Quantum phase transitions and quantum information

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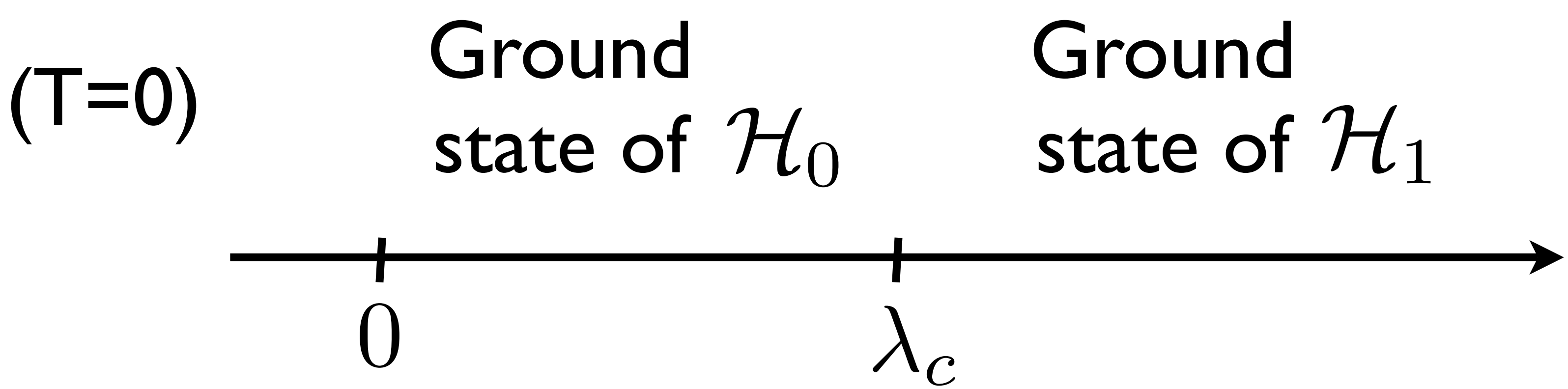
Overview

- Quantum (and classical) phase transitions
  - Critical points, exponents, and universality
- Quantum information perspective
  - Ground state fidelity
  - Time dependent GSF and decoherence
- Algorithms and experimental implementation with NMR & cold atoms

Quantum phase transitions

In general, a QPT occurs in a quantum many body system when there is competition between two parts of the total Hamiltonian:

$$\mathcal{H} = \mathcal{H}_0 + \lambda \mathcal{H}_1$$



Quantum phase transitions

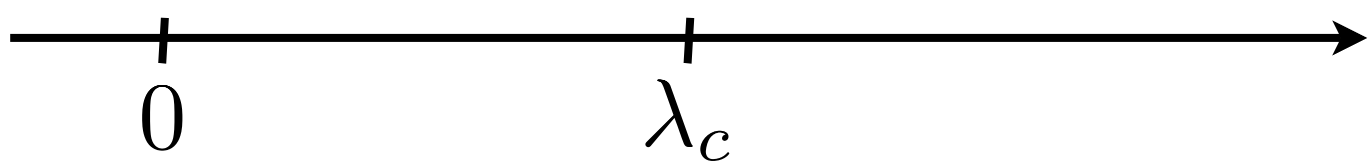
Example: Ising chain with transverse field

$$\mathcal{H} = -J\left(\sum_i \sigma_i^z \sigma_{i+1}^z + \lambda \sigma_i^x\right)$$

$$|\uparrow\uparrow \ldots \uparrow\rangle$$

$$|\downarrow\downarrow \ldots \downarrow\rangle$$

$$|\rightarrow\rightarrow \ldots \rightarrow\rangle$$

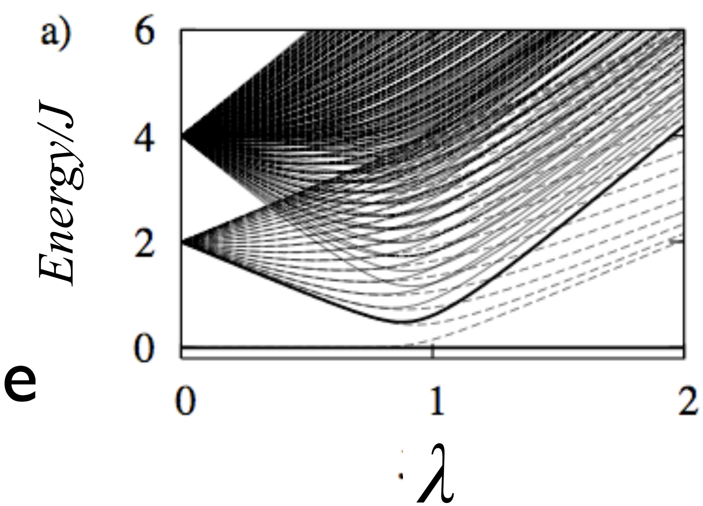


Quantum phase transitions

Example: Ising chain with transverse field

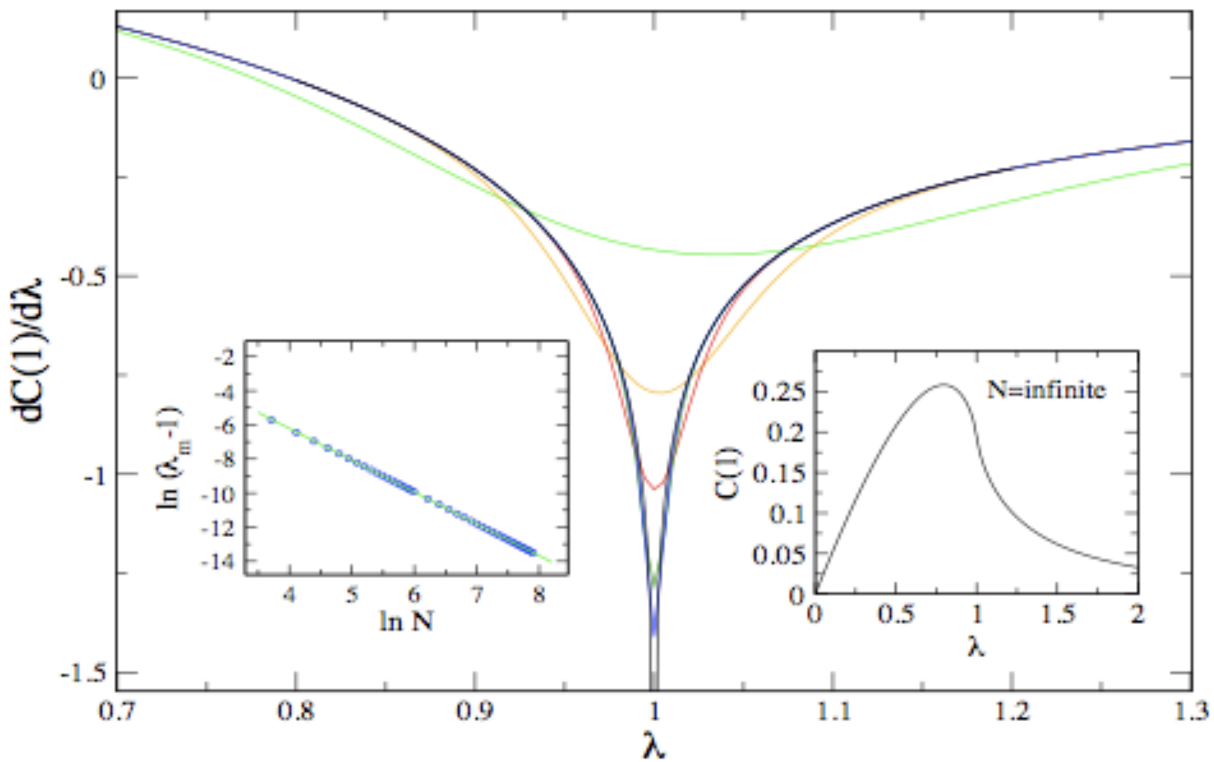
$$\mathcal{H} = -J\left(\sum_i \sigma_i^z \sigma_{i+1}^z + \lambda \sigma_i^x\right)$$

- At the critical point:
  - The gap closes (in thermodynamical limit), equivalent to critical slowing down ($\tau \sim 1/\Delta$)
  - Quantum correlations diverge with critical exponents $\xi \sim |\lambda - \lambda_c|^{-\nu}$
  - Universality (as in classical PT) can be defined and observed



QPTs and quantum information

- Scaling of entanglement at the critical point (not surprising, entanglement ~ correlations)



Osterloh, Amico, Falci, and Fazio, Nature **416**, 608 (2002)

- Alas, we'll take another route

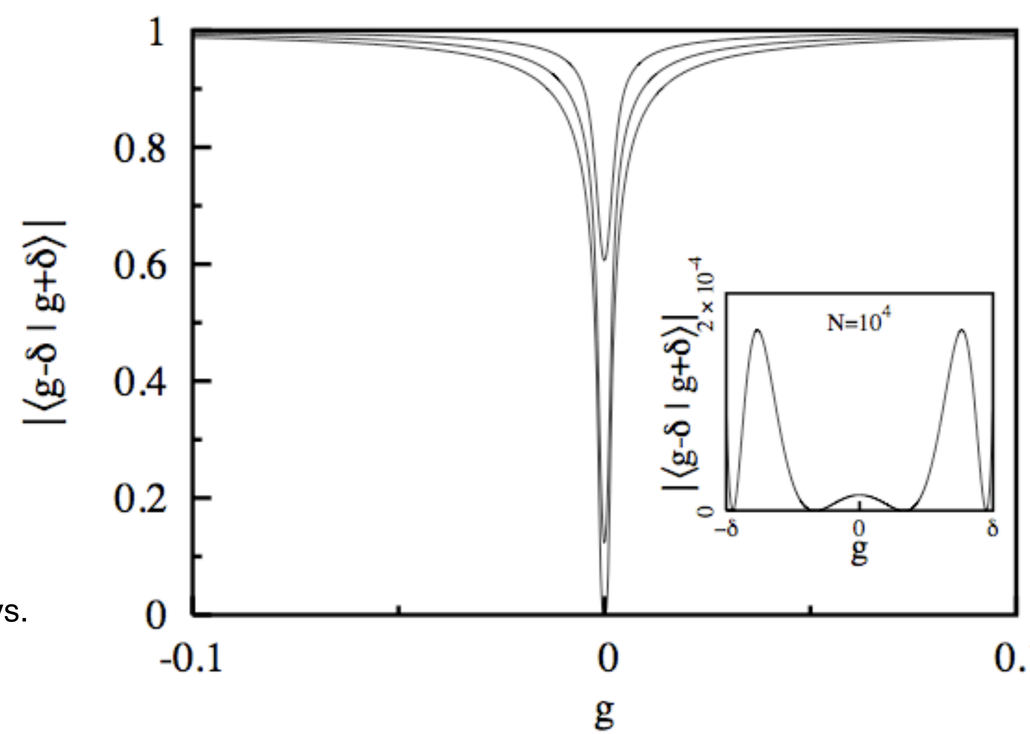
QPTs and fidelity

Ground state fidelity:

$$f_\delta(\lambda) = \langle g(\lambda) | g(\lambda + \delta) \rangle$$

Rationale: two ground states in the same phase are very similar, and orthogonal if in different phases.

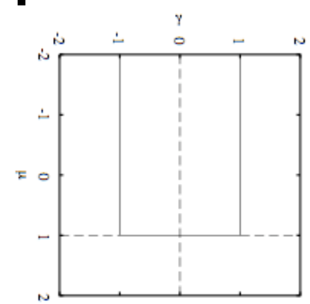
Rationale works, and fidelity contains much more information than just the critical point

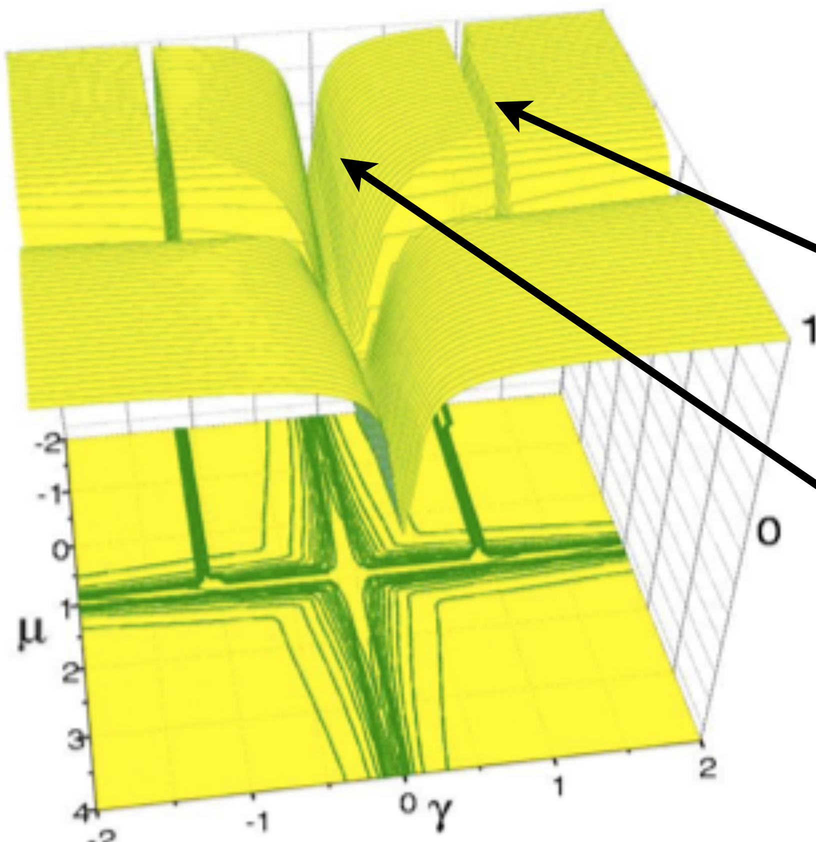


Cozzini, Ionicioiu, Zanardi, Phys. Rev. B 76, 104420 (2007)

Ground state fidelity

Type of discontinuity depends on the order of transition





Cozzini, Giorda, Zanardi
quant-ph/0608059

- A first order QPT (a kink in the GS energy) appears as a discontinuous fidelity.
- Second order QPTs appear continuous with a diverging derivative (in thermodynamic limit)

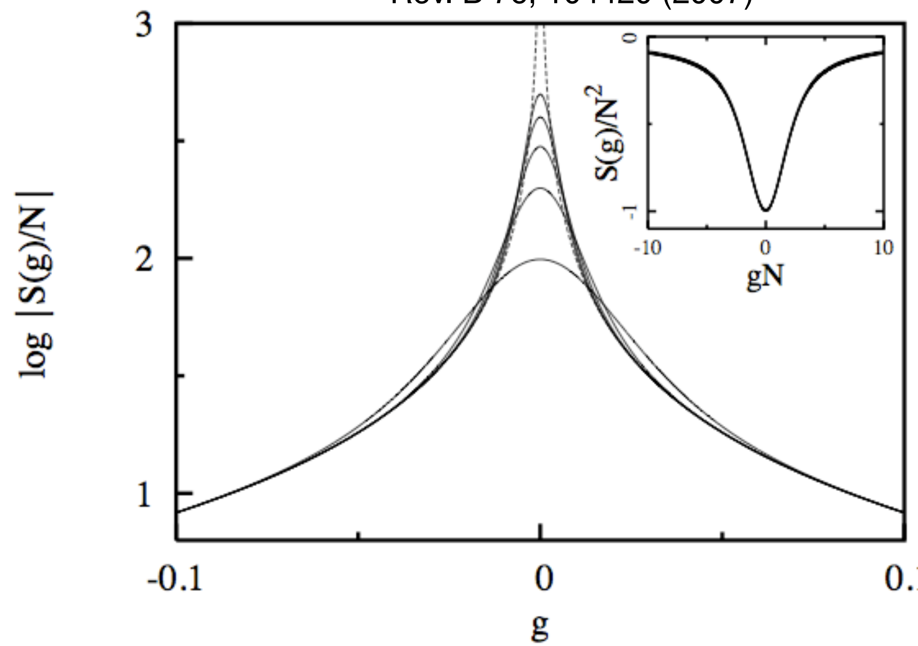
Ground state fidelity

The scaling of the second derivative of the fidelity relates to the critical exponents

$$f_\delta(\lambda) \simeq 1 + \left.\frac{\partial^2 f}{\partial \delta^2}\right|_{\delta=0} \frac{\delta^2}{2} = 1 + S(\lambda)\frac{\delta^2}{2}$$

Susceptibility

Cozzini, Ionicioiu, Zanardi, Phys. Rev. B 76, 104420 (2007)



Argument using matrix product states QPTs:

$$|g> = \sum_{i_1, i_2 \ldots, i_N} Tr(A_{i_1} \ldots A_{i_N})|i_1 \ldots i_N>$$

$$f_\delta(\lambda) = \sum_{k=1}^{D^2} v_k^N \qquad \sum_{i=0}^{d-1} A_i^*(\lambda_1) \otimes A_i(\lambda_2)$$

$$S(\lambda) \simeq N\partial_{\lambda_1}\partial_{\lambda_2} ln\ v_1(\lambda_1, \lambda_2)|_{\lambda_1=\lambda_2=\lambda}$$

$$S_{crit}/N = N^{\rho/\nu} Q(N|\lambda - \lambda_c|^\nu)$$

Also see Venuti and Zanardi, PRL 99, 095701 (2007)

Time dependent GSF

Switch to time domain

$$D_\lambda(\omega) = \langle g(\lambda)|\, \delta(\omega - \mathcal{H}_{\lambda+\delta})\, |g(\lambda)\rangle$$

A local density of states

$$M(t) = \left| \int D_\lambda(\omega) e^{i\omega t} d\omega \right|^2$$

A Loschmidt echo

$$M(t) = \left| \left\langle g(\lambda) \right| e^{i\mathcal{H}_\lambda t} e^{-i\mathcal{H}_{\lambda+\delta} t} \left| g(\lambda) \right\rangle \right|^2$$

- Quantum chaos
- Measures sensitivity to perturbations
- Measures fidelity of a quantum simulation
- Measures decoherence

Loschmidt echo and decoherence

$$\mathcal{H} = \mathcal{H}_1 \otimes I + \lambda \mathcal{H}_2 \otimes I + \delta\, \mathcal{H}_2 \otimes \sigma_Z$$

Product initial state

$$|\psi(0)\rangle = |\epsilon_0\rangle \left(a\,|\uparrow\rangle + b\,|\downarrow\rangle\right) \qquad \mathcal{H}_\lambda \quad \overset{\delta}{\longleftrightarrow}$$

$$\downarrow$$

$$|\psi(t)\rangle = a\,|\uparrow\rangle\,|\epsilon_\uparrow\rangle + b\,|\downarrow\rangle\,|\epsilon_\downarrow\rangle$$

$$\mathcal{H}_\uparrow = \mathcal{H}_1 + (\lambda + \delta)\mathcal{H}_2$$

$$\mathcal{H}_\downarrow = \mathcal{H}_1 + (\lambda - \delta)\mathcal{H}_2$$

$$\rho(0) = \begin{pmatrix} |a|^2 & ab^* \\ a^*b & |b|^2 \end{pmatrix} \longrightarrow \rho(t) = \begin{pmatrix} |a|^2 & ab^*m(t) \\ a^*bm^*(t) & |b|^2 \end{pmatrix}$$

$$m(t) = \langle \epsilon_\uparrow | \epsilon_\downarrow \rangle = \langle \epsilon_0 |\, e^{i\mathcal{H}_{\lambda+\delta}t} e^{-i\mathcal{H}_{\lambda-\delta}t}\, | \epsilon_0 \rangle$$

Decoherence from a critical environment

Quan et al PRL 96, 140604 (2006), FMC et al PRA 75, 032337 (2007)

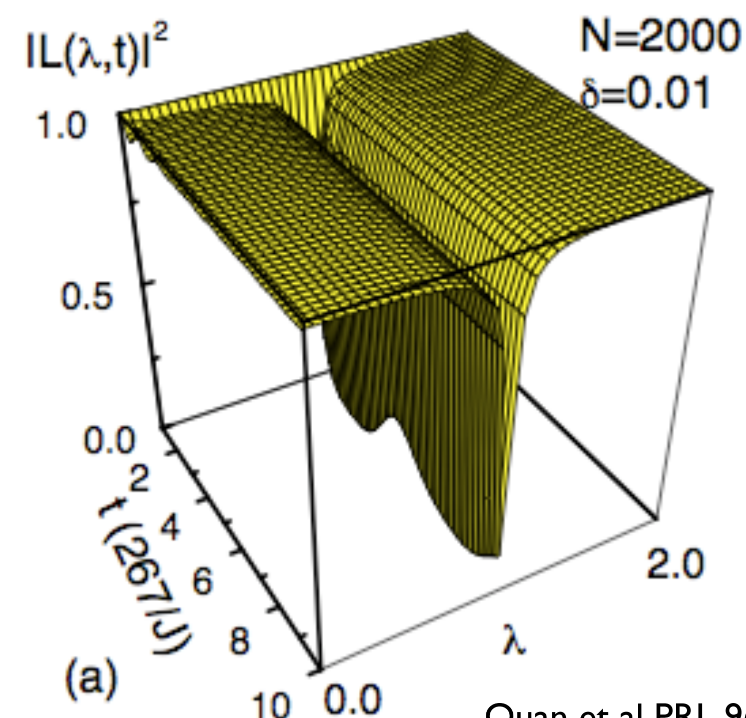
QPTs and the Loschmidt echo

$$M(t) = \left|\langle g(\lambda)|e^{i\mathcal{H}_\lambda t}e^{-i\mathcal{H}_{\lambda+\delta} t}|g(\lambda)\rangle\right|^2$$

Sensitivity to perturbations is used to detect proximity to critical point: far from λ_c, evolutions are similar, near λ_c system undergoes large changes even for small perturbation.

Long time behavior

Result from quantum chaos: M decays to 1/# states needed to represent unperturbed states with perturbed Hamiltonian = strong decay only near critical point.



Quan et al PRL 96, 140604 (2006)

QPTs and the Loschmidt echo

$$M(t) = \left|\langle g(\lambda)|e^{i\mathcal{H}_\lambda t}e^{-i\mathcal{H}_{\lambda+\delta}t}|g(\lambda)\rangle\right|^2$$

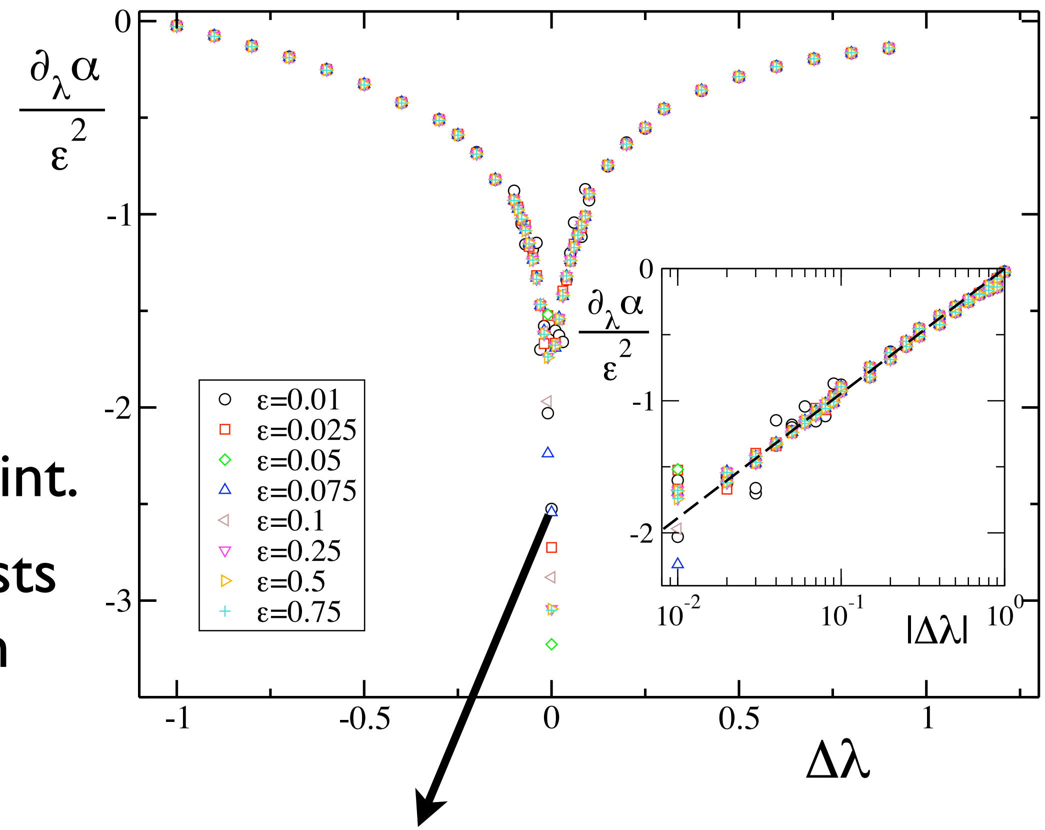
Short time behavior

Perturbation theory gives:

$$M(t) \cong exp[-\alpha(\lambda)\ \delta^2 t^2]$$

Where α is monotonic with λ, first derivative has singularity at critical point.

Numerical evidence (no proof) suggests that critical exponents are encoded in higher derivatives of α.



$$\cong log\ |\lambda\text{-}\lambda_c|$$

QPTs and the Loschmidt echo

- Decay rate has universality features

$$H(\lambda) = \sum_k \epsilon_k(\lambda)\left(\gamma_k^\dagger(\lambda)\gamma_k(\lambda) + 1/2\right)$$

$$\gamma_k(\lambda_1) = \cos(\theta_k)\gamma_k(\lambda_2) - i\sin(\theta_k)\gamma_{-k}^\dagger(\lambda_2)$$

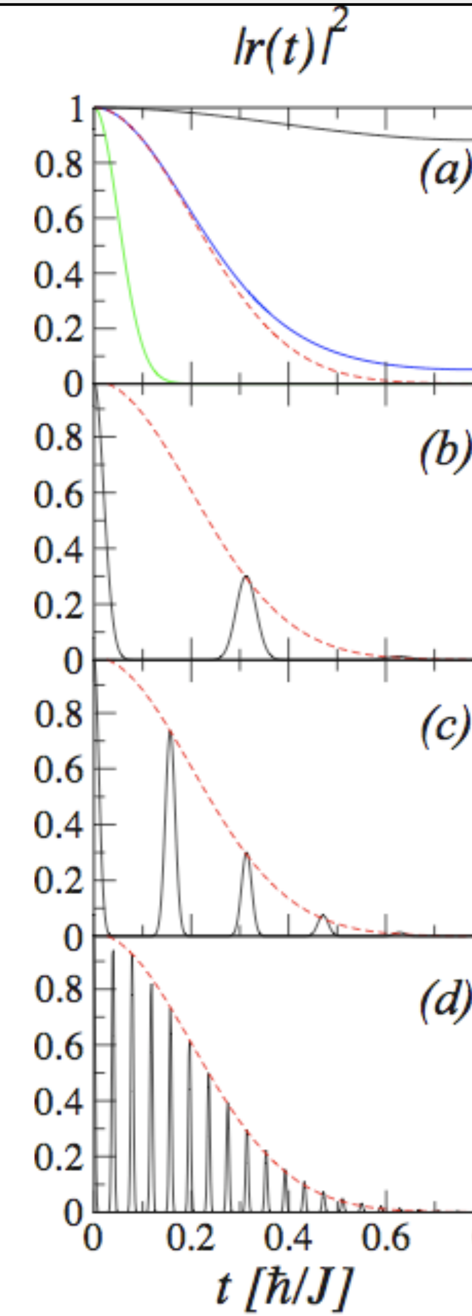
$$m(t) = \prod_{k>0} \cos^2(\theta_k)e^{i\epsilon_k(\lambda_2)t} + \sin^2(\theta_k)e^{-i\epsilon_k(\lambda_2)t}$$

$$f_d(\lambda) = \prod_{k>0}\cos(\theta_k)$$

Weighted angle variance

Energy mean

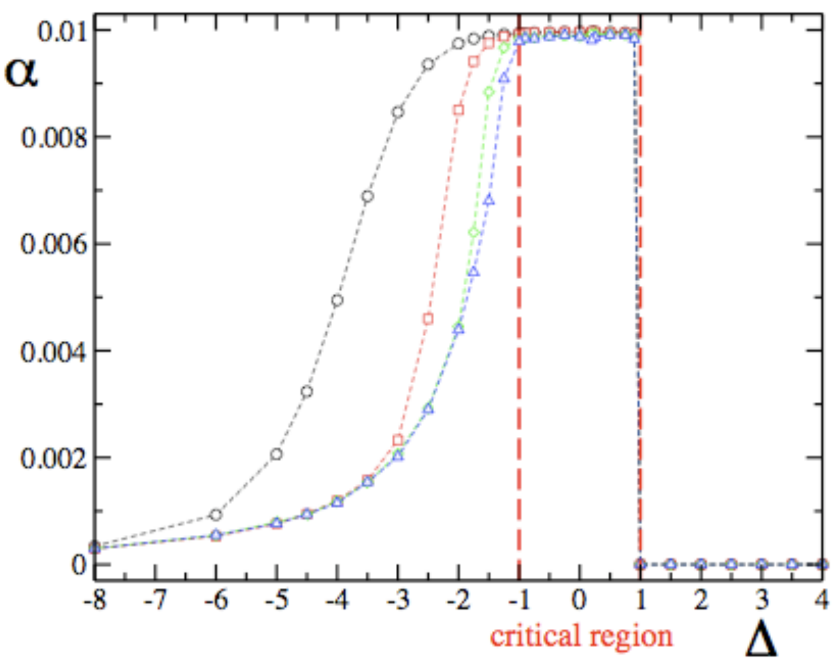
$$|m(t)|^2 = exp(-\sigma_N^2 t^2)|cos(\bar{\epsilon}t)|^N$$



FMC et al PRA 75, 032337 (2007)

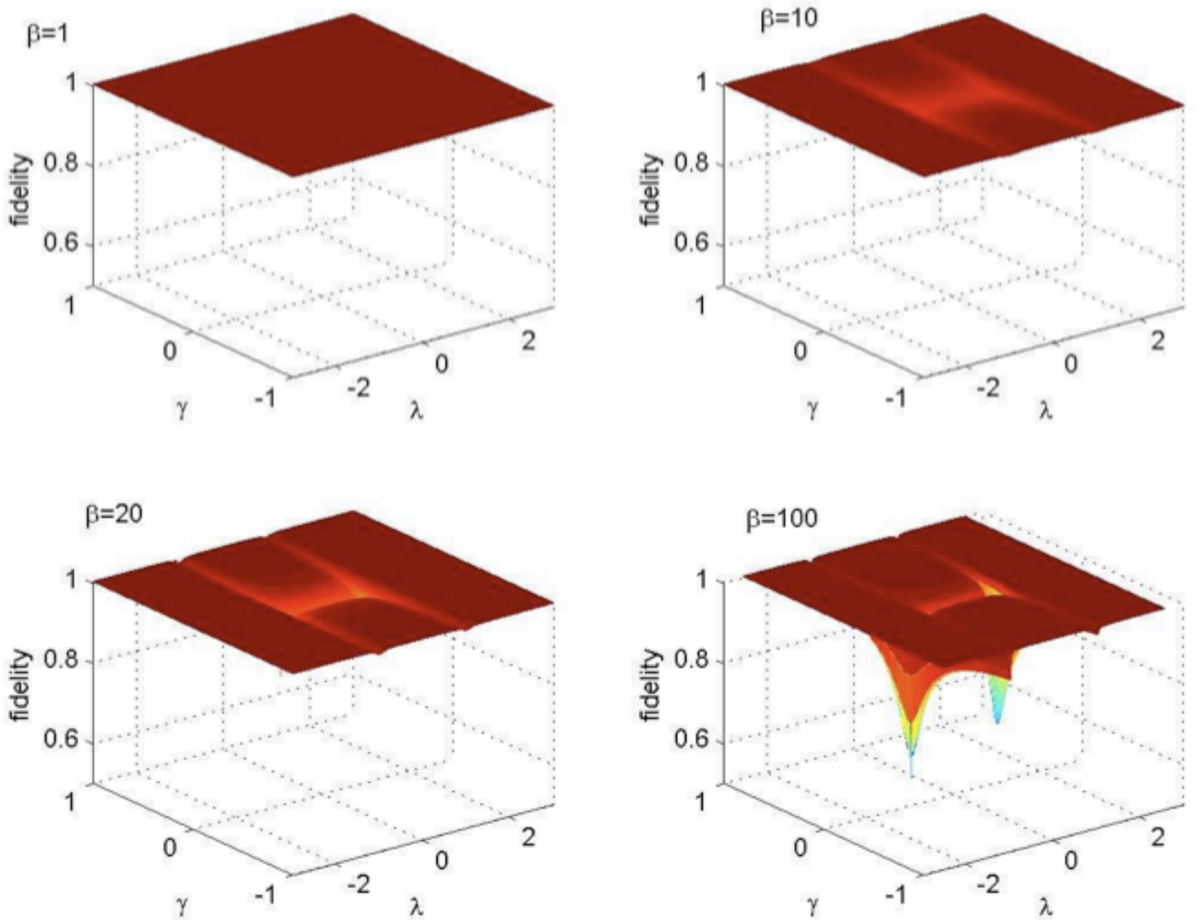
QPTs and the Loschmidt echo

- Coupling to environment (perturbation) can be local



Rossini et al, Phys. Rev. A 75, 032333 (2007)

- Temperature is ok (not so high)

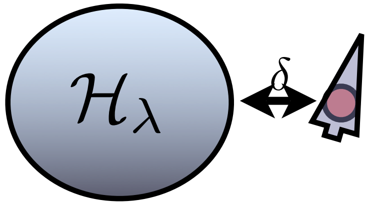


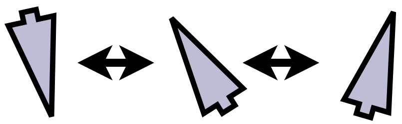
Zanardi et al PRA 75, 032109 (2007)

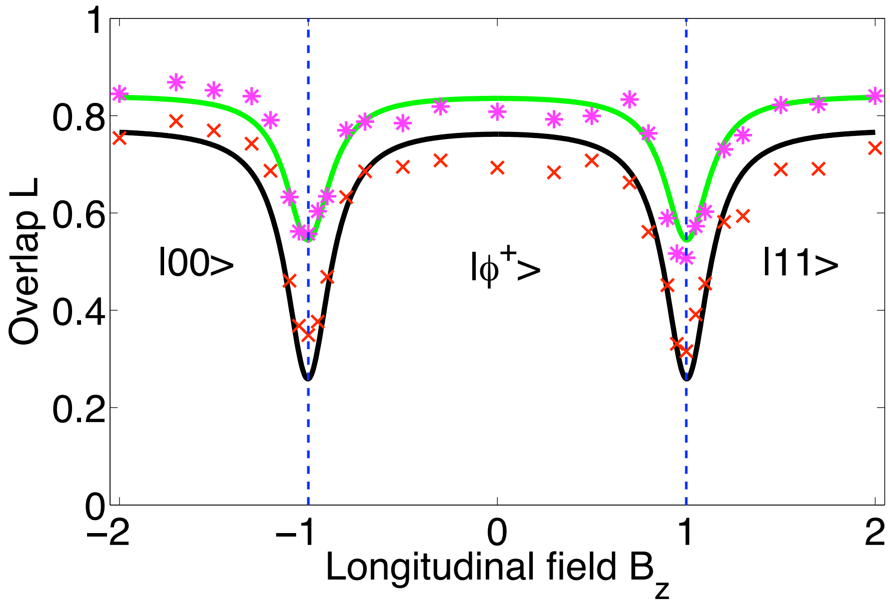
The “Algorithm”

FMC et al PRA 75, 032337 (2007)

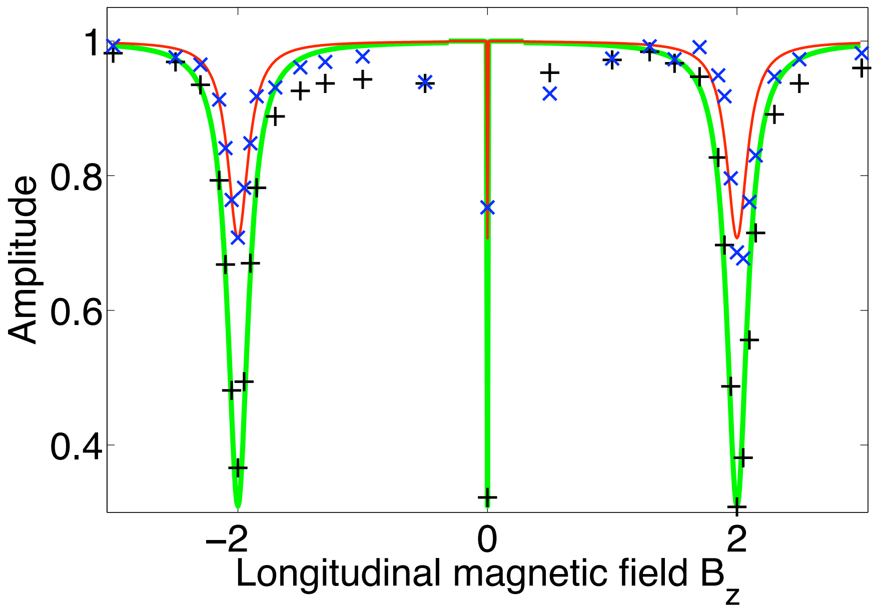
- Prepare initial state (can be T>0)
- Measure decoherence vs λ
- Minimum decoherence signals critical point, derivatives give critical exponents
- An instance of a 1-qubit quantum computer...?
- But we can get rid of the other part using a quantum simulator







Zhang et al, arXiv:0709.3273v2

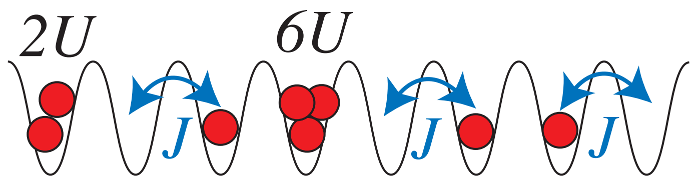


Zhang, FMC, and Laflamme, in preparation

Time reversal in an optical lattice

$$M(t) = \left|\langle g(\lambda)|e^{i\mathcal{H}_\lambda t}e^{-i\mathcal{H}_{\lambda+\delta}t}|g(\lambda)\rangle\right|^2$$

- The LE can be achieved by changing the sign of H (imperfect time reversal)



$$H = -J\sum_{<i,j>} a_i^\dagger a_j + a_j^\dagger a_i \; + U\sum_i n_i(n_i - 1)$$

FMC quant-ph/0609202

Apply linear ramp potential of slope F for a short time τ

$$e^{iF\tau x}a_i^\dagger a_{i+1}\, e^{-iF\tau x}$$

$$\Rightarrow e^{iF\tau}a_i^\dagger a_{i+1}$$

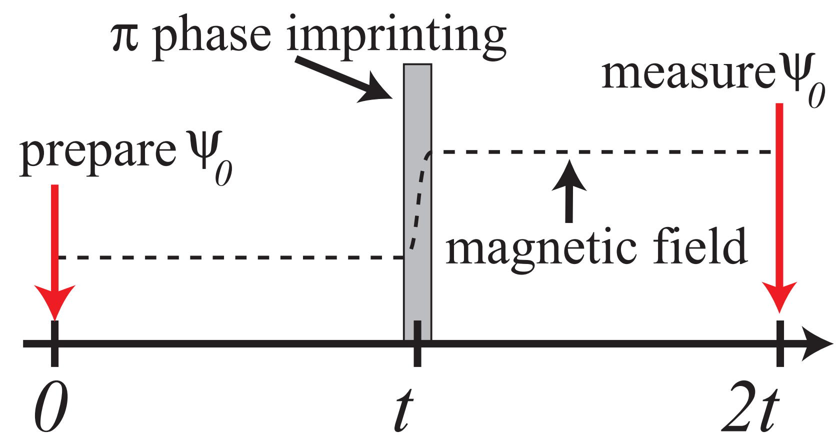
$$F\tau = \pi \equiv J \Rightarrow -J$$

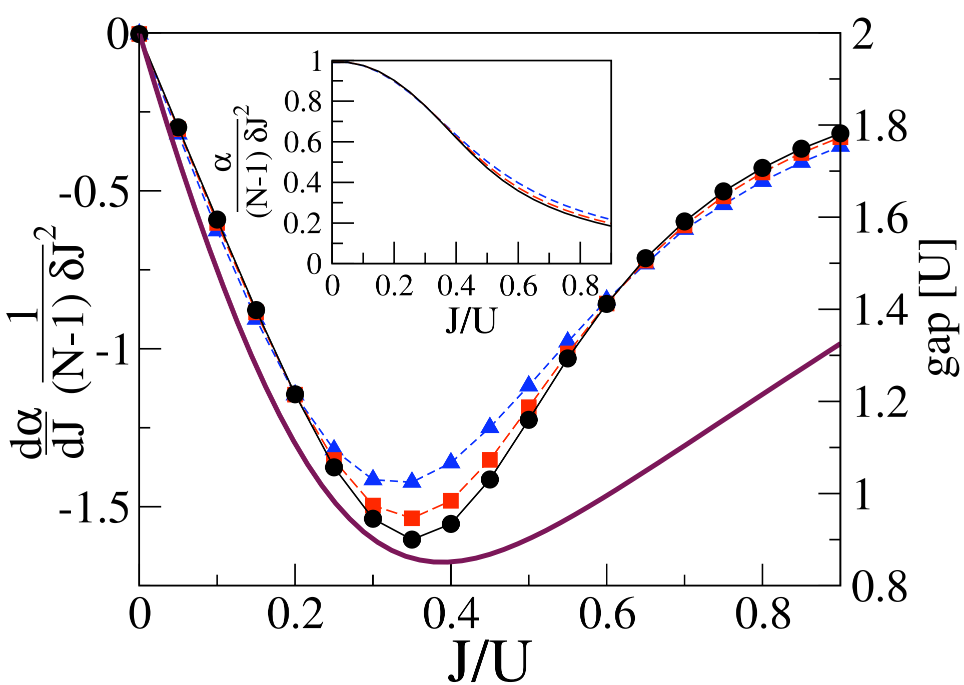
$$U \propto a_S \int |\psi_W(r)|^2\, d^3r$$

A Feshbach resonance is used to tune $a_S \Rightarrow -a_S$

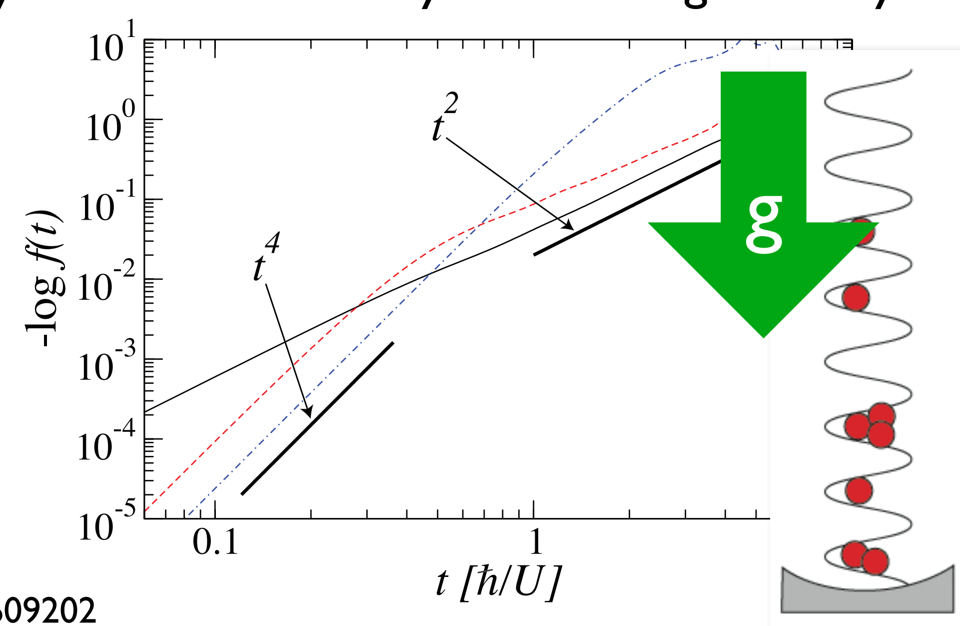
Time reversal in an optical lattice

Perform time reversal with fixed error and look at decay of M as a function of parameters





Further, we can make sensors by putting the system near criticality and looking at decay of M



FMC quant-ph/0609202

Conclusions

- Quantum information brings a fresh perspective to the quantum phase transitions field.
- Fidelity is well suited for certain transitions where study of correlations needs very large systems.
- Fidelity works well with MPS (and PEPS?) classical simulations (can it provide better estimates of critical points, exponents?).
- Study/define non-equilibrium quantum phase transitions
- What is the behavior of classical fidelity in normal PT?