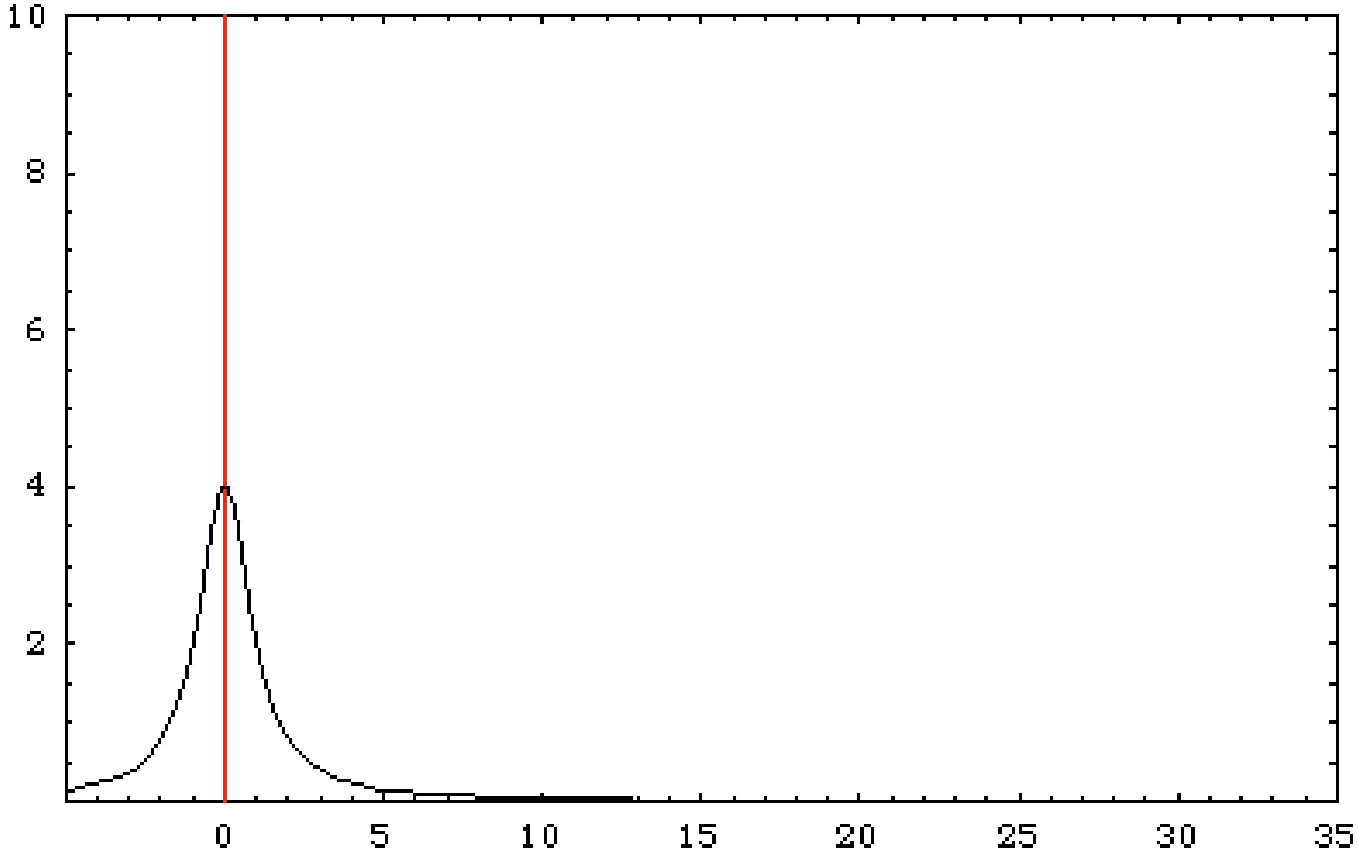
FIG. 8. Numerical solution of the BDO equation illustrating the evolution of a long square wave of elevation into a finite number of amplitude-ordered solitons followed by a subcritical dispersive wave train.



$$x_+ \sim u_0 t, \quad x_- \sim 3\times 2^{-2/3}(u_0 t)^{1/3}$$

Soliton Train

Each Soliton carries a FRACTIONAL CHARGE



10
8
6
4
2
0
5
10
15
20
25
30
35

How to describe quantum dynamics of Edge State?

Step 1: Conjecture:

Edge state (Abelian state) is effectively described by Calogero model

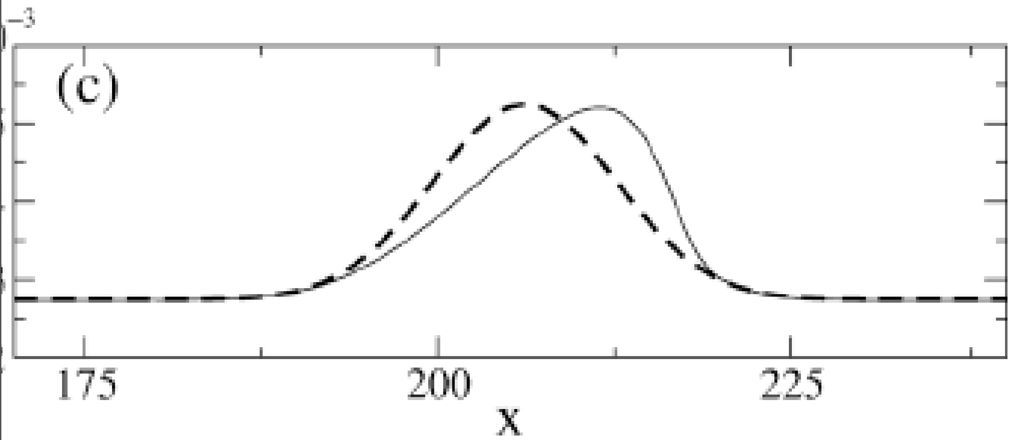
Model Hamiltonian: Calogero model

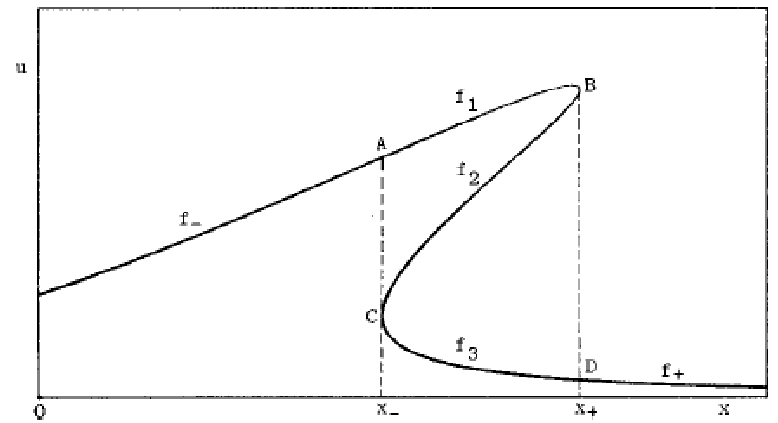
$$V(x) = \wp(x) \to \frac{1}{x^2}, \quad \frac{1}{\sinh^2 x},$$

Interpolates between Lattinger liquid and Calogero model - quantum wires, edge states of FQHE

Shock wave: Riemann-Hopf Equation

$$\boxed{\dot{\varphi} + (\partial_x \varphi)^2 = 0}$$





Classical Riemann equations are ill-defined (no single valued solutions)

Stabilization of shock waves by:

(i) by quantum corrections;

(ii) by interaction

Interaction -- stabilizes shock wave through <u>dispersion</u>

$$H = H_0 + \int c^\dagger(x)c(x)V(x-y)c^\dagger(y)c(y)dxdy$$

$$V(x) = \wp(x) \to \frac{1}{x^2}, \quad \frac{1}{\sinh^2 x},$$

$$\boxed{\dot{\varphi} = \frac{1}{2}(\partial_x \varphi)^2 + \partial_x^2 \tilde{\varphi}}$$

$$\tilde{\varphi} = \frac{1}{\pi L}\int \cot\frac{x-x'}{L}\varphi(x')dx' \qquad \wp(x+iL) = \wp(x)$$

$$\boxed{\dot{\varphi} = \frac{1}{2}(\partial_x \varphi)^2 + \partial_x^2 \tilde{\varphi}}$$

ILW-equation

$$\tilde{\varphi} = \frac{1}{\pi L} \int \cot \frac{x - x'}{L} \varphi(x') dx'$$

$$L \to 0 : \quad \dot{\varphi} = \frac{1}{2}(\partial_x \varphi)^2 + \partial_x^3 \varphi$$

KdV-equation

$$L \to \infty : \quad \dot{\varphi} = \frac{1}{2}(\partial_x \varphi)^2 + \partial_x^2 \int \frac{\varphi(x')}{x - x'} dx'$$

Benjamin-Ono equation

(i) Lattinger Liquid - short range interaction:

$$\dot{\varphi} + (\partial_x \varphi)^2 + \epsilon \underline{\partial_x^3} \varphi = 0 \qquad \text{KdV-equation}$$

(ii) Fractional Hall Edge State - long range interaction (Calogero model)

$$\dot{\varphi} + (\partial_x \varphi)^2 + \nu \partial_x^2 \varphi^H = 0 \qquad \text{Benjamin-Ono equation}$$

$$\varphi^H(x) = \int \frac{\varphi(z)}{x - z} \frac{dz}{2\pi i}$$

(i) KdV-equation: Non-linear waves in shallow water

$$\dot{\varphi} + (\partial_x \varphi)^2 + \epsilon \partial_x^3 \varphi = 0$$

(ii) Benjamin-Ono equation: Non-linear waves in deep water

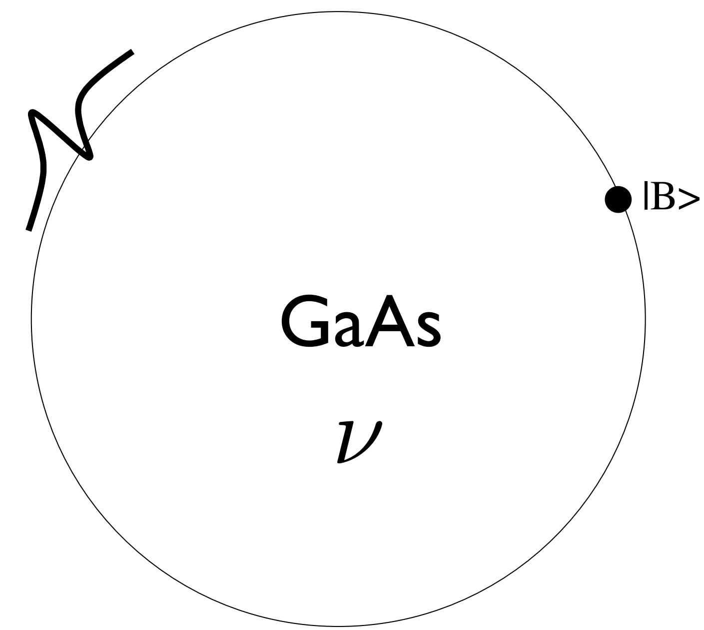
$$\dot{\varphi} + (\partial_x \varphi)^2 + \nu \partial_x^2 \varphi^H = 0$$

$$\varphi^H(x) = \int \frac{\varphi(z)}{x - z} \frac{dz}{2\pi i}$$

Benjamin-Ono equation

Edge State

$$\dot{\varphi} + (\partial_x \varphi)^2 + \nu \partial_x^2 \varphi^H = 0$$



$$\varphi^H(x) = \int \frac{\varphi(z)}{x - z} \frac{dz}{2\pi i}$$

$$\nu - \text{is a fraction}$$

It is a unique equation which solitons have a fixed charge ν

Universal description of non-linear chiral boson at FQHE edge

On the relation between Calogero model and CFT

$$\dot{\varphi} = \nabla_x T(\varphi)$$

$$T = (\partial_x \varphi)^2 + \alpha_0 \partial_x^2 \varphi$$

$$\alpha_0 = \sqrt{\lambda} - 1/\sqrt{\lambda}$$

$$\frac{\lambda(\lambda - 1)}{(x_i - x_j)^2}$$

Period of oscillations is

$$(\text{interaction}) \times (\delta\rho)^{-1} >> k_F^{-1}$$

Quantum Non-linear Equations can be treated semiclassically

Conclusions:

Transport is 1D-electronic systems is essentially *non-linear,*

Any *smooth* semiclassical excitation evolving in time
is unstable, and eventually *sharp*,

will be fragmented to individual *soliton-like* particles,

Fermi-surface is an essentially *semiclassical,* it is *destroyed* in
evolution.

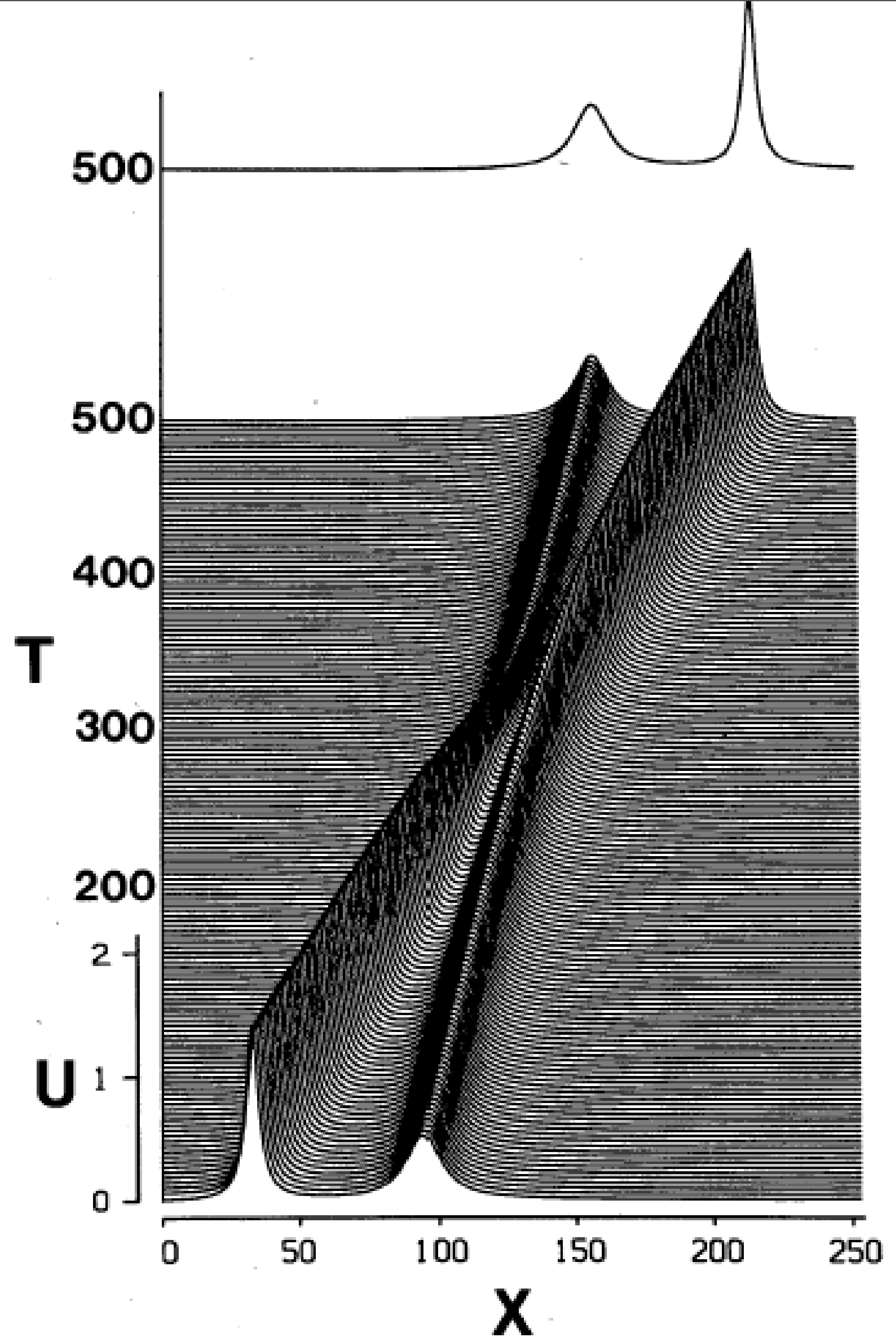


FIG. 6. Interaction of solitary waves governed by the Benjamin–Davis–Ono equation. The last trace has been reproduced separately for clarity. The coordinate system in this illustration and in all subsequent illustrations moves at the critical or linear phase speed. In this frame of reference, solitary waves propagate to the right and subcritical dispersive wave components propagate to the left. Units are nondimensional.

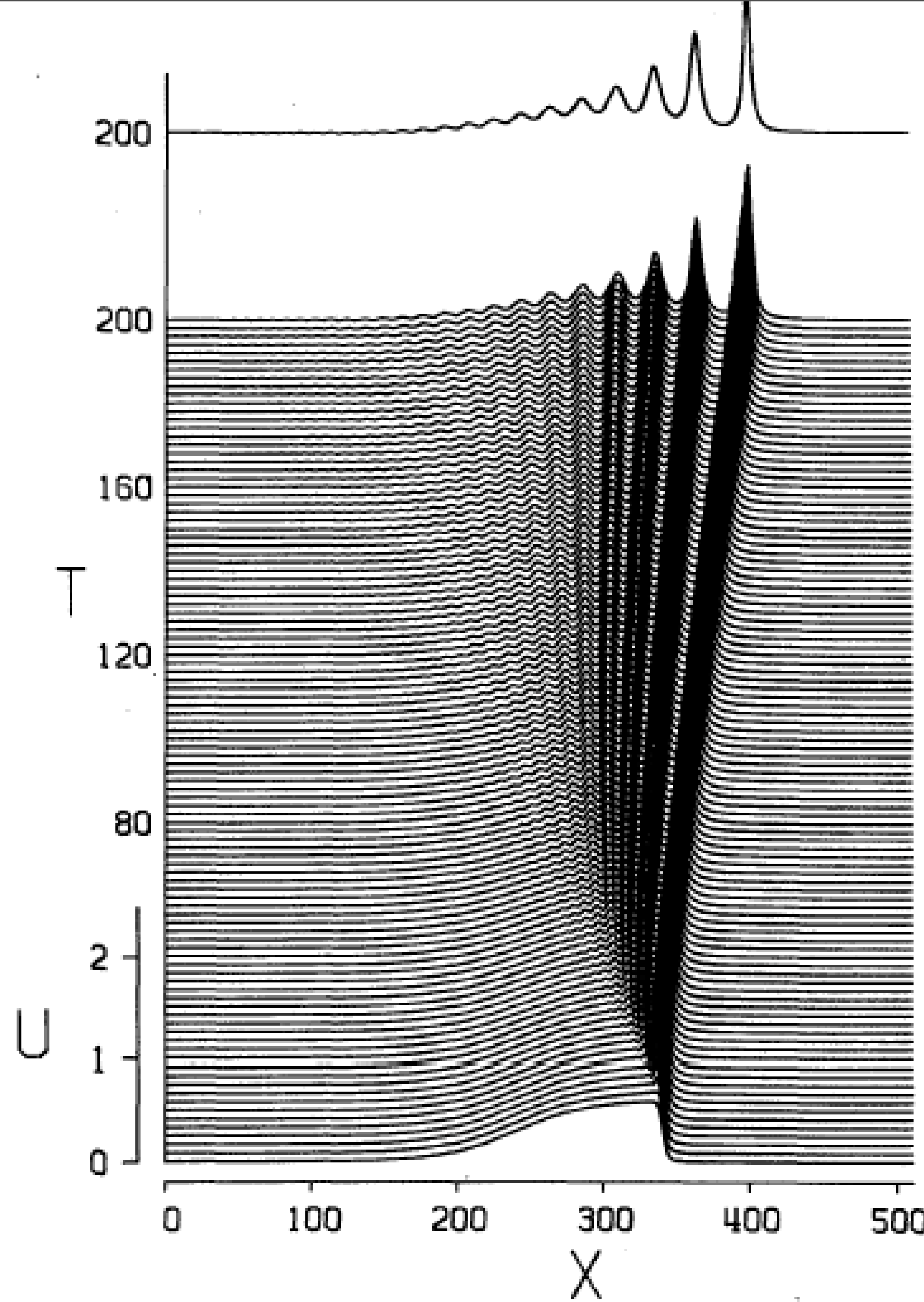


FIG. 11. Solution of the BDO equation for an initial model disturbance for atmospheric waves in the form of a long, but finite-length, wave of elevation. The inversion height behind the leading edge of the disturbance at $T = 0$ decreases slowly, but continuously, to the ambient inversion level.

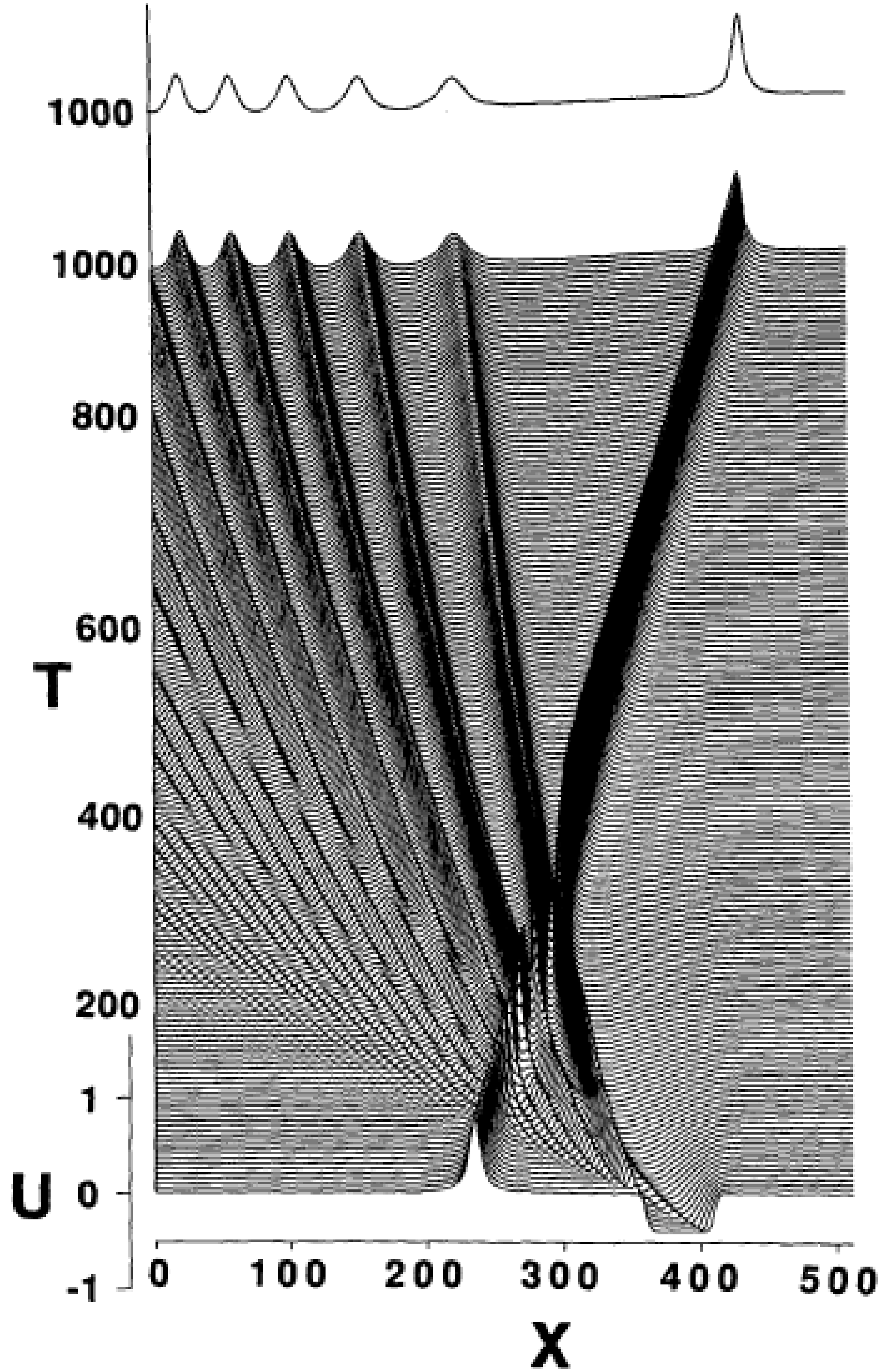


FIG. 7. Numerical integration of the Benjamin–Davis–Ono equation illustrating the stability of a solitary wave under strongly nonlinear interaction with large amplitude subcritical dispersive waves created in the evolution of an initially smooth long wave of depression.

Morning Glory

Morning glory

South Australia

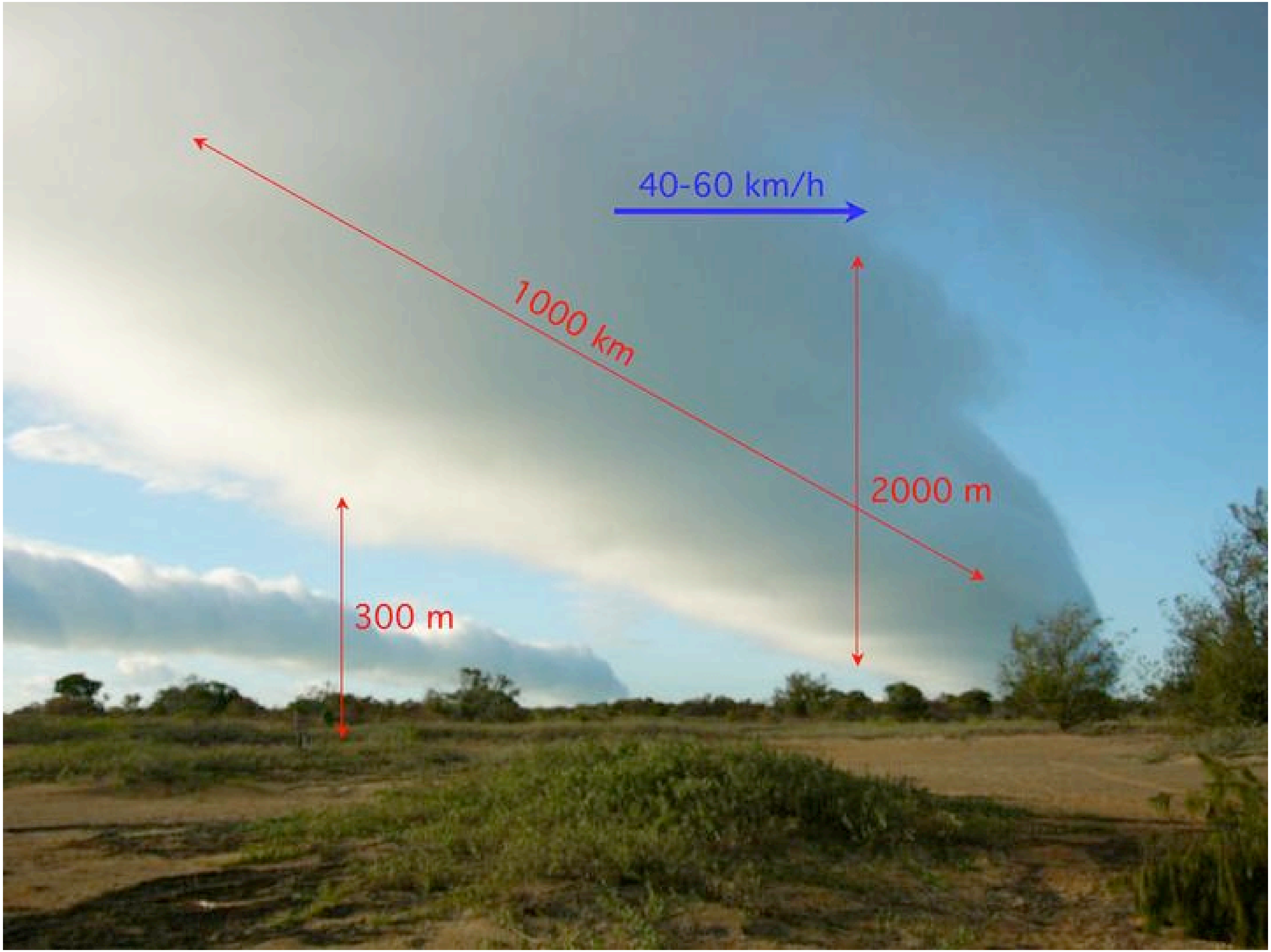
Chain of rolling clouds

Believed to be Benjamin-Ono eq

Morning Glory

Rolling clouds in
the Gulf of Carpenteria,
Northern Australia

www.dropbears.com


40-60 km/h
1000 km
2000 m
300 m