Cavity Method for Quantum Spin Glasses

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(arXiv:0706.4391; related arXiv:0712.3540)

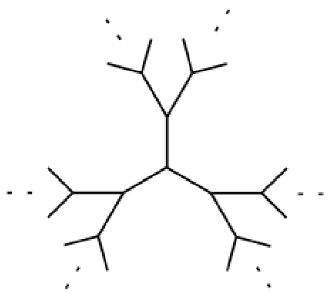
Outline

- What we’ve studied
- Why is this interesting
- Some results
- Future directions

Ising Spin Glass on Bethe Lattice

- Infinite limit of Cayley (q-regular) trees

$$H = -\sum_{(ij)} J_{ij}\sigma_i^z\sigma_j^z - B_t\sum_i \sigma_i^x$$

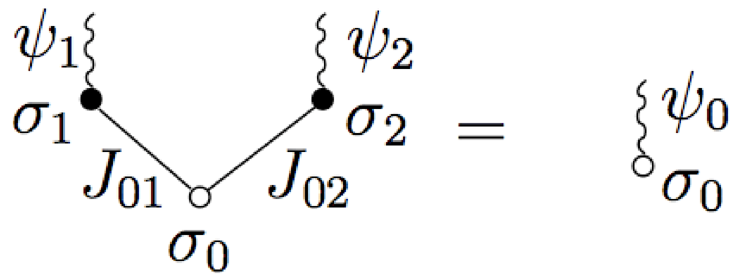


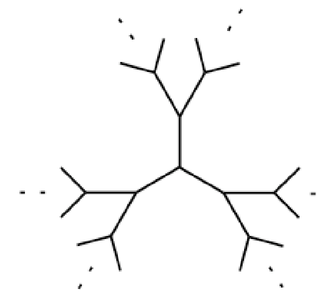
Non-commuting (i.e. *Quantum*)

$$P(J_{ij}) = \frac{1}{2}\delta(J_{ij} - J) + \frac{1}{2}\delta(J_{ij} + J)$$

Classical Cavity Method (B_t = 0)

Iteration Equations for Cavity Fields





$$\psi_0(\sigma_0) = \frac{1}{Z} \sum_{\sigma_1,\ldots,\sigma_{q-1}=\pm 1} \exp\left(\beta \sum_i J_{0i}\sigma_0\sigma_i\right) \psi_1(\sigma_1)\ldots\psi_{q-1}(\sigma_{q-1})$$

equivalent

$$h_0 = \frac{1}{\beta} \sum_{i=1}^{q-1} \tanh^{-1}(\tanh(\beta J_{0i}) \tanh(\beta h_i))$$

where $$\psi_i(\sigma_i) = \frac{e^{\beta h_i \sigma_i}}{2\cosh(\beta h_i)}$$

Classical Cavity Method (cont'd)

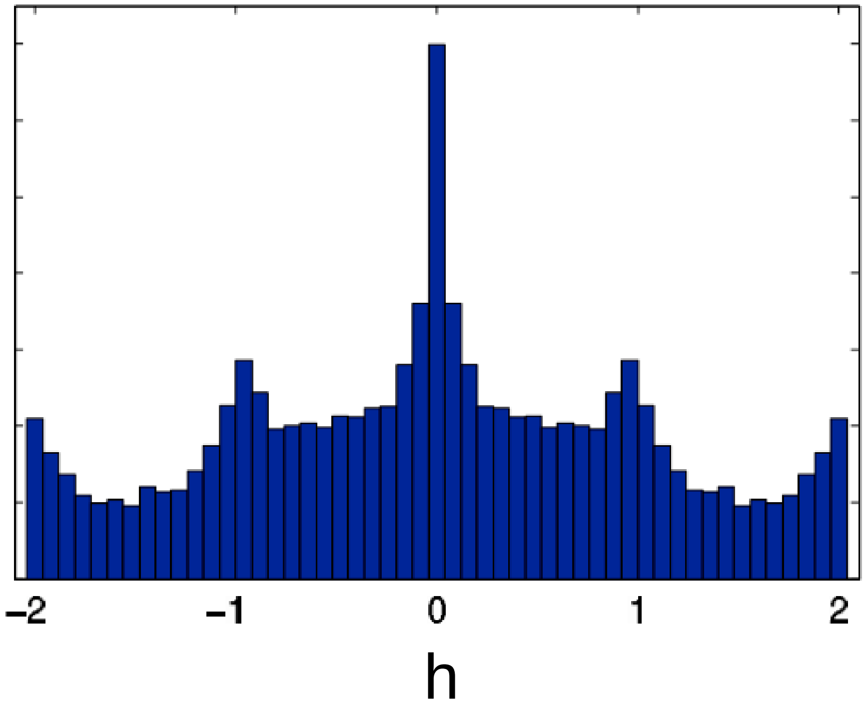
- For the non random case, fixed points of the iteration equation yield cavity fields well away from the outer layer of the tree
- For the random case, consider a distribution of cavity fields which reproduces itself in the interior of the tree (generational average)

Classical Cavity Method (cont'd)

Replica Symmetric Fixed Point for Cavity Field Distribution:

$$P(h) = \int \prod_{i=1}^{q-1} dh_i P(h_i) \left\langle \delta(h - U(\{h_i\}, \{J_{0i}\})) \right\rangle_J$$

Solution in SG phase; above T_c weight only at h=0



Quantum formulation

- Useful to think in path integral language
- Integrating out ancestor spins generates a cavity effective action for given spin
- At next step combination of “bare” action and cavity action give rise to a functional recursion relation
- For spin glass, study distribution of cavity actions

Why is this interesting?

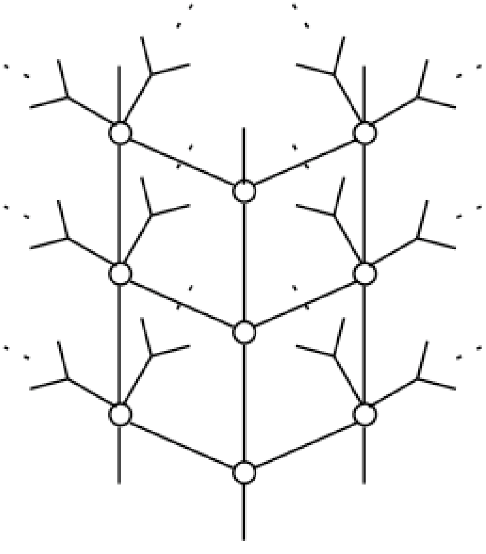
- Nature of ordering in spin glass unsettled (Catholics vs Protestants)
- Quantum effects in ordered phase could use more microscopic investigation
- Leads to approximate theory of systems with sparse loops, such as random graphs with fixed connectivity
- Corresponding algorithm is belief propagation

<u>QMAC problems</u>

<u>Quantum BP</u>

Details: Trotter Decomposition

- Transverse field B_t generates ferromagnetic coupling Γ in imaginary time
- Disordered within hyperplanes; correlated along imaginary time



Cavity Actions

Cavity Fields

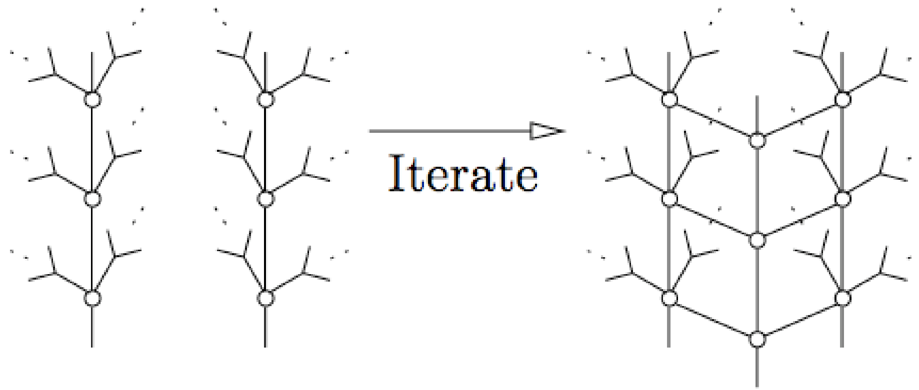
$$\psi_i(\sigma_i) = \frac{e^{\beta h_i \sigma_i}}{2\cosh(\beta h_i)}$$

Cavity Actions

$$\psi[\sigma(t)] = e^{-S[\sigma]}$$

$$S[\sigma] = -\log Z - h\Delta t \sum_t \sigma(t) - \sum_{t,t'} \Delta t^2 C^{(2)}(t'-t)\sigma(t)\sigma(t') - $$
$$- \sum_{t,t',t''} \Delta t^3 C^{(3)}(t'-t, t''-t')\sigma(t)\sigma(t')\sigma(t'') + \ldots$$

Quantum Fixed Point



$$P_{FP}\left[\psi[\sigma(t)]\right] = \left\langle \delta\left[\psi[\sigma(t)] - \psi_0[\sigma(t);\{J_{0i},\psi_i\}_{i=1}^{q-1}]\right]\right\rangle_{J_{0i},\psi_i}$$

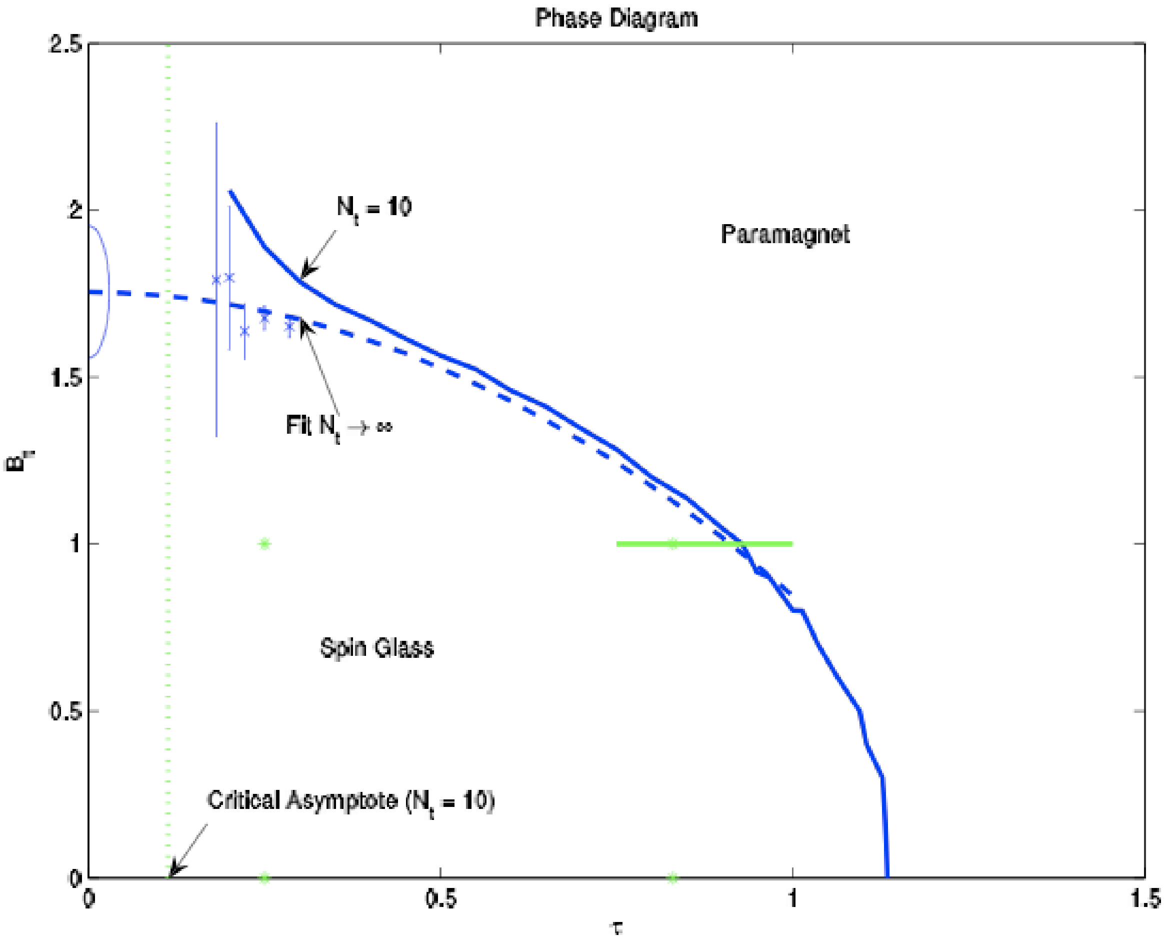
$$= \int \left(\prod_{i=1}^{q-1} D\psi_i P_{FP}\left[\psi_i\right] dJ_{0i} P(J_{0i})\right) \delta\left[\psi[\sigma(t)] - \psi_0[\sigma(t);\{J_{0i},\psi_i\}_{i=1}^{q-1}]\right]$$

Use population dynamics to generate distribution

Elementary treatment

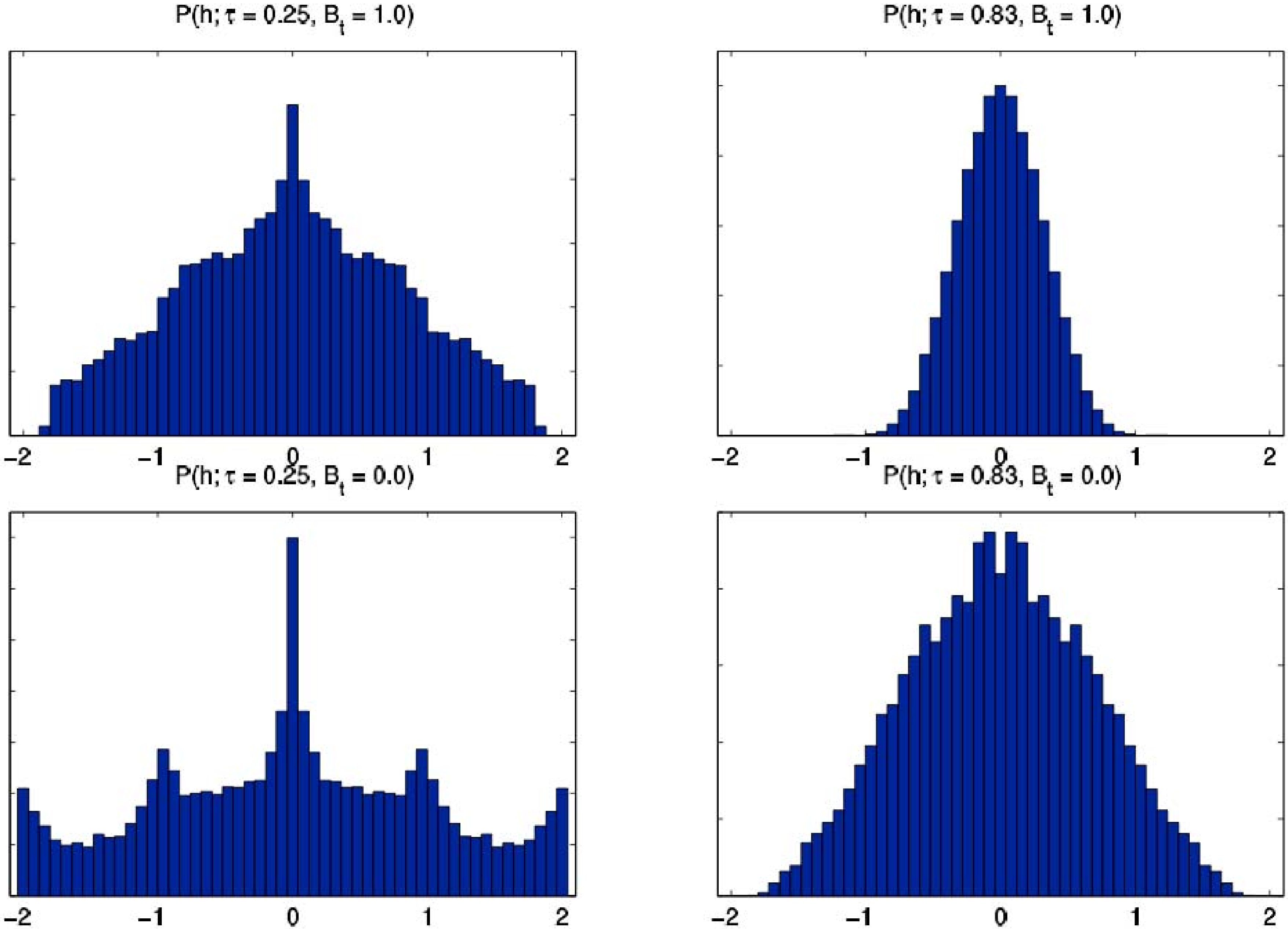
- 6-11 time slices
- 2500 cavity actions
- 2500 * 1000 iterations
- Keep only two spin interactions in effective action, but between all time slices
- Still, nontrivial results, cf Usadel & Co.

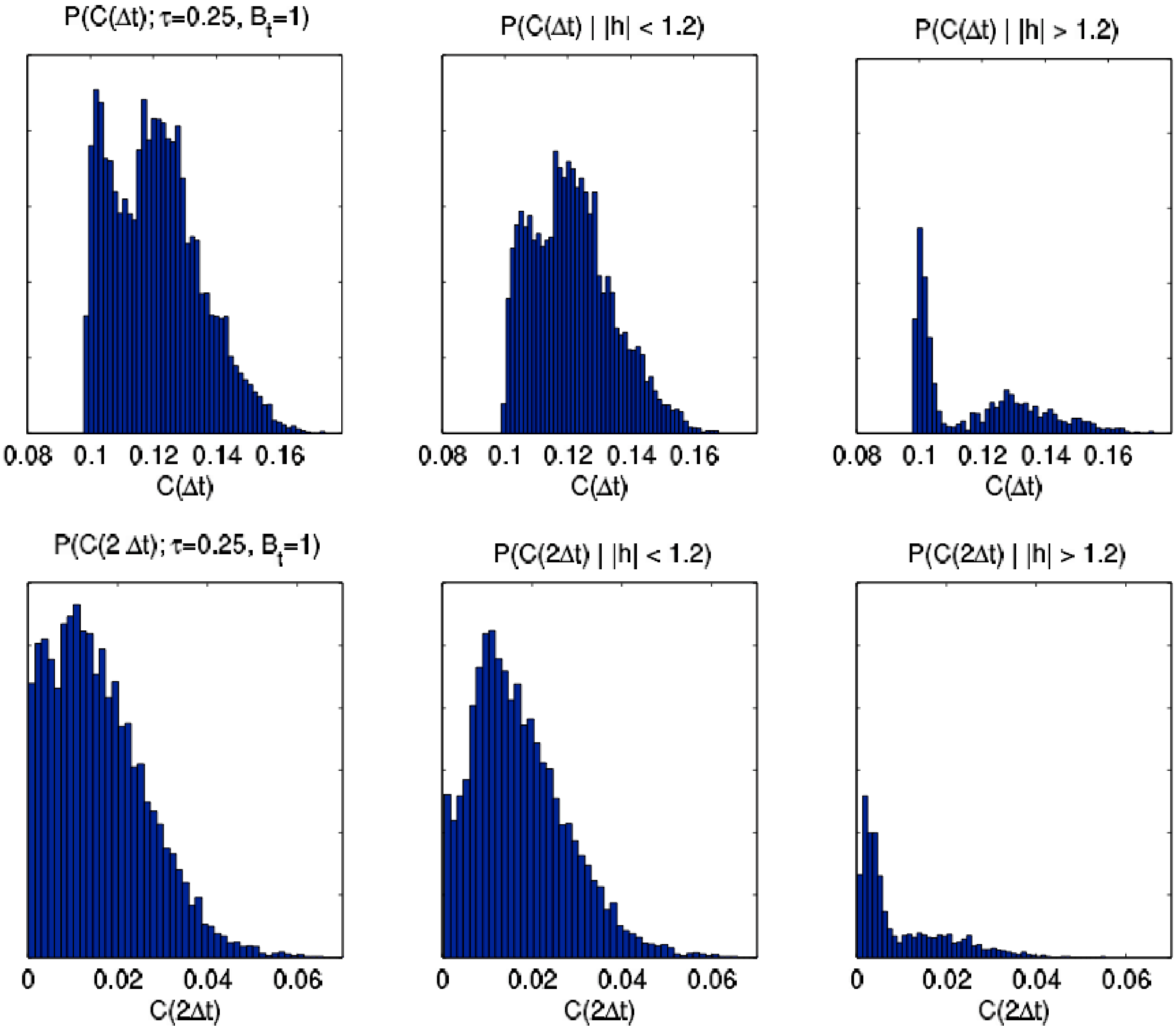
Phase diagram: q = 3



Fixed point distributions

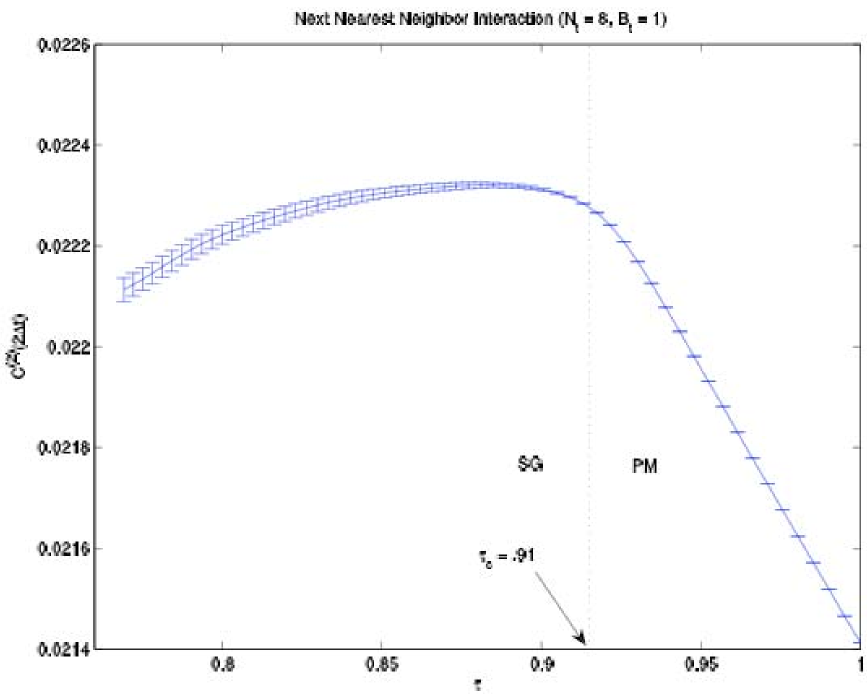
- In PM phase there is no local magnetic field; spin-spin interaction has unique value (generally, cavity action is unique in PM phase)
- In SG phase field has a distribution – needed to produce EA op - as does the interaction

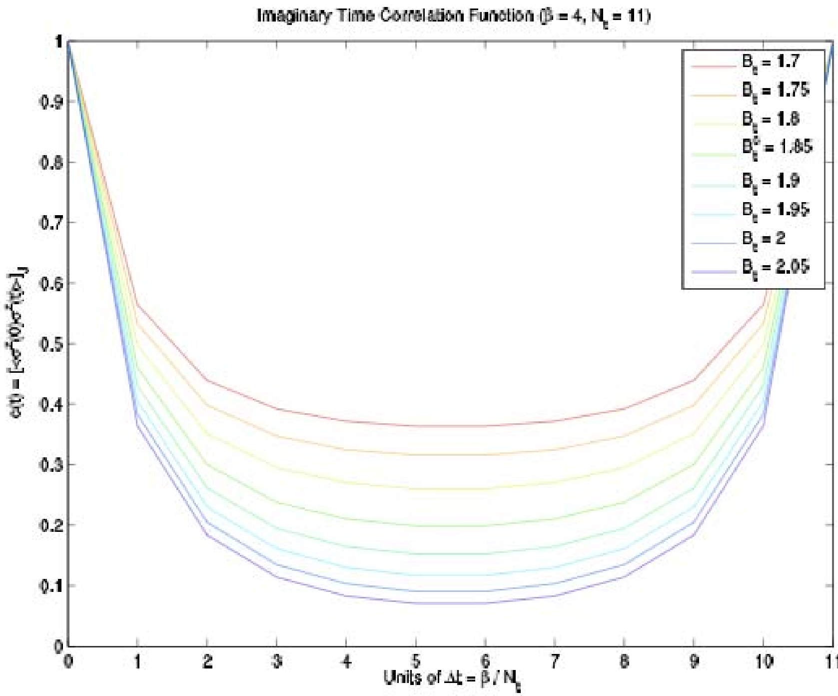

P(h; τ = 0.25, B_t = 1.0)
P(h; τ = 0.83, B_t = 1.0)
P(h; τ = 0.25, B_t = 0.0)
P(h; τ = 0.83, B_t = 0.0)
−2
−1
0
1
2



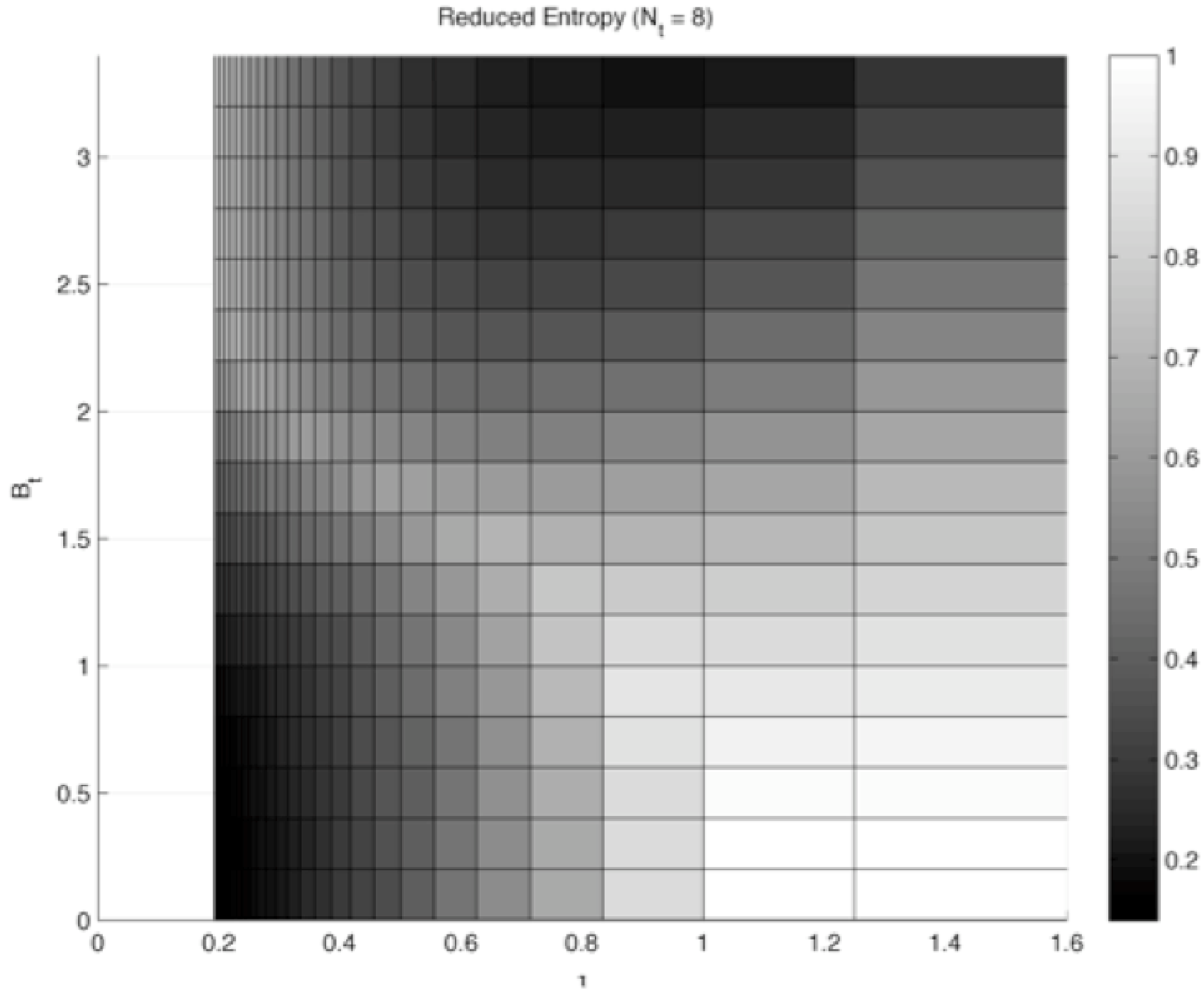
P(C(Δt); τ=0.25, B_t=1)
P(C(Δt) | |h| < 1.2)
P(C(Δt) | |h| > 1.2)
0.08
0.1
0.12
0.14
0.16
C(Δt)
P(C(2 Δt); τ=0.25, B_t=1)
P(C(2Δt) | |h| < 1.2)
P(C(2Δt) | |h| > 1.2)
0
0.02
0.04
0.06
C(2Δt)

Imaginary Time Interactions & Correlations





Single Spin von Neumann Entropy



Future directions

- Run the iteration on a fixed instance of a random graph (QBP)
- Continuous time formalism
- MC evaluation of cavity action (MCRG)
- Analytic limits
- What would a truly quantum BP look like - one that would run on a quantum computer? (Fixed point behavior versus unitary evolution, QMAC by QC)