Comment on “Dynamic Scaling in the Spatial Distribution of Persistent Sites”

E. Ben-Naim[1] and P. L. Krapivsky[2]
[1] *Theoretical Division and Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos NM 87545*
[2] *Center for Polymer Studies and Department of Physics, Boston University, Boston MA, 02215*

Recently, Manoj and Ray [1] investigated the spatial distribution of unvisited sites in the one-dimensional single-species annihilation process $A + A \to 0$. They claimed that this distribution is characterized by a new length scale $\mathcal{L}(t) \sim t^z$, and that the dynamical exponent z depends upon the initial concentration. We show numerically that this assertion is erroneous. Regardless of the initial concentration, this spatial distribution is characterized by the diffusive length scale $\mathcal{L}_D(t) \sim (Dt)^{1/2}$.

The spatial distribution of unvisited sites is described by $F_l(t)$, the probability density that two consecutive unvisited sites are separated by l sites. This quantity satisfies $P_0(t) = \sum_l F_l(t)$ and $1 = \sum_l (l+1)F_l(t)$, with $P_0(t)$ the fraction of unvisited sites. The latter condition follows from length conservation.

In the diffusive annihilation process $A + A \to 0$, the particle density $n(t)$ decays algebraically according to $n(t) \simeq (8\pi Dt)^{-1/2}$, where D is the hopping rate. This behavior is independent of the initial concentration. Furthermore, writing $n(t) \sim 1/\mathcal{L}_D(t)$ suggests that the diffusive length is the only asymptotically relevant length scale. As shown in Fig. 1, the following scaling form holds

$$F_l(t) \sim t^{-1}\mathcal{F}\left(lt^{-1/2}\right), \tag{1}$$

indicating that $F_l(t)$ is characterized by $\mathcal{L}_D(t)$ alone. The scaling form (1) is consistent with the normalization $1 = \sum_l (l+1)F_l(t)$. We stress that the scaling function $\mathcal{F}(x)$ is also independent of the initial conditions.

The known decay of the number of unvisited sites $P_0(t) \sim t^{-\theta}$ ($\theta = 3/8$ is the persistence exponent) together with the requirement $P_0(t) = \sum_l F_l(t)$ can now be used to infer the small x divergence of $\mathcal{F}(x)$

$$\mathcal{F}(x) \sim x^{-2(1-\theta)} \qquad x \to 0. \tag{2}$$

In the complementary $x \to \infty$ limit, we observed an exponential decay $\mathcal{F}(x) \sim \exp(-Ax)$, indicating independence of distant unvisited sites. Similar small argument divergence and exponential tail underly a related quantity $P_n(t)$ [2], the probability that a site has been visited n times. In both cases, the corresponding exponent follows directly from θ. Furthermore, Eqs. (1)-(2) extend to persistent sites in the q-state Potts-Glauber model if θ is replaced by $\theta(q)$ [3].

In summary, the spatial distribution of unvisited sites is characterized by the diffusive length scale. Although the initial concentration affects the transient behavior, it is irrelevant asymptotically (see Fig. 2). The scaling form suggested in Ref. [1] is correct only if the choices $z = 1/2$, $\omega = \theta$, and $\tau = 2(1-\theta) = 5/4$ are made. Thus, no additional independent exponents emerge from $F_l(t)$.

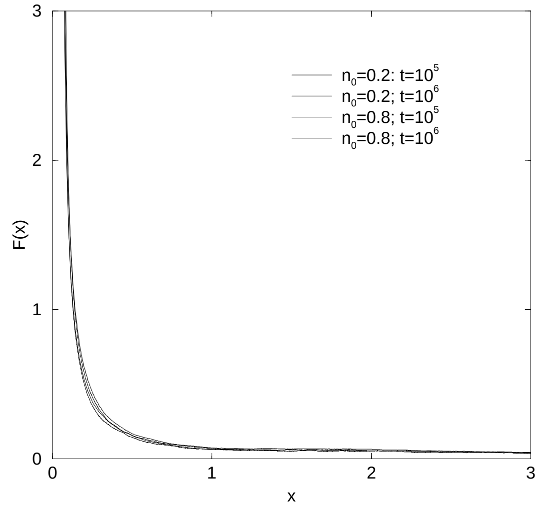

Fig. 1 The scaling distribution $\mathcal{F}(x) \equiv tF_l(t)$ versus $x = lt^{-1/2}$ for two different times $t = 10^5$, 10^6, and two different initial densities $n_0 = 0.2$, 0.8. Numerical simulations were performed in a system of size $L = 10^7$ and the results represent an average over 10 different realizations.

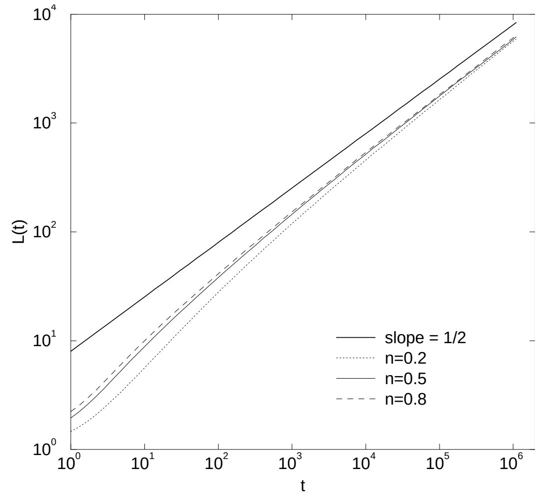

Fig. 2 Time dependence of the typical domain size $L(t) = \sum_l l^2 F_l(t) / \sum_l lF_l(t)$.

[1] G. Manoj and P. Ray, cond-mat/9901130.
[2] E. Ben-Naim, L. Frachebourg, and P. L. Krapivsky, *Phys. Rev. E* **53**, 3078 (1996).
[3] B. Derrida, V. Hakim, and V. Pasquier, *Phys. Rev. Lett.* **75**, 751 (1995).