Domain Statistics in Coarsening Systems

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References

1. P. L. Krapivsky and E. Ben-Naim, *Phys. Rev. E* **56**, 3788 (1997).

2. E. Ben-Naim and P. L. Krapivsky, *J. Stat. Phys.*, in press.

Domain Number Distribution

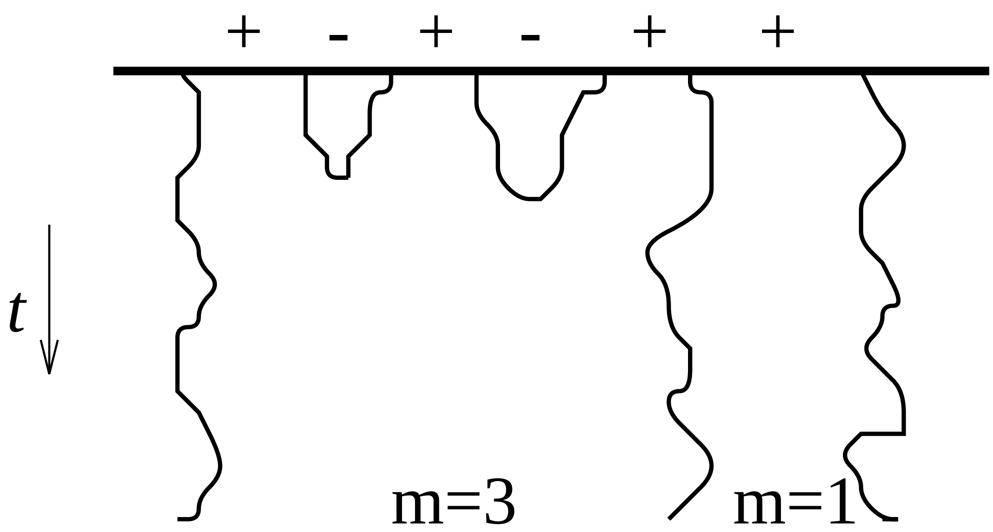


Fig. 1. Domain motion in the Ising-Glauber model. Surviving domains are marked by $+$, annihilated domains by $-$. The domain number m at a later time is also indicated.

- **The Domain Distribution:** Let $Q_m(t)$ be the distribution of domains with m ancestors. Well defined in arbitrary 1D coarsening processes. Gives the following quantities:

- **The Domain Density:** $N(t) = \sum_m Q_m(t)$

- **The Domain Survival Probability:** $S(t) = \sum_m mQ_m(t)$

- **Unreacted ("single parent") Domain Density:** $Q_1(t)$

1D Ising model with nonconserving Glauber dynamics

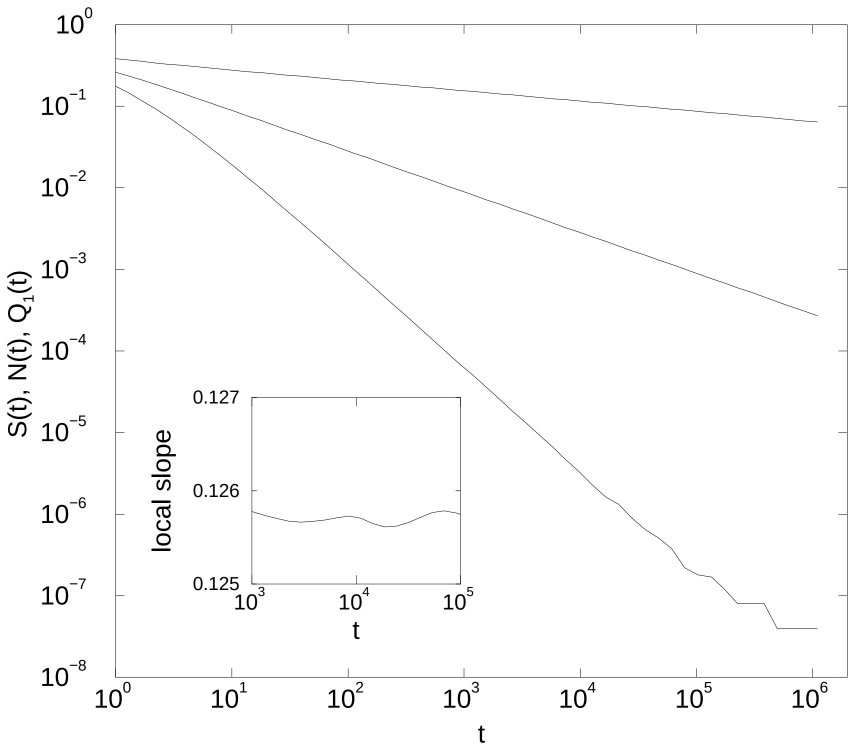


Fig. 2. Monte Carlo data for the Ising-Glauber model. The domain survival probability $S(t)$, the domain density $N(t)$, and the density of unreacted domains $Q_1(t)$ are shown (top to bottom). The inset plots the local slope $-d\ln S(t)/d\ln t$. Size of spin chain is $L = 10^7$.

$$\boxed{\textbf{All densities decay algebraically}}$$

Scaling Properties

The domain density ($\nu = 1/z$, z the dynamical exponent)

$$N(t) \sim t^{-\nu}$$

The domain survival probability

$$S(t) \sim t^{-\psi}$$

The density of unreacted Domains

$$Q_1(t) \sim t^{-\delta}$$

The domain distribution

$$Q_m(t) \simeq t^{\psi-2\nu}\mathcal{Q}(mt^{\psi-\nu})$$

Bounds on exponent (since $Q_1 \leq \Sigma_m Q_m \leq \Sigma_m mQ_m$)

$$\psi \leq \nu \leq \delta$$

Relation to persistence exponent (since $S(t) \leq P(t) \sim t^{-\theta}$)

$$\psi \leq \theta$$

Numerical Verification

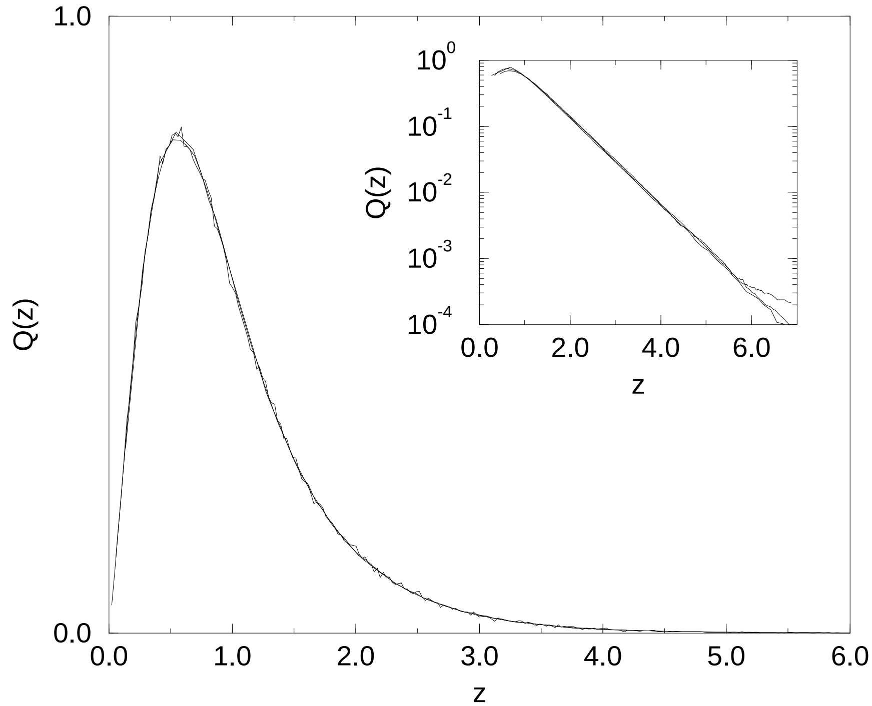


Fig. 3. The scaling distribution $\mathcal{Q}(z)$ vs. $z = m/\langle m \rangle$ for three different times $t = 10^2, 10^3, 10^4$ in the Ising-Glauber case. The inset demonstrates the exponential behavior of the large-z tail. 100 systems of size $L = 10^5$.

Extremal properties of scaling function

$$\mathcal{Q}(z) \sim \begin{cases} z^{\sigma} & z \ll 1, \\ \exp(-\kappa z) & z \gg 1. \end{cases}$$

Scaling relation (obtained by considering Q_1)

$$\delta - \nu = (\nu - \psi)(1 + \sigma)$$

$$\boxed{\textbf{Only } \psi \textbf{ and } \delta \textbf{ are independent exponents}}$$

Independent Interval Approximation (IIA)

- **Domain Length-Number Distribution:** Let $P_{n,m}(t)$ be the distribution of domains of length n and m ancestors. Gives the domain number distribution $Q_m(t) = \sum_n P_{n,m}(t)$, and the domain length distribution $P_n(t) = \sum_m P_{n,m}(t)$.

1D T=0 Ising-Potts model with Glauber dynamics: Single spin flip dynamics. Domains walls perform random walk and annihilate/coalesce upon contact.

Rate Equation:

$$\begin{aligned}\frac{dP_{n,m}}{dt} &= P_{n-1,m} + P_{n+1,m} - 2P_{n,m} \\ &+ \frac{P_1}{(q-1)N^2}\left[\sum_{i,j} P_{i,j}P_{n-1-i,m-j} - N(P_{n,m} + P_{n-1,m})\right]\end{aligned}$$

Exact exponents: (D_α the cylinder parabolic function)

$$\begin{aligned}\delta &= \frac{1}{2} + \frac{1}{q} \\ 0 &= \int_0^\infty dx\, x^{-2\psi} D_{1/q}(x) D'_{1/q}(x)\end{aligned}$$

$$\boxed{\textbf{IIA assumes neighboring domains are uncorrelated}}$$

Features of the approximation

	MC			IIA	
q	ψ	δ	σ	ψ	δ
2	0.126	1.27	1.05	0.136612	1
3	0.213	0.98	0.67	0.231139	5/6
8	0.367	0.665	0.24	0.385019	5/8
50	0.476	0.525	0.03	0.480274	13/25
∞	1/2	1/2	0	1/2	1/2

- **Gives exact $\nu = 1/2$, good approximation for ψ, δ:**
- **Approximation is exact for $q = 1$ and $q = \infty$**
- **Correct scaling behavior of $Q_m(t)$ and $P_n(t)$**

$\boxed{\textbf{IIA provides a very close description}}$

Ising Model with Conserving Kawasaki Dynamics

- **Kinetics: spin-exchange.** In $T \downarrow 0$ limit, domains of length L diffuse with rate L^{-1}.

- **Independent Interval Approximation:**

$(\langle n^{-1} \rangle = \Sigma_{n,m}\, n^{-1} P_{n,m})$

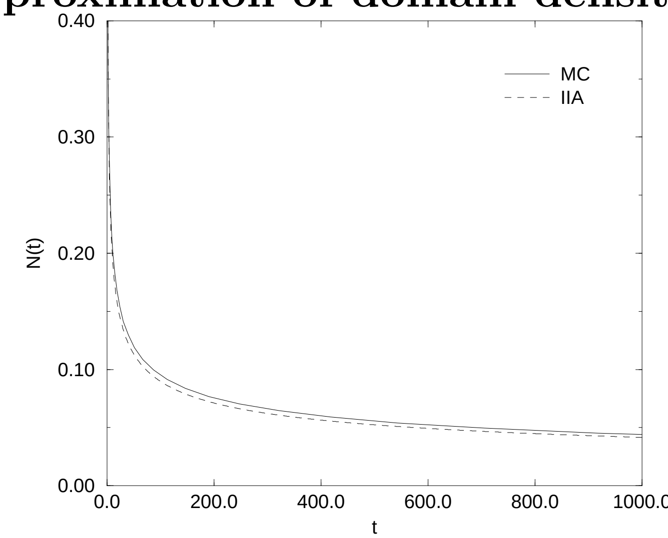
$$\frac{dP_{n,m}}{dt} = \langle n^{-1} \rangle (P_{n-1,m} - 2P_{n,m} + P_{n+1,m}) + \frac{P_1}{N^2}\left[\sum_{i+j=n}\sum_{k+l=m} i^{-1} P_{i,k} P_{j,l} - N(n^{-1} + \langle n^{-1} \rangle) P_{n,m}\right].$$

- **Gives the exact $\nu = 1/3$**

- **Good estimates for domain exponents:**

Monte Carlo: $\psi = 0.130$, $\delta = 0.705$; IIA: $\psi = 0.147$, $\delta = 645$

- **Good approximation of domain density:**



Conclusions

- **Two additional nontrivial decay exponents found.**
- **Independent Interval Approximation** provides correct qualitative behavior of domain distribution, good estimates for domain exponents.
- **Behavior is independent of dynamics/model.**

Outlook

- **How many more exponents exist?** Probably an infinite number. For example, consider the survival probability of consecutive domains.
- **Are these exponents really independent?** Yes. Exact solution for the random field Ising model gives $\psi = (3 - \sqrt{5})/8$ while $\theta = 1/2$ [D. S. Fisher et al, cond-mat/9710270].