Opinion Dynamics: Rise and Fall of Political Parties

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Complex Systems (T-13)

With: Paul Krapivsky, Sidney Redner *(Boston/CNLS)*

Thanks: Lev Tsimring (San Diego), Michael Cross (Caltech), Harvey Rose (T-13)

papers, talk available at http://cnls.lanl.gov/~ebn

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Plan

1. Motivation: modeling social dynamics
2. Noisy opinion dynamics
   - -- Single party dynamics
   - -- Two party dynamics
   - -- Multiple party dynamics
3. Noiseless opinion dynamics

E. Ben-Naim, cond-mat/0411427
E. Ben-Naim, P. Krapivsky, S. Redner, Physica D **183**, 190 (2003)

Modeling social dynamics

- Ultimate goal: predictive models of human opinions
- Relevance: politics, economics, consumer, sports

Questions

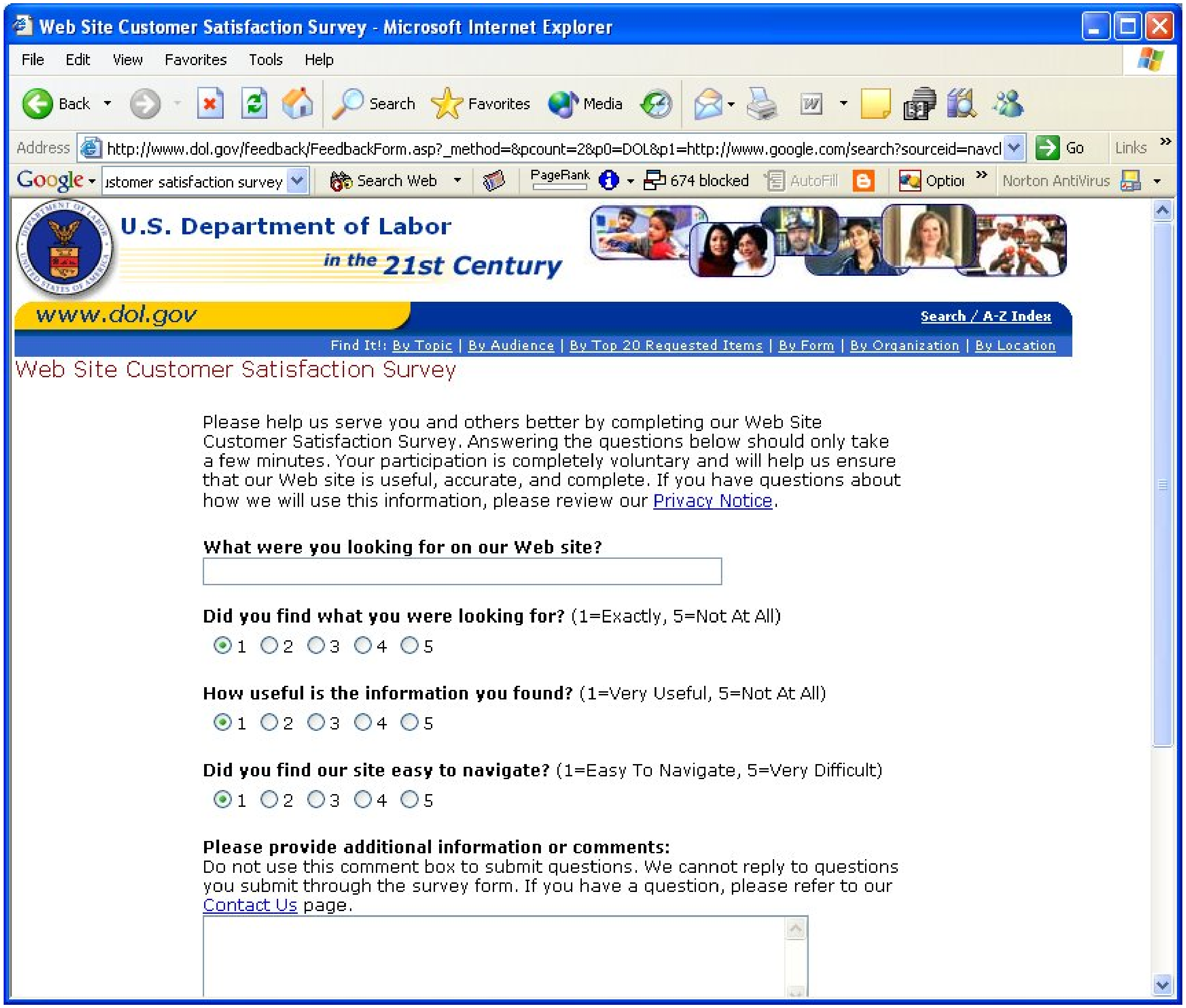
- Are "physics" concepts useful?

  Microscopic interactions → collective phenomena
- Are humans predictable?

This should help

- Large data sets available
- Large number of humans $N \sim 10^9$
- Human opinions can be quantified

Quantifying opinions



Humans interact, opinions evolve

NCAA Football Bowl Seaso

Rankings: Week 17

Division I-A Polls

AP Top 25		USA Today/ESPN	
1.	USC (44)	1.	USC (35)
2.	Oklahoma (14)	2.	Oklahoma (16)
3.	Auburn (7)	3.	Auburn (9)
4.	California	4.	California
5.	Utah	5.	Texas
6.	Texas	6.	Utah
7.	Louisville	7.	Georgia
8.	Georgia	8.	Louisville
9.	Virginia Tech	9.	Virginia Tech
10.	Boise State	10.	Boise State

tendency to reach consensus?

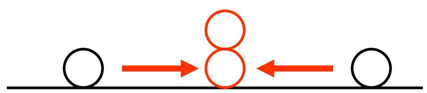
The Compromise Process

- **Opinion measured by a single variable**

$$-\Delta < n < \Delta$$

- **Compromise: reached via pairwise interactions**

$$(n_1, n_2) \rightarrow \left(\frac{n_1 + n_2}{2}, \frac{n_1 + n_2}{2} \right)$$



- **Conviction: restricted interaction range**

$$|n_1 - n_2| \le \delta$$

- **Minimal, one parameter model**
- **Mimics competition between compromise and conviction**

R Axelrod, J Conf. Res. 41, 203 (1997)

G. Deffuant, G Weisbuch et al, Adv. Comp. Sys 3, 87 (2000)

Diffusion (noise)

- Individuals may change opinion spontaneously

$$n \xrightarrow{D} n \pm 1$$

- Adds noise (“temperature”)
- Linear process: no interaction
- Mimics unstable, varying opinion
- Influence of environment, news, editorials, events

Rate equations

simplest compromise process
total opinion, total population conserved

$$(n-1, n+1) \to (n, n) \qquad \delta = 2$$

Probability distribution $P_n(t)$

Kinetic theory: nonlinear rate equations

$$\frac{dP_n}{dt} = 2P_{n-1}P_{n+1} - P_n(P_{n-2} + P_{n+2}) + D(P_{n-1} + P_{n+1} - 2P_n)$$

☑ **Numerical integration of probability distribution**

☒ **Monte Carlo simulation of stochastic process**

Single party dynamics

- **Initial condition: large isolated party**

$$P_n(0) = m\left(\delta_{n,0} + \delta_{n,-1}\right)$$

- **Steady-state: compromise and diffusion balance**

$$DP_n = P_{n-1}P_{n+1}$$

- **Core of party: localized to a few opinion states**

$$P_{-1} = P_0 = m \qquad P_1 = D \qquad P_2 = D^2 m^{-1}$$

- **Compromise negligible for n>2**

Well defined core

The Tail

- **Diffusion dominates outside the core**

$$\frac{dP_n}{dt} = D(P_{n-1} + P_{n+1} - 2P_n) \qquad P \ll D$$

- **Standard problem of diffusion with source**

$$P_n \sim m^{-1}\Phi\left(nt^{-1/2}\right)$$

- **Tail mass**

$$M_{tail} \sim m^{-1}t^{1/2}$$

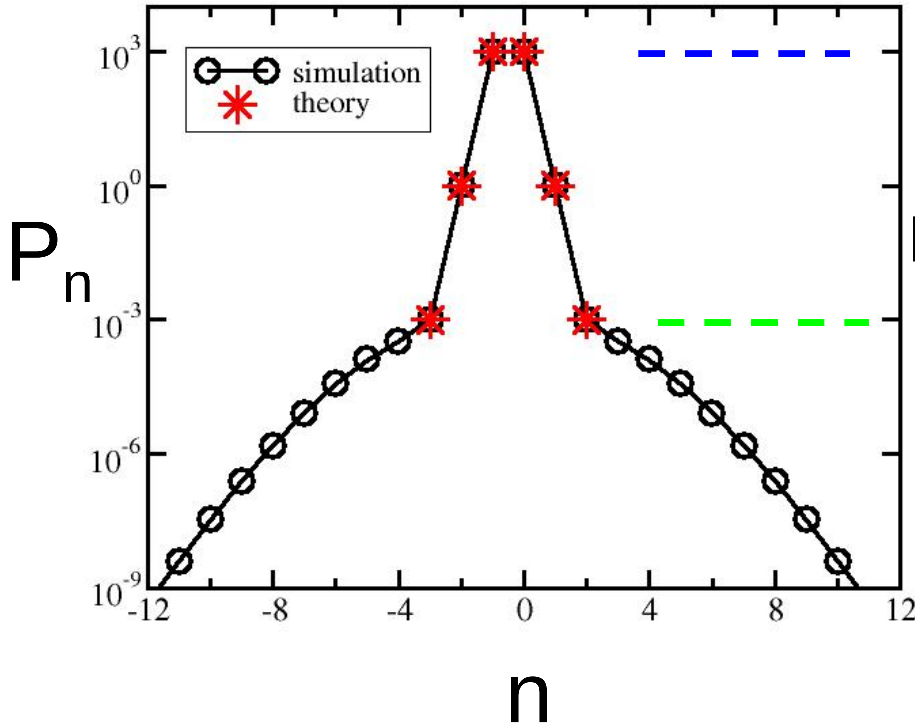
- **Party dissolves when**

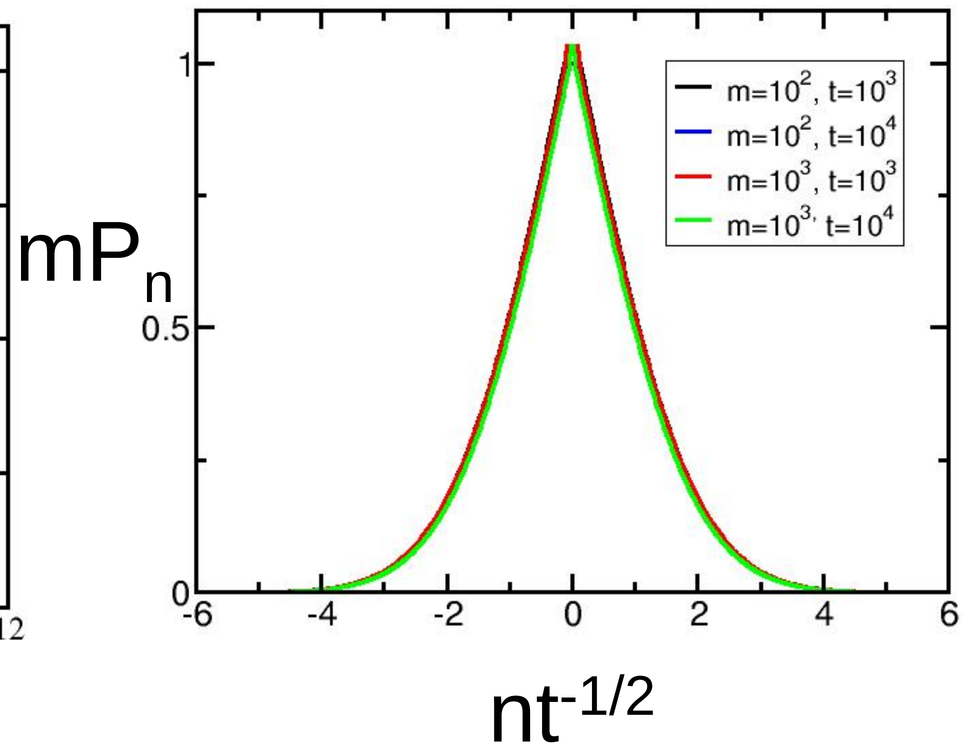
$$M_{tail} \sim m \qquad \Rightarrow \qquad \boxed{\tau \sim m^4}$$

Party lifetime grows fast with its size

Core versus Tail

$m=10^3$





Party height=m

Party depth~m^{-1}

Self-similar shape

Gaussian tail

Qualitative features

- Exists in a quasi-steady state
- Tight core localized to a few sites
- Random opinion changes of members do not affect party position
- Party lifetime grows very fast with size
- Ultimate faith of a party: demise
- Its remnant: a diffusive cloud
- Depth inversely proportional to size, the larger the party the more stable

Two party dynamics

- **Initial condition: two large isolated parties**

$$P_n(0) = m_1\left(\delta_{n,0} + \delta_{n,-1}\right) + m_2\left(\delta_{n,l+2} + \delta_{n,l+3}\right)$$

- **Interaction between parties mediated by diffusion**

$$0 = P_{n-1} + P_{n+1} - 2P_n$$

- **Boundary conditions set by parties depths**

$$P_0 = 1/m_1 \qquad P_l = 1/m_2$$

- **Steady state: linear profile**

$$P_n = \frac{1}{m_1} + \left(\frac{1}{m_2} - \frac{1}{m_1}\right)\frac{n}{l}$$

Merger

- Steady flux from small party to larger one

$$J \sim l^{-1}(1/m_< - 1/m_>) \sim (lm_<)^{-1}$$

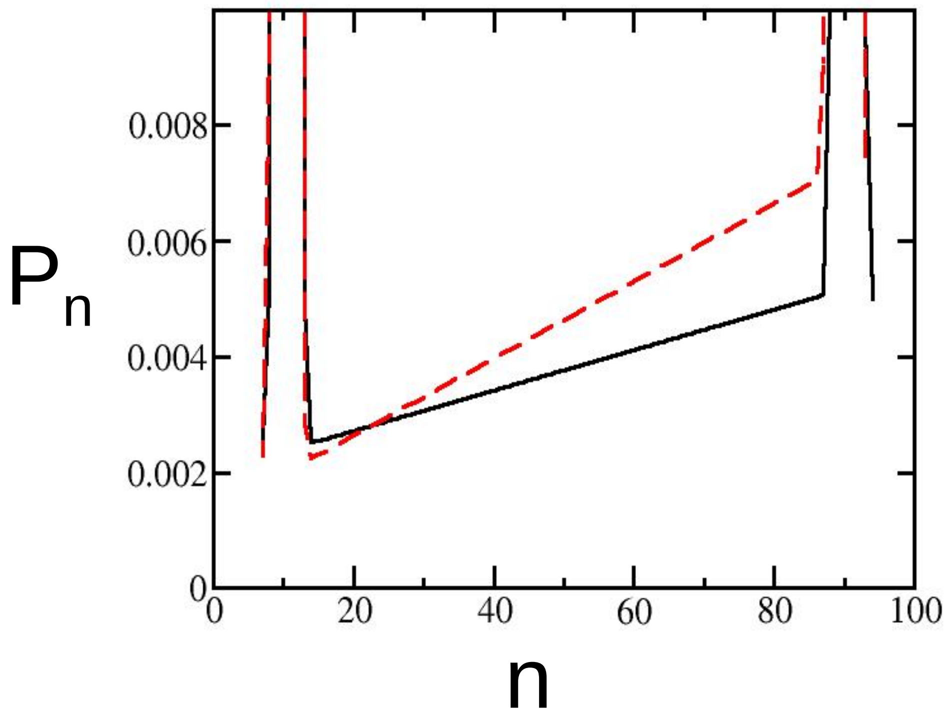
- Merger time

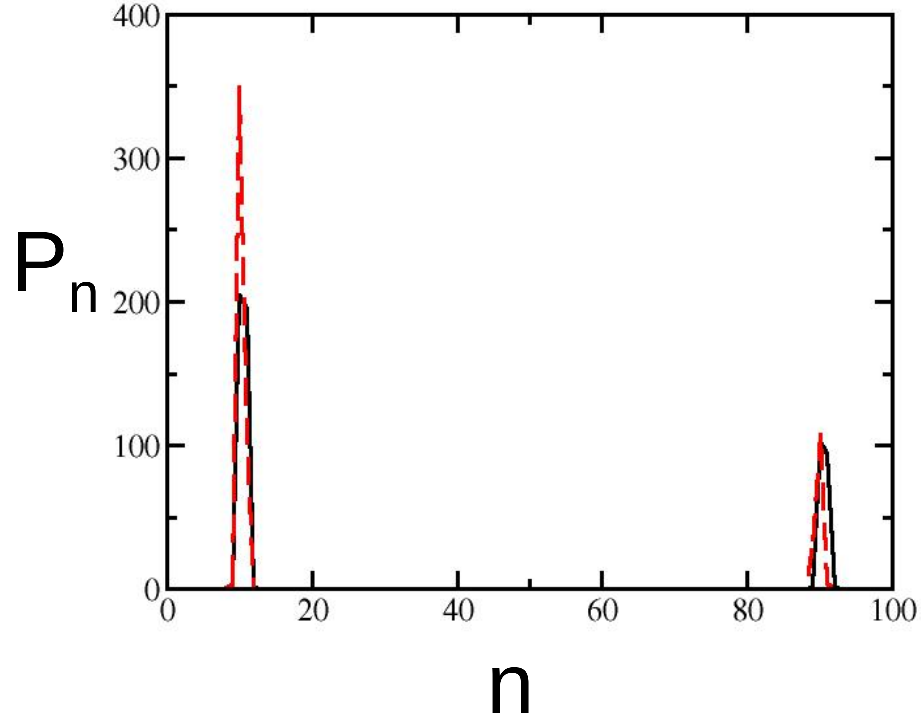
$$T \sim m_< / J \sim l(m_<)^2$$

- Lifetime grows with separation ("niche")
- Outcome of interaction is deterministic
- Larger party position remains fixed throughout merger process

Small party absorbed by larger one

Merger: numerical results





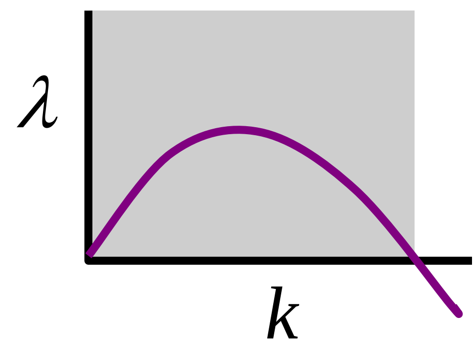
Multiple party dynamics

- **Initial condition: large isolated party**

$$P_n(0) = \text{randomly chosen number in } [1-\varepsilon : 1+\varepsilon]$$

- **Linear stability analysis**

$$P_n - 1 \sim \exp[ikn + \lambda t]$$



- **Growth rate of perturbations**

$$\lambda = 2(2\cos k - \cos 2k - 1) + 2D(\cos k - 1)$$

- **Long wavelength perturbations unstable**

$$k < k_0 \qquad \cos k_0 = D/2$$

P=1 stable only for strong diffusion $D>D_c=2$

Strong noise ($D>D_c$)

- Regardless of initial conditions

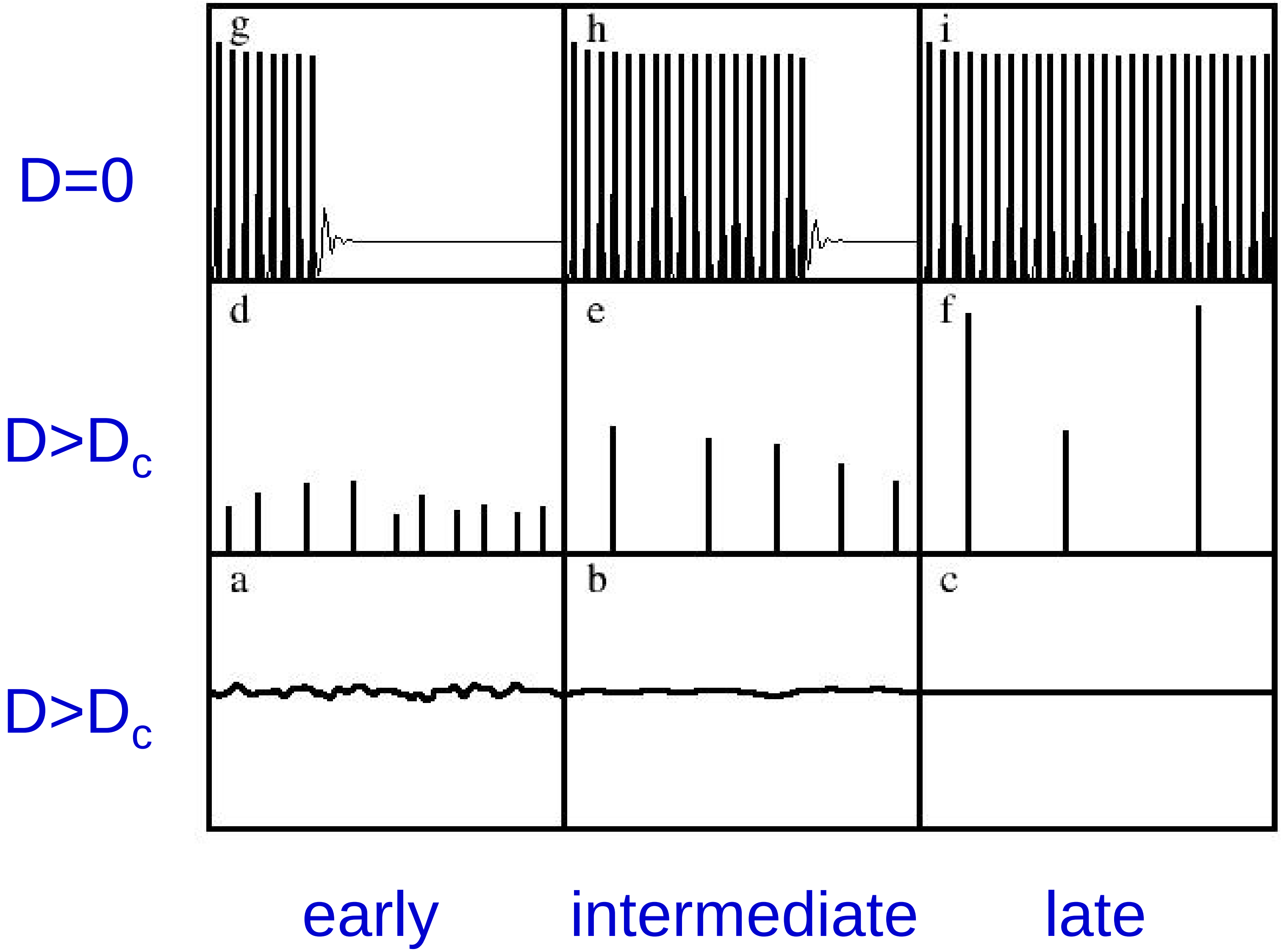
$$P_n \to \langle P_n(0) \rangle = 1$$

- Relaxation time

$$\lambda \cong (D_c - D)k^2 \quad \Rightarrow \quad \tau \sim (D - D_c)^{-2}$$

No parties, disorganized political system

Three scenarios



Weak noise ($D<D_c$): Coarsening

- Smaller parties merge into large parties
- Party size grows indefinitely
- Assume a self-similar process, size scale m
- Conservation of populations implies separation

$$l \sim m$$

- Use merger time to estimate size scale

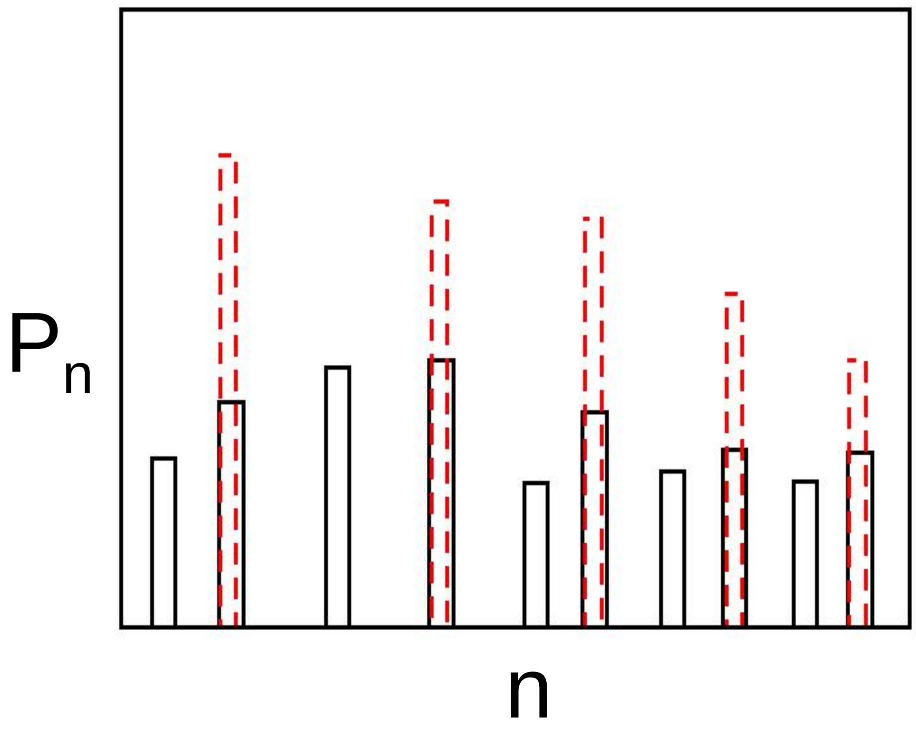
$$t \sim lm^2 \sim m^3 \qquad \Rightarrow \qquad \boxed{m \sim t^{1/3}}$$

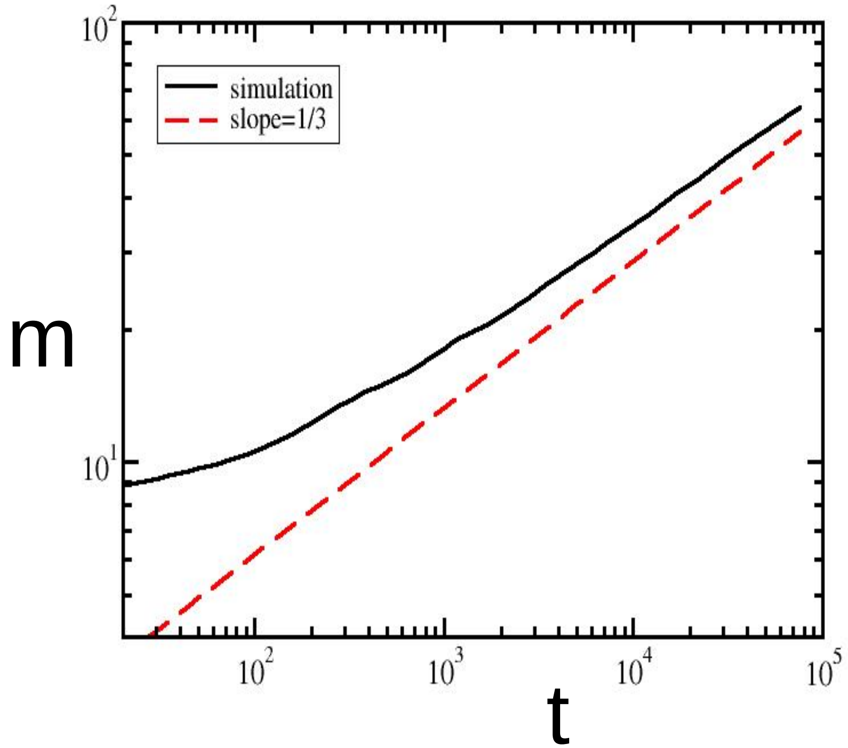
- Self-similar size distribution

$$P_m \sim t^{-1/3} F\left(mt^{-1/3}\right)$$

Lifshitz-Slyozov ripening

Coarsening: numerics





- **Parties are static throughout process**
- **A small party with a large niche may still outlast a larger neighbor!**

Conclusions: noiseless dynamics

- Isolated parties
  - Tight, immobile core and diffusive tail
  - Lifetime grows fast with size
- Interaction between two parties
  - Large party grows at expense of small one
  - Deterministic outcome, steady flux
- Multiple parties
  - Strong noise: disorganized political system, no parties
  - Weak noise: parties form, coarsening mosaic
  - No noise: pattern formation

Pure compromise dynamics (D=0)

problem setup

- **Given initial distribution (continuous opinions)**

$$P_0(x) = \begin{cases} 1 & |x| < \Delta \\ 0 & |x| > \Delta \end{cases}$$

- **Find final distribution (frozen)**

$$P_\infty(x) = ?$$

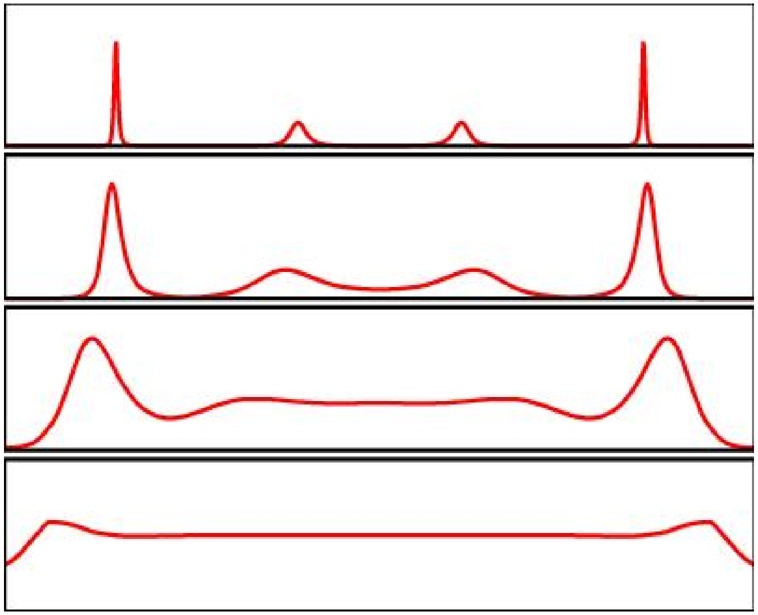
- **Multitude of final states**

$$P_\infty(x) = \sum_{i=1}^{N} m_i \delta(x - x_i) \qquad |x_i - x_j| > 1$$

- **Dynamics selects one (deterministically)**

Multiple localized clusters (parties)

kinetic theory



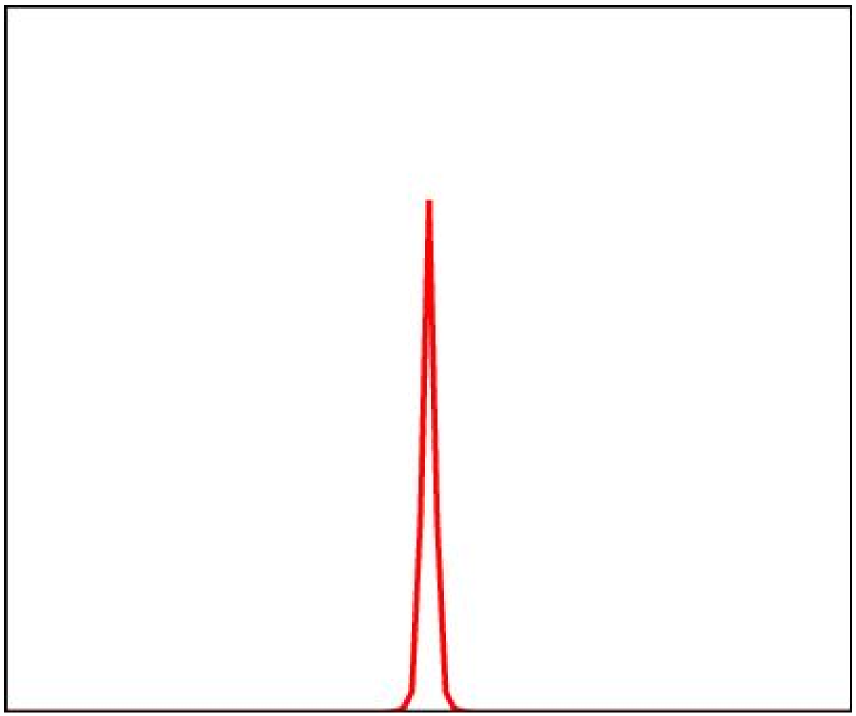
☑ **Numerical integration of probability distribution**

$$\frac{\partial}{\partial t}P(x,t)=\iint_{|x_1-x_2|<1}dx_1dx_2P(x_1,t)P(x_2,t)\left[2\delta\left(x-(x_1+x_2)/2\right)-\delta(x-x_1)-\delta(x-x_2)\right]$$

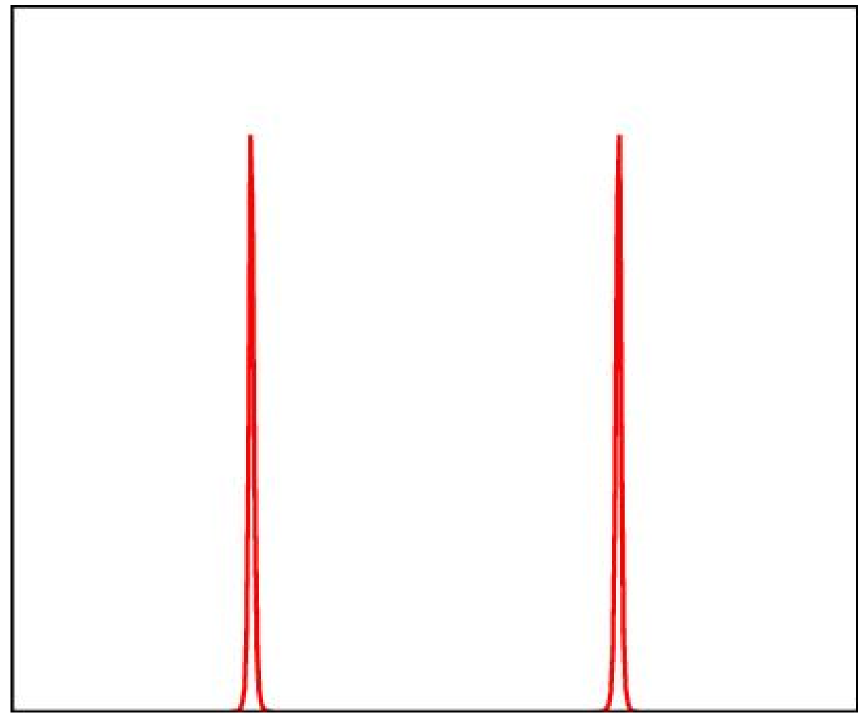
☒ **Direct simulation of stochastic process**

Rise and fall of central party

$0 < \Delta < 1.871$



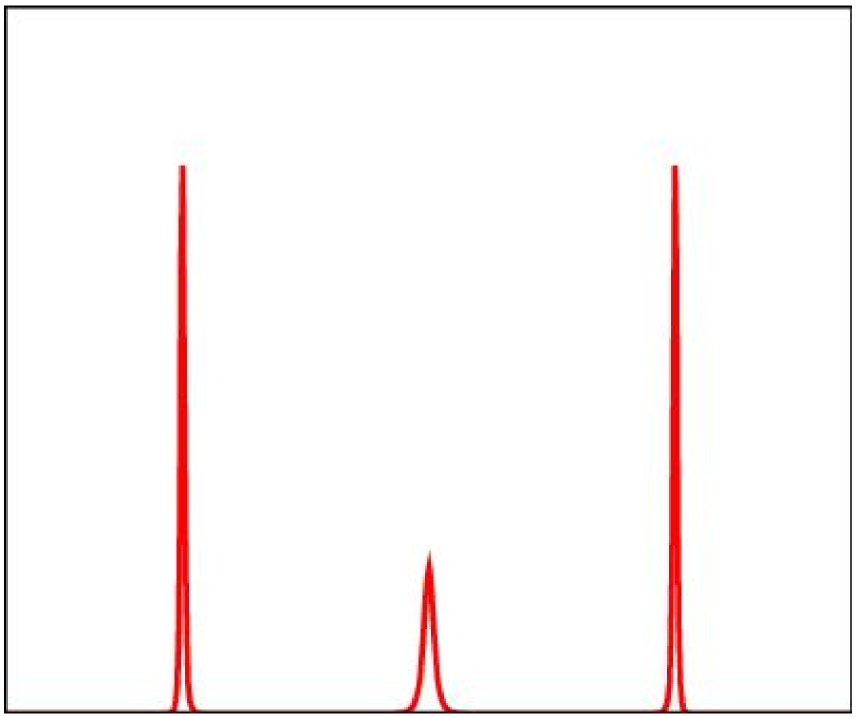
$1.871 < \Delta < 2.724$



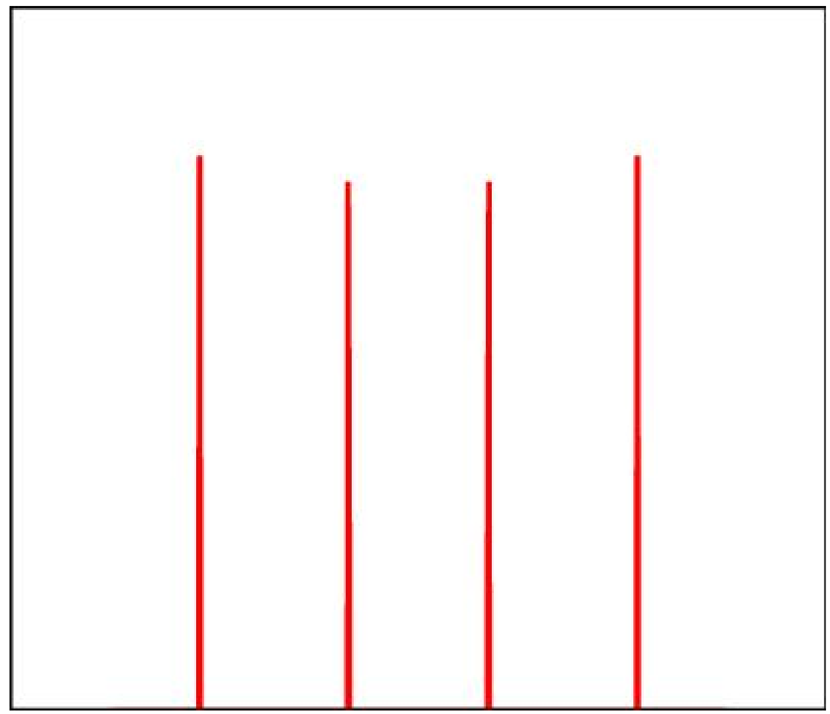
Central party may or may not exist!

Reemergence of central party

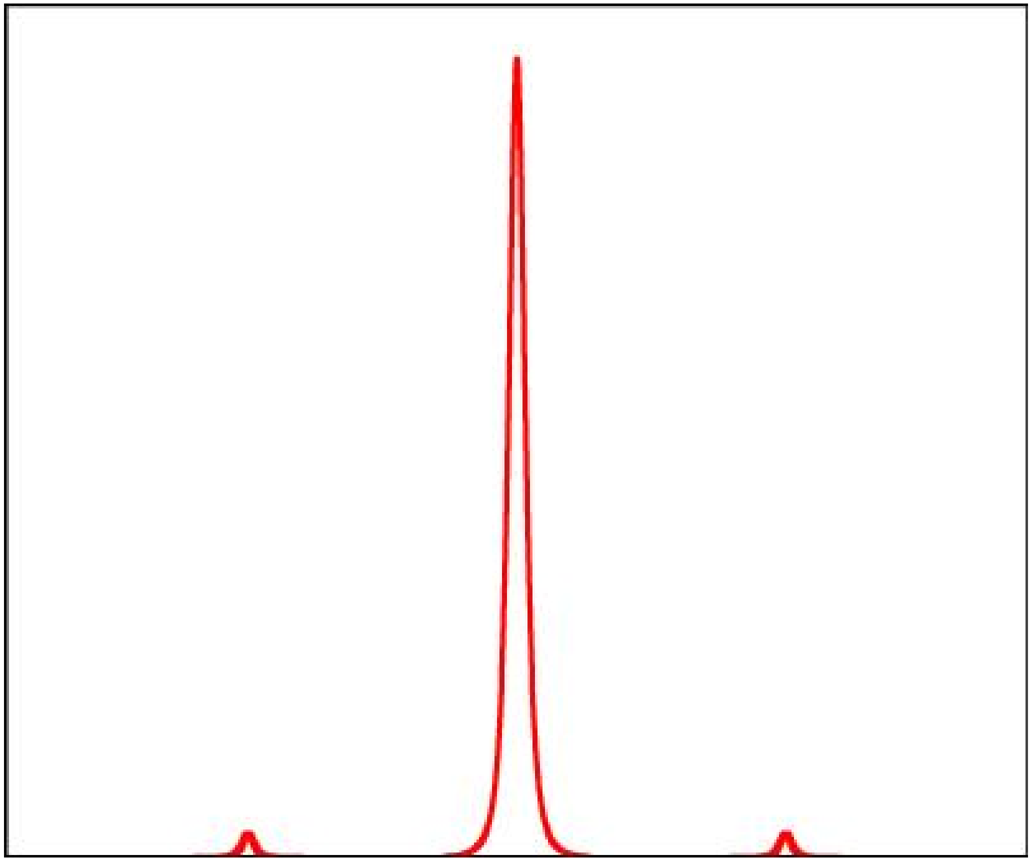
$2.724 < \Delta < 4.079$



$4.079 < \Delta < 4.956$

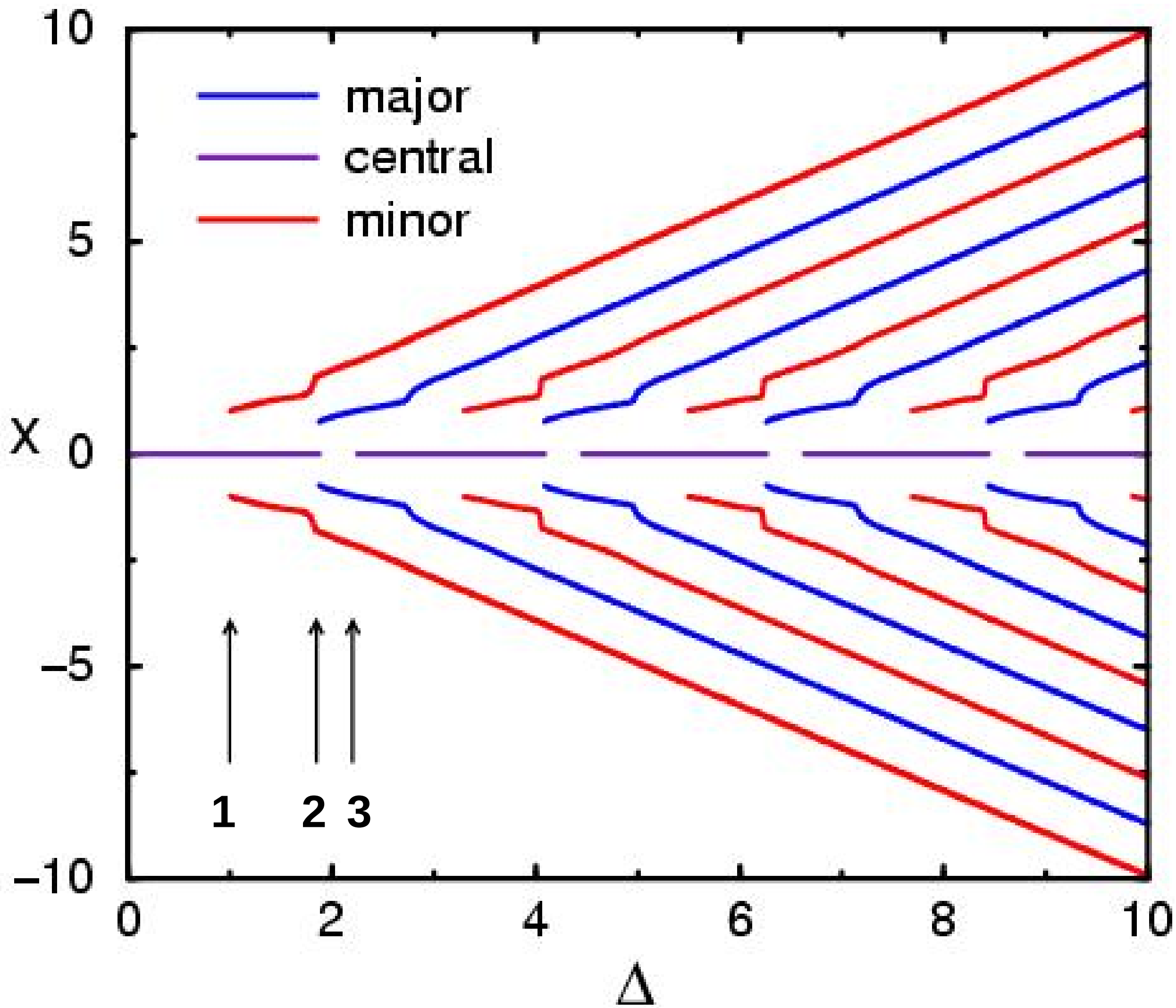


Emergence of extremists



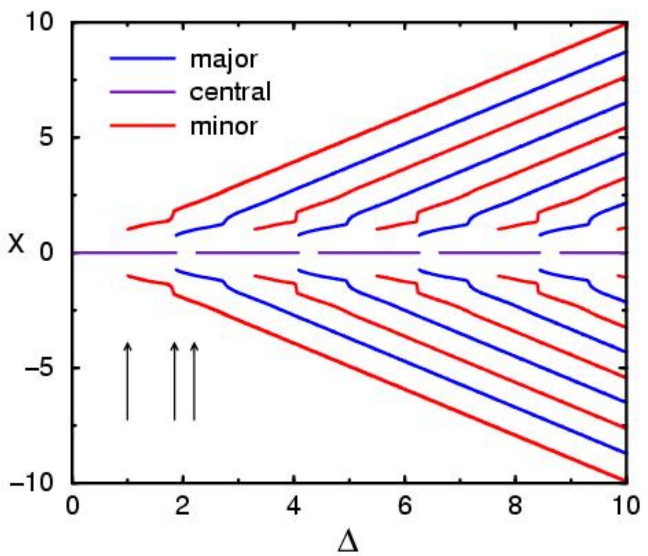
Tiny parties (mass <10^{-3})

Bifurcations and Patterns



Self-similar structure, universality

- **Periodic sequence of bifurcations**
  1. **Nucleation of minor cluster branch**
  2. **Nucleation of major cluster branch**
  3. **Nucleation of central cluster**

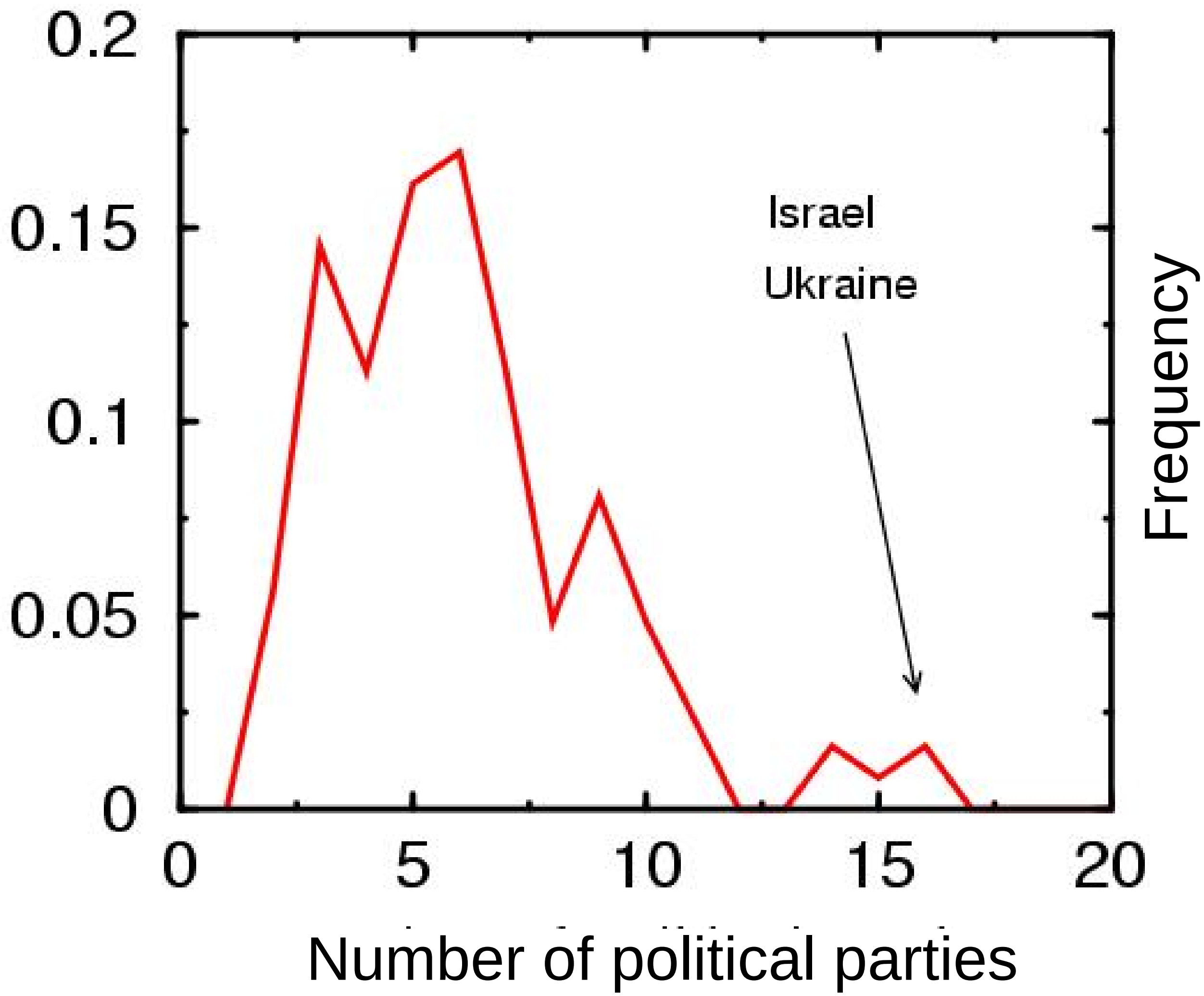


- **Alternating major-minor pattern**
- **Clusters are equally spaced**
- **Period gives major cluster mass, separation**

$$x(\Delta) = x(\Delta + L) \qquad L = 2.155$$

How many political parties?

- Data: CIA world factbook 2002
- 120 countries with multi-patry parliaments
- Average=5.8 standard deviation=2.9



Cluster mass

- **Masses are periodic**

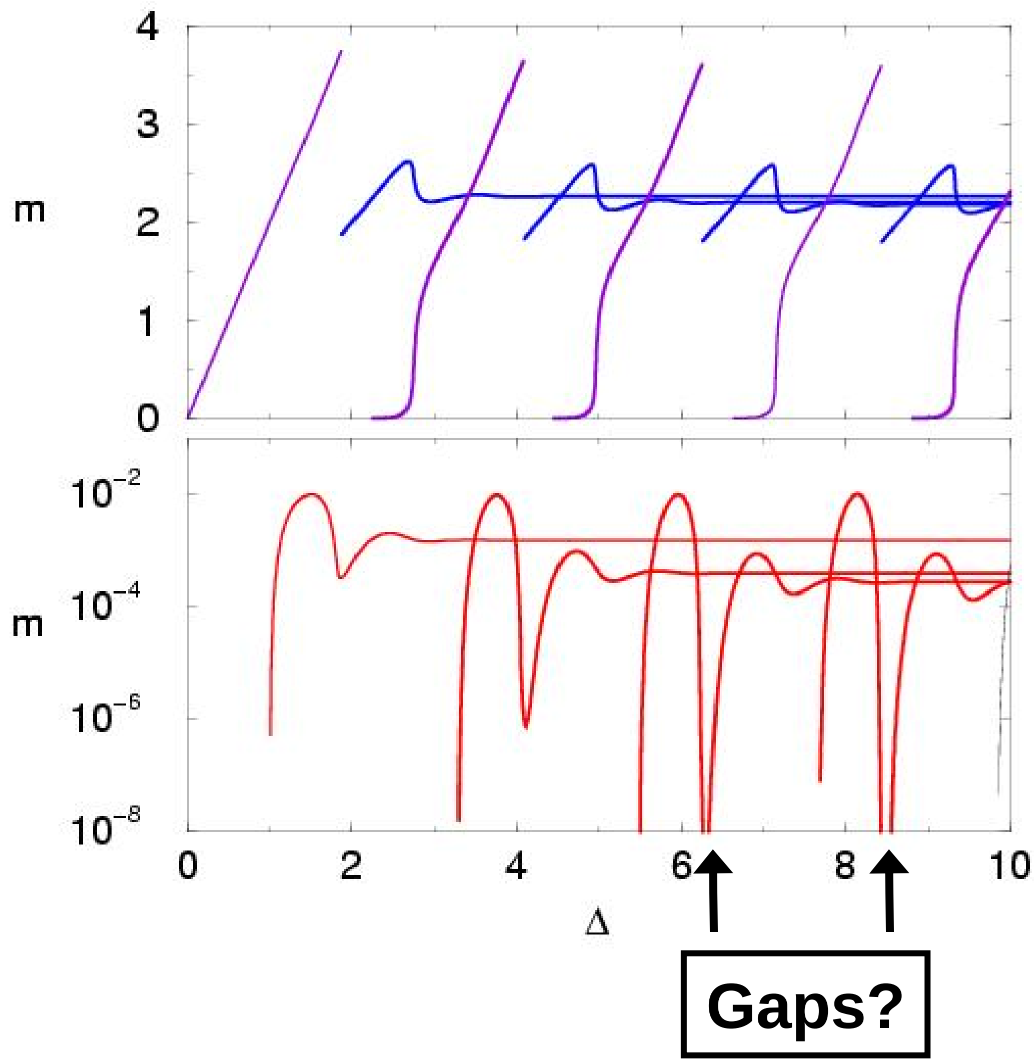
$$m(\Delta) = m(\Delta + L)$$

- **Major mass**

$$M \rightarrow L = 2.155$$

- **Minor mass**

$$m \rightarrow 3\times10^{-4}$$



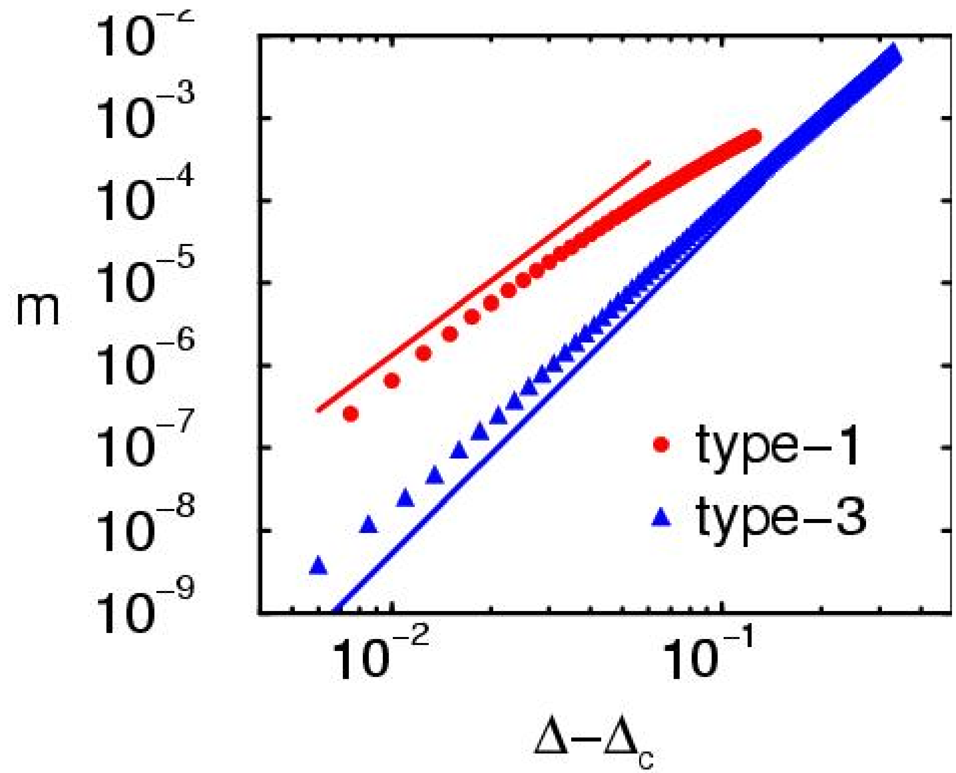
Scaling near bifurcation points

- **Minor mass vanishes**

$$m \sim (\Delta - \Delta_c)^{\alpha}$$

- **Universal exponents**

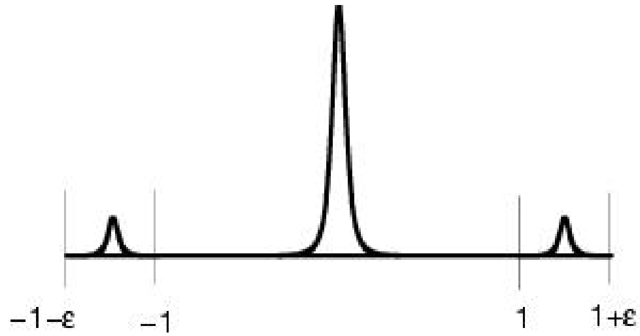
$$\alpha = \begin{cases} 3 & \text{type } 1 \\ 4 & \text{type } 3 \end{cases}$$



L-2 is the small parameter
explains small saturation mass

Heuristic derivation of exponents

- **Perturbation theory** $\Delta = 1 + \varepsilon$
- **Central cluster** $x(\infty) = 0$
- **Extremist minor cluster** $x(\infty) = 1 + \varepsilon / 2$



◆ Rate of transfer from minor cluster to major cluster

$$dm / dt = -mM \quad \rightarrow \quad m(t) \sim \varepsilon e^{-t}$$

◆ Process stops when

$$x \sim e^{-t_f / 2} \sim \varepsilon$$

◆ Final minor cluster mass

$$m(\infty) \sim m(t_f) \sim \varepsilon^3$$

Consensus

- **Integrable for** $\Delta < 1/2$

$$\langle x^2(t)\rangle = \langle x^2(0)\rangle e^{-\Delta t}$$

- **Final state: localized**

$$P_\infty(x) = 2\Delta\delta(x)$$

- **Rate equations in Fourier space**

$$P_t(k) + P(k) = P^2(k/2)$$

- **Self-similar collapse dynamics**

$$\Phi(z) \propto (1+z^2)^{-2} \qquad z = \frac{x}{\sqrt{\langle x^2(t)\rangle}}$$

The Inelastic Maxwell Model, Ben-Naim & Krapivsky, Lecture Notes in Physics **624,** 65 (2003)

Pattern selection

- **Linear stability analysis**

$$P-1 \propto e^{i(kx+\omega t)} \quad \Rightarrow \quad \omega(k)=\frac{8}{k}\sin\frac{k}{2}-\frac{2}{k}\sin k-2$$

- **Fastest growing mode**

$$d\omega/dk=0 \quad \Rightarrow \quad L=2\pi/k=2.2515$$

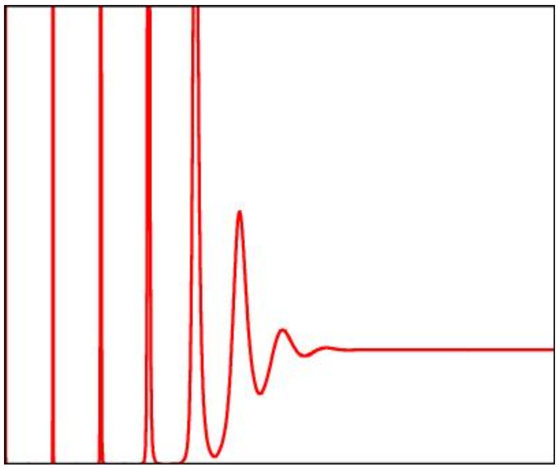
- **Traveling wave (FKPP extremal selection)**

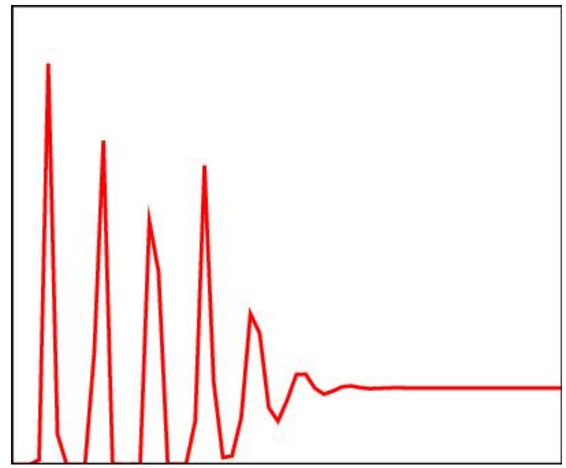
$$d\omega/dk=\mathrm{Im}(\omega)/\mathrm{Im}(k) \quad \Rightarrow \quad L=2\pi/k=2.0375$$

Patterns induced by wave propagating from boundary.
However, emerging period is different L=2.155!

Pattern selection intrinsically nonlinear

Traveling waves



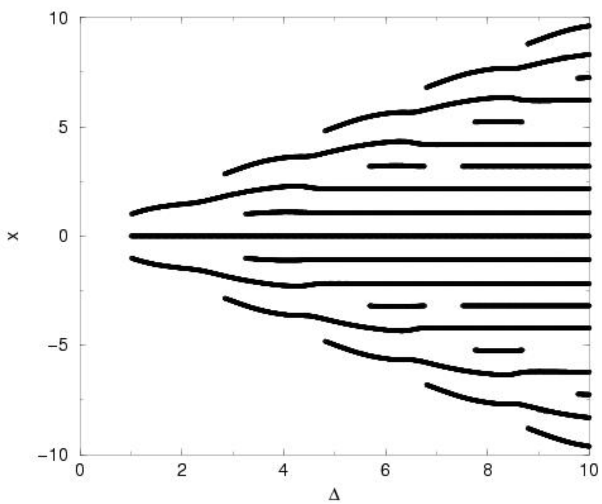


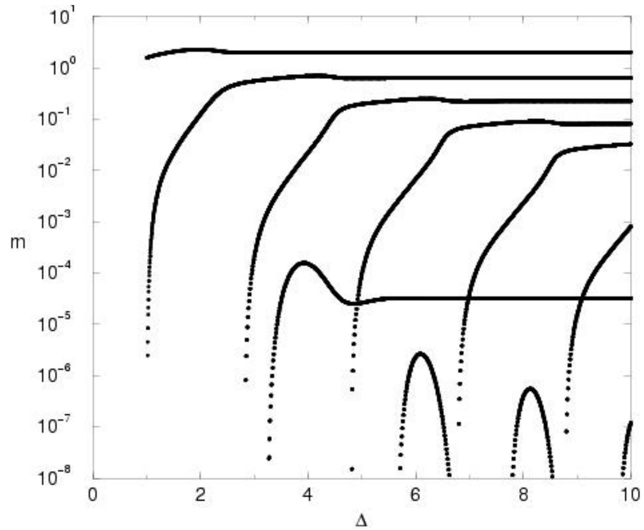
$$P - 1 \propto \exp\left[-\lambda(x - vt) + i(kx + wt)\right]$$

Discrete opinions

$$L_{\max} = 6 \qquad L = 5.67 \qquad L_{\text{trav wave}} = 5.31$$

Exponential initial conditions

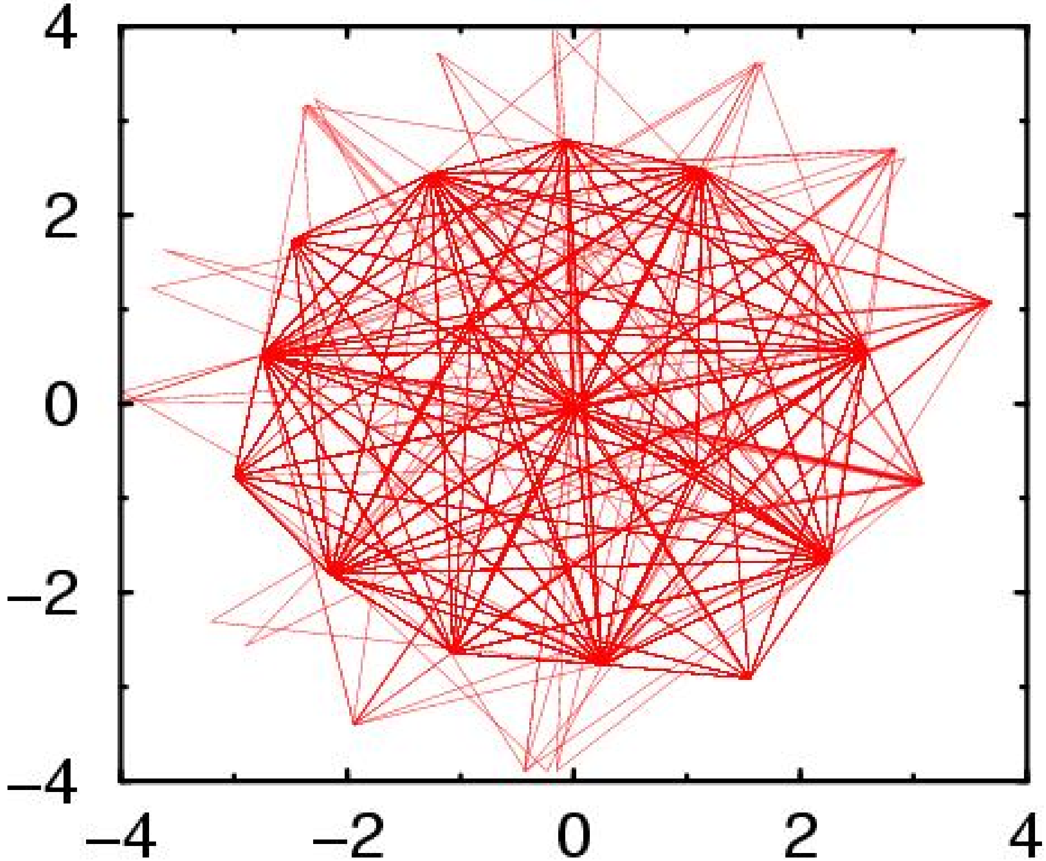




- **Bifurcations induced at the boundary**
- **Periodic structure, nontrivial period**
- **Two types of bifurcations**
  1. **Nucleation of major branch**
  2. **Nucleation of minor branch**

Central cluster is stable

Two kinds of opinions



symmetry breaking, packing

Conclusions: noiseless dynamics

- Clusters form via bifurcations
- Periodic structure
- Alternating minor-major pattern
- Central party not always exists
- Power-law behavior near transitions

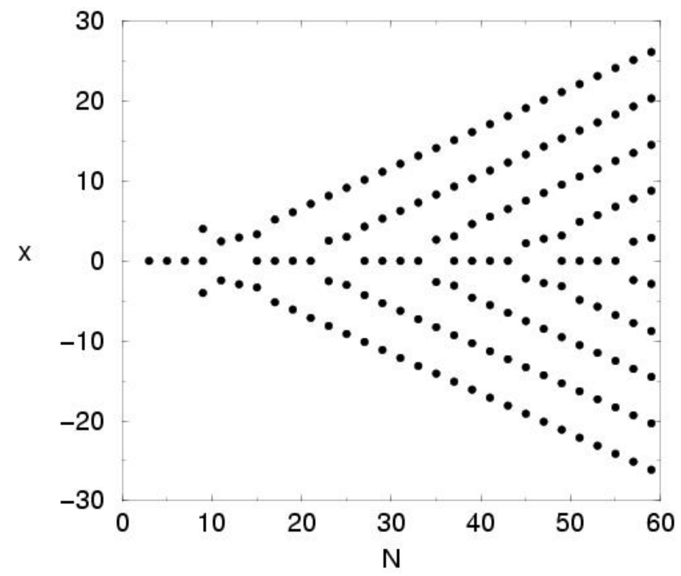
Outlook

- ✓ Pattern selection criteria
- ◆ Gaps
- ◆ Role of initial conditions, classification
- ◆ Role of spatial dimension, correlations
- ◆ Disorder, inhomogeneities
- ◆ Tiling/Packing in 2D
- ◆ Discord dynamics (seceder model, Halpin-Heally 03)

Many open questions

General features

- Dissipative system, volume contracts
- Energy (Lyapunov) function exists: <x^2>
- No cycles or strange attractors
- Uniform state is unstable (Cahn-Hilliard)



$$P_i = 1 + \phi_i \qquad \phi_t + \left(\phi + a\phi_{xx} + b\phi^2\right)_{xx} = 0$$

Discrete case yields useful insights

Discrete opinions

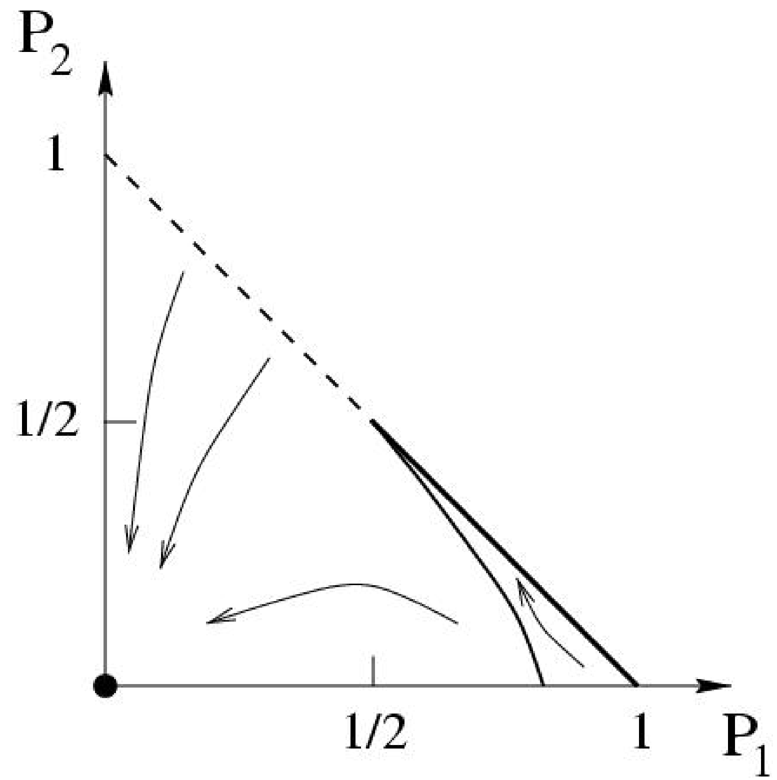
- **Compromise process**

$$(i-1, i+1) \to (i, i)$$

- **Master equation**

$$dP_i / dt = 2P_{i-1}P_{i+1} - P_i(P_{i-2} + P_{i+2})$$

- **Example: 6 states**
- **Symmetry + normalization: two-dimensional problem**



Initial conditions determine final state

Isolated fixed points, lines of fixed points